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ABSTRACT

This resource was written to provide students with an awareness of critical issues facing the world today. In courses for college students, it can motivate their study of mathematics, teach them how to solve mathematical problems related to current global issues, provide coherence to mathematical studies through a focus on issues of human survival, and make students aware of the magnitude of current global problems and how they can help find solutions. The mathematical topics covered include the four basic mathematical operations: percents, ratio, and proportions; graphs and charts; sequences; statistics; and probability. Among the topics related to global survival are hunger, development, rapid population growth, depletion of resources, waste, water resources, the arms race, and pollution. Each chapter includes problems and exercises, plus a list of references. Solutions to selected problems, a review of selected mathematical topics, a bibliography, a list of environmental groups, a questionnaire, and possible term theme topics are appended. (MNS)

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Mathematics and Global Survival

Richard H. Schwartz

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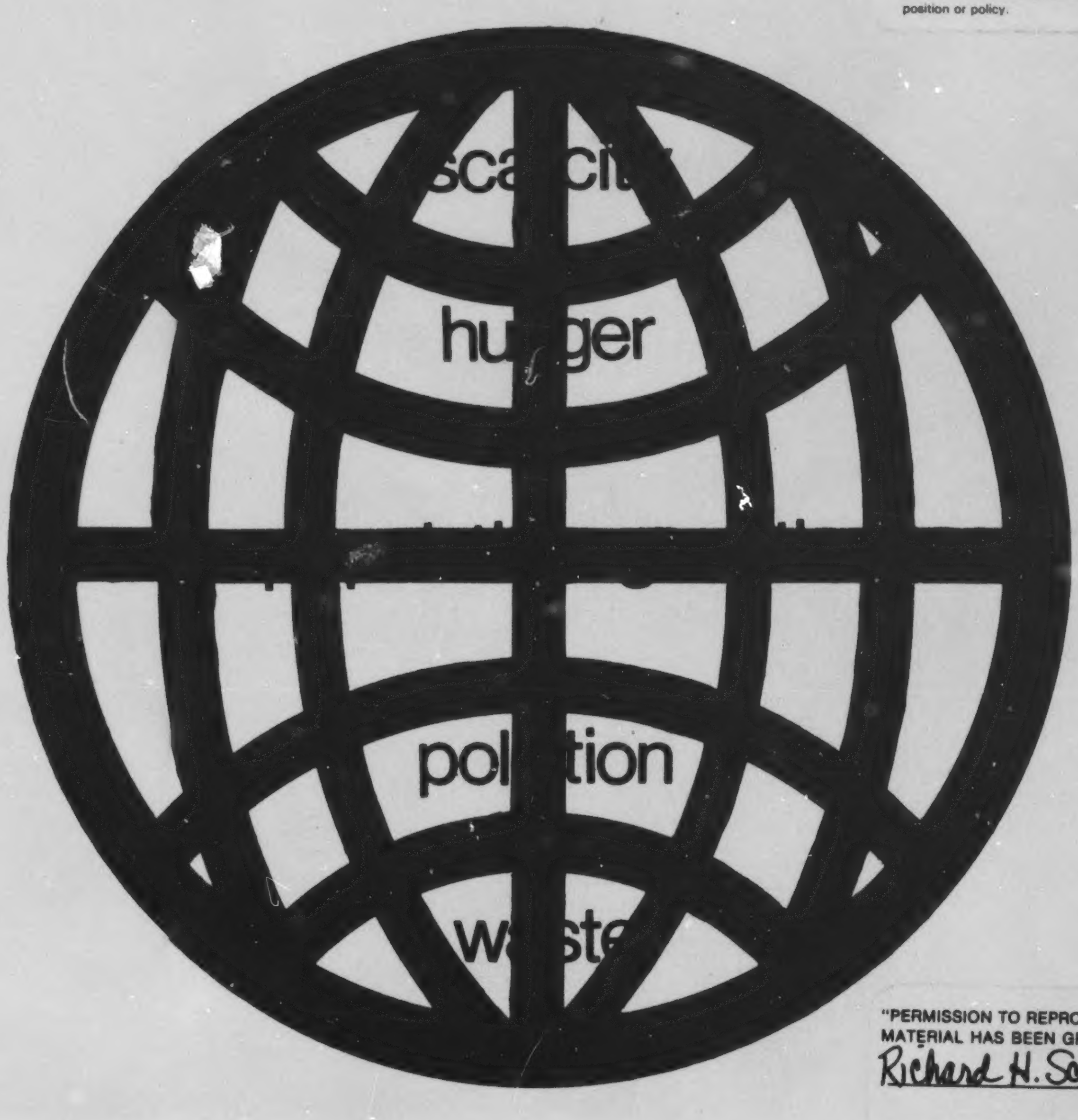

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MATHEMATICS

AND

GLOBAL SURVIVAL

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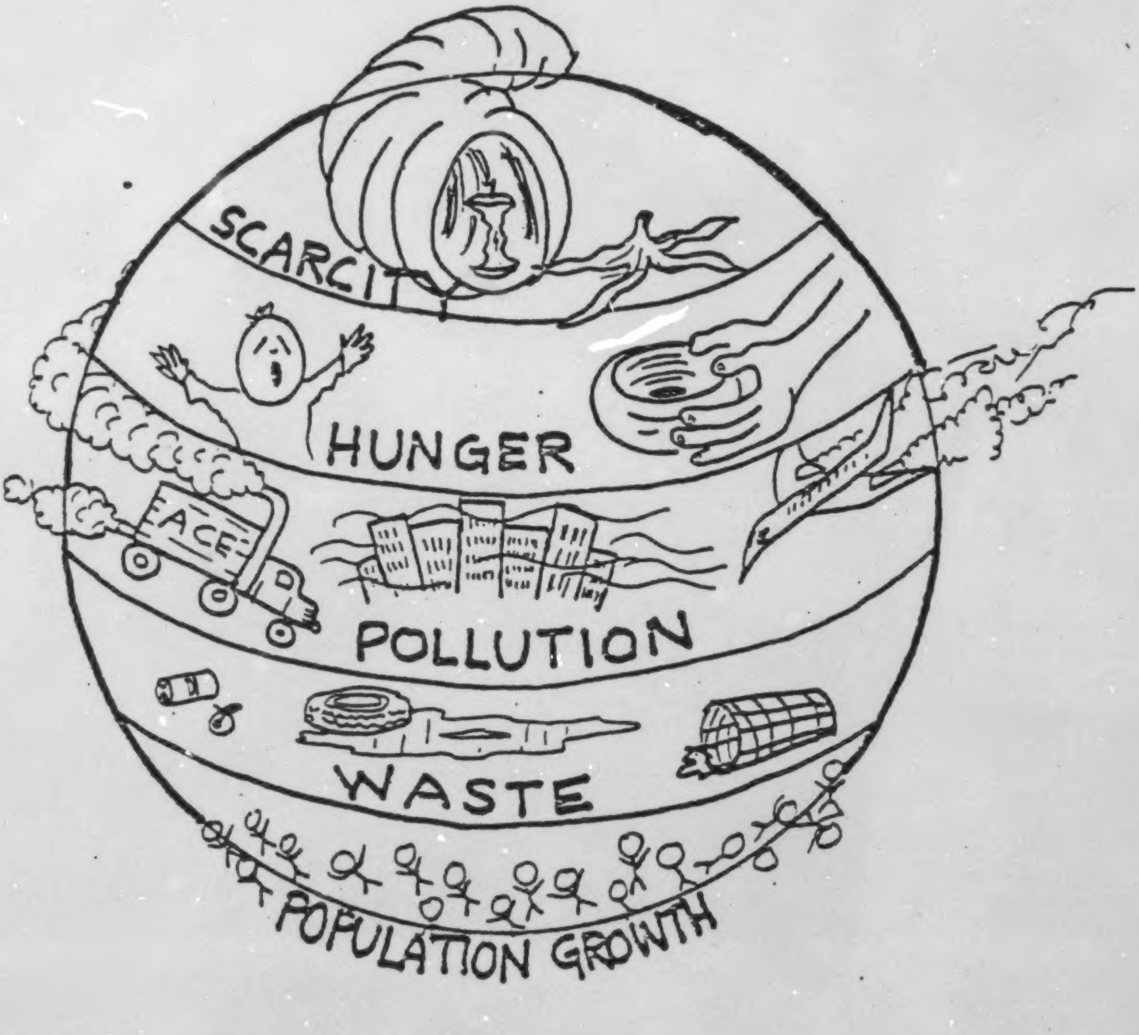
SCARCITY
HUNGER
ACE
POLLUTION
WASTE
POPULATION GROWTH

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Preface

Prince Kropotkin wrote in "A Letter to the Young," "Ask what kind of world do you want to live in? What are you good at and what do you want to work at to build that world? What do you need to know? Demand that your teachers teach you that." Mathematics and Global Survival has been written to provide students with an awareness of the critical issues facing the world today, and the tools to help them respond effectively to these problems.

This book is the outgrowth of a 3 credit course that I have taught at the College of Staten Island for the past 4 years. The course has been successful in meeting the following objectives:

1. motivating students in their study of mathematics;
2. teaching students how to solve mathematical problems related to current global issues;
3. providing coherence to mathematical studies by focusing all the mathematical topics on issues of human survival;
4. making students aware of the magnitude of current global problems and showing them how their studies can help them find solutions to these problems.

Mathematics and Global Survival can be used effectively in courses for liberal arts students. It provides coherence and relevance to such courses by building every mathematical topic and nearly every mathematical problem around human survival issues.

The material would also be valuable for ecology, technology, engineering, business, and education majors for use in an elective course. It would provide useful background material for their careers. The book is interdisciplinary (related to several fields including mathematics, demography, geography, economics, and ecology); it can be useful in showing such students the need for using information from several disciplines.

Students who need review of basic mathematical concepts and skills, and who have been unsuccessful with conventional approaches, would find the practical problems in this book helpful.

The following features make this book valuable for teaching mathematics and awareness of global issues.

1. The material is addressed directly to the student. Many sample problems are given. Review material is presented as necessary. Nothing is assumed with regard to the student's mathematical knowledge.

2. Complete detailed answers to many examples are given. Included in most answers are comments relating to the relevance of the particular problem to global survival.

3. The use of the book for effective learning is considered in several ways. There is a preliminary note to the student on how to use the book most effectively. Each step in problem solutions is shown and carefully explained. The relevance of problems is indicated. A review of basic mathematical concepts is included in the appendix. Solutions to most homework exercizes is given so that students can check their work.

4. Every problem is related in some way to an issue of global survival. Thus, they are relevant to the future lives of the student. They don't use meaningless numbers, but are built on current real-world statistics. A wide variety of problems use the information given on the 1979 World Population Data Sheet of the Population Reference Bureau, Inc., a complete, relatively current set of information on population, plus recent graphs and charts related to global problems.

A wide variety of mathematical topics are covered. These include basic mathematical operations (addition, subtraction, multiplication, and division), percents, ratios and proportions, solving word problems, graphs, circle diagrams, bar charts, histograms, arithmetic and geometric sequences, sampling, measures of central tendency (averages), variability, equations, introduction to probability, including tree-diagrams and Pascal's triangle, the normal curve, chi-square analysis of hypotheses, and correlation. Considerable practice is given in obtaining data from graphs and charts.

Many topics related to global survival are covered. These include hunger, development, rapid population growth, depletion of resources, waste, water resources, the arms race, and various types of pollution. Since these topics are treated through specific mathematical problems, many with commentary, a valuable introduction to these issues should result from studying this book.

The last 5 chapters relate previously discussed mathematics to specific issues: Smoking (chapter 16), The Arms Race (chapter 17), Staten Island (chapter 18), The Year of the Child (chapter 19), and Vegetarianism (chapter 20).

The book has enough material for two semester's work. A first course could focus on basic computational and graphical methods: chapters 1 through 10 and 16 through 20. A second course could consider statistical problems related to global issues: chapters 9 through 20.

Our world is in trouble; it faces many interrelated crises. Continued population growth at anything near current rates will have disasterous effects. The United States is the world's most wasteful nation; we must change to more efficient methods and simpler lifestyles to reduce the threats that our affluence poses to global stability. A primary cause of hunger and poverty (and population growth) is, as the National Council of Churches put it, "institutionalized injustice."

It is essential that people become aware of the many crises that the world faces and start to take action to avert disaster. It is hoped that this book will be useful as a start in this regard, and that many other concerned people will join in to add similar approaches.

Acknowledgments

I wish to thank those people who reviewed the first draft of this book and made constructive comments: College of Staten Island Professors Frank Battaglia, Joe Fishman, Sherman Heller, Ercument Ozizmir, and Mike Sormani; Prof. Martin Weissman of Essex Community College; and high school mathematics teachers, Bob Frankel, Hy Goldkrantz, and Irv Reich.

Much appreciation is owed the authors and publishers who gave permission to reprint tables and graphs. The Population Reference Bureau, Inc. was especially helpful in providing background material.

The staff of the College of Staten Island library gave much assistance in locating background information.

The reprographics staff, especially Gene Rasmussen, was very helpful in the process of getting this material into the present form.

The picture on both the cover and the front page was drawn by Louis Blois, based on ideas of Mrs. Arlene Goldberg.

Finally, much thanks to my wife, Loretta, and our children, Susan, David, and Deborah, without whose patience and encouragement, this book would not be written.

A Message to the Teacher

Do you have liberal arts students who wonder (often aloud) why they have to study mathematics? Do you have weak students who need instruction and practice at basic mathematical operations but have not succeeded with traditional approaches? Are you looking for applications of mathematics to meaningful problems that can challenge and interest your students? If your answer to any or all of these questions is yes, this book can be useful to you.

Please don't hesitate to consider using this material even though you may not feel completely at ease on issues considered, such as population growth, world hunger, pollution, and resource scarcity. You can rapidly educate yourself in these areas through the solution of the problems in this book, and the use of some of the valuable books and other sources listed in the Bibliography.

You need not pretend to be an expert on every topic at the start. You can tell your students that some areas of global issues are unfamiliar to you too, and that you are going to learn along with them. Students will appreciate such honesty, and the enthusiasm behind your newly discovered facts and concepts will more than make up for any initial lack of knowledge.

While you will need to put in more time in this course, especially the first time you teach it, than in more conventional mathematics courses, the potential rewards should make the time well spent. You will be teaching relevant material, increasing your awareness and understanding, helping yourself and others to become better citizens. The facts and concepts that you gain can assist you to play a major role in helping move our society toward changes necessary for stability and survival.

Here are some ideas that you may wish to use to enhance your course:

1. Relate the course to newspaper and magazine articles and TV and radio reports and programs. Use of current news concerning global issues shows the

students the relevance of what they are learning. Perhaps you or a student might find it valuable to organize a scrapbook of articles on topics being considered.

2. Use films, filmstrips, and slide shows to broaden coverage of issues. An annotated list of some audiovisual resources is in the Appendix.

3. Use guest speakers who have expertise on some pertinent topic.

4. Take a field trip to some local site; for example, a garbage disposal area, a sewage treatment plant, or an environmental center. Every community has resources and people willing and able to conduct tours.

5. Have students gather data from their own surveys. Use this plus data from the class to make calculations, draw graphs, compute averages, etc. Students are more highly motivated when they know that their opinion or information is part of the data being analyzed. There are examples of possible survey questions in the chapter on Sampling and in the Appendix.

6. Require students to do research and report to the class on some aspect of a global issue. This can possibly be used instead of a final exam. This helps broaden the scope of the issues covered in class, and gives students the opportunity to use the mathematics they've learned, in the analysis of a problem. Some possible topics for these reports are listed in the Appendix.

7. Have students conduct a debate using the material that they've learned. Possible topics include:

Is population growth a prime cause of hunger?

Should there be a mandatory deposit on beverage containers?

Does U. S. wastefulness help cause world hunger?

Should we pursue a nuclear energy path, a solar energy path, or some combination of the two?

Is there really an energy crisis?

Should we give more aid to underdeveloped countries? If so, under what conditions and what kind of aid?

Many of these ideas would be especially valuable if the course is interdisciplinary, combining, for example, mathematics and communications skills, or mathematics and social studies.

This book is just a beginning in a field that must continue to grow. Please send me corrections, suggestions, and ideas for additional problems, topics, and projects. You may send them to Prof. Richard Schwartz
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715 Ocean Terrace
Staten Island, New York 10301

Best of luck in using this book and in working for a better future world.

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Chapter 1 Introduction

Why a book on Mathematics and Global Survival? Simply because the threats to the survival of our planet are so great that every means that can be used to make people aware of these threats and the actions necessary to reduce them must be taken.

Here are a few of the types of issues and problems that are considered in this book:

How serious is population growth?

Is there an energy crisis?

Are we running out of natural resources?

How much impact does the style of life in the United States have on pollution and resource usage?

How many people die of hunger and its effects every day, every minute, every second?

Has the United States reached zero population growth?

How many trees have to be cut down to produce the New York Times and New York Daily News on an average Sunday?

The nature of the threats that we face are indicated by the statistics used in this book as a basis for mathematics problems. Consider the following:

1. It took all of the world's history for human population to reach 4 billion people in 1976. At current rates it will double to 8 billion people in about 40 years. Yet even at our present population, there is much hunger, pollution, poverty, etc.

2. Mexico's population is doubling every 20 years. At that pace, in a century there would be 32 Mexicans for every one today.

3. Many third world cities are growing explosively. Mexico City had 3 million people in 1950, but is projected to have over 30 million by the end of this century.

4. Between 1/3rd and 1/2 of the world's people receive inadequate nourishment. About 1/2 billion people are chronically malnourished and close to starvation.

5. Between ten and twenty million people die due to hunger and its effects annually. In some areas, 40% of the children die before the age of 5. Millions of survivors suffer blindness or are permanently retarded physically and mentally.

6. Over $400 billion is spend annually by the nations of the world for weapons. This figure exceeds the income of 2 billion of the world's poorest citizens.

7. U. S. electrical energy usage has been doubling every ten years, until recently. We waste about half of our energy.

8. It takes an average of about ten calories of fuel energy to produce one calorie of food energy in the U. S. In other countries they get as much as 50 calories of food energy per calorie of fuel energy.

The list could go on and on. Through use of this book you will become familiar with these and many more staggering statistics related to issues of human survival. It is hoped that you will be able to use this information to prevent our present apparent journey to disaster.

1.1 A Message to the Student

This book is about your world and the critical problems that it is facing. Hence it is about the conditions related to the environment and quality of life in your future, and that of children and grandchildren that you may have.

This book uses mathematical concepts and problems to help you gain an understanding of global problems and to provide you with the facts and tools that can help you work toward improving your world.

Here are some ideas that can help you to get the most out of this material:

1. Relate the material that you are studying to your life, your experiences, your other courses. Look for applications of issues raised here in newspaper and magazine articles, television and radio reports, and films.

2. Use the book effectively. Read it with a pencil and paper. Write directly in the book. Underline key ides. Restate concepts in your own words. Refer to the review of basic mathematics concepts in the back, if necessary. Solve many problems, and check your solutions with those given in the appendix.

3. Review the book before coming to class. Decide what strategy to use to learn most effectively in class. Prepare questions on concepts and problems on which you need further help.

4. Frequently ask why. Try to get the key facts about global problems. Realize though that a vast field is being considered and not all answers are known. Perhaps you will make contributions ~~toward~~ finding some of these missing answers.

5. Stay up to date. Much of the material is sequential and builds on previous material.

6. Try to see the interrelationships between the various issues considered.

7. The use of a hand held calculator is strongly recommended. It will save you much time and add to the accuracy of your solutions. It is advisable, but not essential that you get a calculator which provides logarithms; this will be helpful for chapter 8.

8. With the information and concepts from this book and related books and courses, consider taking action to make others aware of survival issues and steps necessary for their solution. There are many things that you can do that would be helpful: write letters to newspapers and public officials; organize and circulate petitions; form a campus club; speak to family, friends, neighbors, and

other students; become the local Ralph Nader, Paul Ehrlich (population activist), or Barry Commoner (energy and environmental activist) on your campus. You can make a difference.

Please send any ideas or suggestions that you may have about additional mathematical concepts and problems, or for improving the book to

Prof. R. Schwartz
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Staten Island, N.Y. 10301

Best of luck in using this book and in working for a better future world.

Chapter 2 Basic Computations

2.1 Basic mathematical operations

Much insight into global issues can be obtained by considering relatively easy problems which require only the basic mathematical operations of addition, subtraction, multiplication, and division.

Many problems can be set up by using a cancellation of units approach. For example, if we travel for 4 hours at an average speed of 50 miles per hour, the total distance traveled is

$$\frac{50 \text{ miles}}{\text{hour}} \cdot 4 \text{ hours} = 200 \text{ miles}$$

(The dot represents multiplication.) Note that the units of hours cancel and we are left with the distance in miles.

Other problems require a sum or total, in which case we add, or a difference, which requires subtraction.

Example 2.1

It has been estimated that the average American has 50 times the impact on the environment (in terms of resource consumption and pollution)[1] as does a person in India, Nigeria, or another developing country (also sometimes called underdeveloped, undeveloped, less developed, poor, or third-world country[2]). How many people in these countries would have the same impact as 220.3 million Americans?[3]

Solution

The units cancellation method provides the key to the solution. Fifty Indians (or others in developing countries) have the same impact on the environment as one American. Hence, we use a fraction (discussed in the math review in the appendix) of 50 Indians/1 American and multiply it by the U. S. 1979 population of 220.3 million Americans.

$$\frac{50 \text{ Indians}}{1 \text{ American}} \cdot 220{,}300{,}000 \text{ Americans} = 11{,}015{,}000{,}000 \text{ Indians}$$
$$= 11.0 \text{ billion Indians}$$

(rounded off)

(The review of basic mathematical concepts in the appendix considers calculations using a hand calculator, and the interpretation of large numbers such as millions and billions.)

Note the cancellation of the unit of Americans. This provides a check that we have set up the solution properly.

Consider our result, 11.0 billion Indian equivalents, with the total population of world which is now (1979) about 4.3 billion people. The U. S. population, because of affluence and much consumption has an impact (in Indian equivalents) more than that of twice the present population of the world. The American life style has much convenience and material goods, but it does have a major impact on the environment. This will be explored further in other mathematical problems.

Example 2.2

In 1978 the world spent about $400 billion for the military,[4] an amount approximately equal to the annual income of the world's poorest 2 billion people (almost half of the world's population).[5] Estimate the average income of the world's poorest 2 billion people.

Solution

We must equate (set equal) two quantities: the $400 billion spent for the military and the annual income of the world's poorest 2 billion people, which can be expressed as average income (in $ per person) times the 2 billion people. Hence,

$$\$400 \text{ billion} = \text{Income of world's poorest 2 billion people}$$

$$\$400 \text{ billion} = \text{average income } \frac{\$}{\text{person}} \cdot 2 \text{ billion people}$$

(Consideration of units in this equation shows that it has been properly set up; that is, people cancels out with person.)

To solve for the average income, we divide both sides by 2 billion people.

$$\text{average income (\$ per person)} = \frac{\$400 \text{ billion}}{2 \text{ billion people}}$$

$$\text{average income} = \underline{\$200 \text{ per person}}$$

(By contrast the average American income is about \$7,000 annually.[6])

This problem illustrates two essential factors related to global survival:

1) tremendous military expenditures at a time when so many human needs are not being met. (Chapter 17, Mathematics and the Arms Race has many problems related to military expenditures and their effects.)

2) widespread poverty among so many of the world's people.

Example 2.3

The cost of collecting the residential, commercial, and institutional wastes was about \$3.5 billion in 1969, an average of \$18 per ton.[7] About how many tons of such wastes were collected in 1969?

Solution

We apply the units cancellation method.

$$\$3.5 \text{ billion} = 18 \frac{\$}{\text{ton}} \cdot (\text{no. of tons})$$

$$\$3{,}500{,}000{,}000 = 18 \cdot \frac{\$}{\text{ton}} \cdot (\text{no. of tons})$$

$$\text{No. of tons} = \frac{\$3{,}500{,}000{,}000}{18 \text{ \$/ton}} = 194 \text{ million tons} \qquad \text{(rounded off)}$$

(round off of numbers is considered in the review of mathematical concepts in the appendix.)

Solid waste disposal is becoming a critical problem. More than half of U. S. cities are running out of space to dump their garbage. The Fresh Kills landfill in Staten Island, New York City, the world's largest garbage dump, is presently scheduled to be completed by 1985. Efforts to recycle materials and to use garbage as an energy source still need much research and planning.

Example 2.4

One proposal to alleviate gasoline shortages is to use gasohol, a mixture of 1 part ethanol to 9 parts gasoline. To obtain 1 gallon of ethanol requires an input of 41,000 B.T.U.'s (British thermal units, a common measure of heat energy) to grow the corn from which the ethanol will be distilled, and 131,000 B.T.U.'s to ferment and distill the ethanol and to produce a by-product, a grain which can be fed to animals. The output from the process is 84,000 B.T.U.'s of energy from the 1 gallon of ethanol plus 50,000 B.T.U.'s, the energy value of the feed grain by-product.[8] What is the net energy gained or lost in this process?

Solution

We must compute the energy input and the energy output and compare them.

Energy input = 41,000 B.T.U.'s + 131,000 B.T.U.'s = 172,000 B.T.U.'s

Energy output = 84,000 B.T.U.'s + 50,000 B.T.U.'s = 134,000 B.T.U.'s

Energy input - Energy output = 172,000 B.T.U's - 134,000 B.T.U.'s = 38,000 B.T.U.'s

Note that the input energy is greater than the output energy; we have a net loss in energy, or an energy deficit. Some may feel that there is also a moral deficit here as we are using corn to make fuel for our automobiles and feed for our cattle, at a time when there is widespread hunger in the world.

This problem has significance with regard to some proposed solutions to the world's problems. When people say that we can use synthetic (manufactured) food to supplement our food supply, we must ask how much energy output we are getting compared to the energy input. When it is claimed that we can send people to live in space if the earth's population gets too large, we should consider the energy necessary to accomplish this task. When it is suggested that we convert high sulphur coal to a less polluting liquid or gaseous form, we must find the energy necessary to do this, the water required in the process, and the energy lost to the fuel due to conversion. This is not to imply that gasohol, perhaps with another input to make ethanol, has no role, or that technology cannot provide valuable answers to help solve global problems.

Before trying to solve some of the following exercise problems, consider these remarks:

1. These mathematical problems are not difficult, but they are all word problems. To solve them you must go from the words to a mathematical formulation and decide whether you must add, multiply, etc. Hence, read the problems carefully and decide on the essential parts - what is given, what is required, and what conditions are involved. Use the units cancellation method when applicable.

2. Each problem has some significance relevant to issues of global survival. Do not just obtain a numerical result but try to interpret the significance of your answer with regard to global issues.

3. The problems are organized by issues such as hunger, development, population, pollution, etc. to make them more meaningful. Remember that the purpose of this book is not only to help you perform mathematical operations better, but also to give you basic understandings of critical global issues.

4. As you read newspapers and magazines and see data related to global issues, see if you can make up some examples of your own.

Exercises

Hunger and Food Related Problems

H1. The Department of Agriculture recently estimated that in one year 420,000 acres were converted into urban development (houses, stores, etc.), 160,000 acres were converted into highways and airports, 420,000 acres were converted into use for reservoirs and flood control, and 1,000,000 acres were set aside for wilderness areas, parks, recreation areas, and wildlife refuges.[9] How many acres in total were lost in that year for potential agricultural use?

H2. More than 100,000 of the world's children go blind each year for lack of vitamin A.[10] How many go blind on the average daily?

H3. There have been many estimates of the number of the world's people dying of hunger and its effects. A common estimate is that 10 to 20 million people die annually due to lack of adequate food.[11] How many people is this per (a) week, (b) day, (d) hour, (d) minute, (e) second?

H4. U. S. Census Bureau surveys recently showed that over 1000 farmers go out of business per week. How many would this be in one year? ten years?

Development Problems

(Development issues are related to the economic development of the world's countries; these issues include trade, health, education, and the arms race.)

D1. In 1974, developing countries spent the following for imports of manufactured fertilizers:[12]

nitrogenous fertilizer	$900 million
phosphate fertilizer	$380 million
potash fertilizer	$170 million.

What was the total amount spent by these countries for imported fertilizer in 1974?

D2. From mid-1970 to mid-1976, American companies sold about $25.5 billion worth of arms to foreign countries.[13] How much is this on the average per year?

D3. The combined annual budgets of the U. N. World Health Organization and the Food and Agriculture Organization would fund the U. S. Department of Defense for about 16 hours.[14] About how many times per year does the U. S. Department of Defense spend the amount of money in these two U. N. budgets?

Population Problems

P1. In 1975, it is estimated that there were 15,879,000 Americans under 5 years of age, 17,325,000 Americans between 5 and 10 years of age, and 20,409,000 Americans between 10 and 15 years of age.[15] How many Americans were there under 15 years of age in 1975?

P2. In 1967, in America the life expectancy for females was 74.2 years and for males it was 67.0 years. How much longer could a female baby expect to live than a male baby?

Waste Problems

W1. At the Second International Congress of the World Wildlife Fund held in London in November, 1970, British Naturalist Peter Scott stated that the average American accounts for more toxic wastes poured into rivers and oceans than 1,000 Asians.[16] Using this figure the toxic wastes poured into waterways by 220 million Americans (in 1979) is equivalent to that of how many Asians?

W2. In 1969, 36 billion glass container units were produced, approximately 178 containers per American.[17] Using this information, estimate the U. S. population in 1969.

W3. The average American uses about 250 cans per year.[18] How many cans would be used annually by a U. S. population of 220 million people?

W4. One ton of recycled waste paper saves about seventeen trees. In 1969, 11.5 million tons of paper were recycled.[19] How many trees were saved?

W5. The average American uses about 575 pounds of paper per year.[20] About how many tons would 220 million Americans use in one year? One ton = 2,000 pounds.

W6. In 1969, 4,340,000,000 tons of garbage of all types were generated in America.[21] How many pounds per day is this? How many pounds per person per day is this if the U. S. population was about 202 million people?

W7. It takes about 17 trees to produce 1 ton of paper.[19] How many trees must be cut down to produce the 14 million pounds of paper used by the New York Times and New York Daily News on an average Sunday?[22]

Transportation Problems

T1. There is about one vehicle in use for every two people in the U. S.[23] For a U. S. population of about 220 million, about how many vehicles are there in the U. S.?

T2. How many gallons of gas are used in one year by a car that travels 9,000 miles and gets 15 miles per gallon of gas? Repeat if a more fuel efficient car getting 30 miles per gallon was used instead.

T3. There are currently an average of nearly 50,000 deaths per year from automobile traffic. About how many deaths does this represent per day? per hour?

Water Resources and Pollution

WR1. To reduce the water shortage, the Saudi Arabian government wants to tow a 100 million-ton rectangular iceberg roughly 1 mile long, 1,000 feet wide, and 900 feet deep.[24] (1 mile = 5,280 feet)

a) What is the approximate volume of the iceberg in cubic feet?

b) What is the approximate density of the iceberg in pounds per cubic foot? (Density = weight divided by volume.)

WR2. Because of pollution, the natural life of Lake Erie was cut by some 15,000 years in just over half a century.[25] How many years, on the average, were cut off its natural life in just one year?

Energy

E1. New York State recently was considering the possibility of a 700,000 kilowatt facility on Staten Island to generate electricity for mass transit. What would be the capacity of the facility in (a) watts, (b) megawatts? (1 kilwatt = 1,000 watts; 1 megawatt = 1,000,000 watts.)

E2. If a large power plant is cooled with water from a cooling pond, about an acre of pond is required for each megawatt of power produced.[25] If a football field has an area of about 3/4 acre, how many football fields would be needed to cool the 700 megawatt power plant which was proposed for S. I.?

E3. A cooling tower (large structure which transmits waste heat to the atmosphere) for a large electric power plant is about 400 feet in diameter.[27]

This area is equal to about how many football fields (100 ft. by 300 ft.)? The area of a circle is about 3.14 times the diameter of the circle squared divided by 4, or $A = (3.14 \cdot D^2)/4$.

E4. How much will it cost to run a 1200 watt toaster for 1 year if the toaster is used an average of 4 minutes a day and electricity costs 10 cents per Kilowatt-hour? (1 kilowatt = 1,000 watts.) Note that we pay for energy which is power (in kilowatts) times time (in hours).

E5. Water flows through the cooling condensers of a large power plant at a rate of about 30,000 gallons per second.[28] If water for all uses amounts to 150 gallons per day for a person in a city, the water needed to cool a large power plant is equivalent to the needs of a city of how many people?

E6. A 1000-megawatt electric power plant using a "wet-tower" cooling method disperses about 30 cubic feet of water to the atmosphere during each second of operation.[29] Show that this would correspond to an inch of rainfall daily if it was uniformly distributed over a square mile (5,280 feet x 5,280 feet) of land.

2.2 Ratio and Proportion

A ratio is a way to compare two quantities, by dividing one quantity into the other. For example, the ratio of the population of the People's Republic of China to that of the United States in 1979 was

$$\frac{950 \text{ million Chinese}}{220.3 \text{ million Americans}} = \frac{4.31 \text{ Chinese}}{1 \text{ American}}, \text{ or } 4.31 \text{ Chinese per American}$$

If a country has 60 million people and 50,000 doctors, the ratio of people per doctor is

$$\frac{60{,}000{,}000 \text{ people}}{50{,}000 \text{ doctors}} = \frac{1{,}200 \text{ people}}{1 \text{ doctor}} \text{ or } 1{,}200 \text{ people per doctor}$$

An equation which states that two ratios are equal is called a proportion if it is a true statement. Thus, $3/4 = 6/8$ is a proportion.

In general, $\frac{a}{b} = \frac{c}{d}$ is a proportion if $a \cdot d = b \cdot c$.

Note that $\frac{5}{7} = \frac{7}{9}$ is not a true statement (hence, not a proportion) since $7 \cdot 7 = 49$, and $5 \cdot 9 = 45$ (the required products, $b \cdot c$ and $a \cdot d$, are not equal).

Further practice at using ratios and proportions is given in the review of mathematical concepts in the appendix.

In this article we will use ratios and proportions in many problems which have significance with regard to global issues.

Example 2.5

In 1977, the Gross National Product (total value of all goods and services in a country in one year) in the United States was $8640 and that for India was $150. 30 What was the ratio of the Gross National Product of the U. S. compared to that of India?

Solution

To get the required ratio, we have to divide the Gross National Product of India ($150) into the Gross National Product of the U.S. ($8640). We get

$$\frac{\$8640}{\$150} = 57.6 \text{ American dollars per Indian dollar}$$

Example 2.6

According to the 1979 World Population Data Sheet, in 1979, the world's population was about 4,321 million people and there were about 28 births per 1,000 people in the world. Approximately how many babies were born in 1979? (Note that this is written in 1979, but the past tense is used because many people will read it after 1979.)

Solution

Let x be the approximate number of babies born in 1979. Don't be confused by the use of x. In this chapter, it will always represent an unknown quantity. We have to use two ratios and equate them to give a proportion.

One ratio is 28 babies per 1,000 people, the given birth rate in 1979.

The second ratio is x babies per 4,321 million people (4,321,000,000 people). These two ratios are equal. Hence, our proportion is

$$\frac{28}{1000} = \frac{x}{4,321,000,000}$$

Cross multiplying (since we have a proportion), we get $1000 \cdot x = 28(4,321,000,000)$ Dividing both sides by 1000, to solve for the unknown x, gives

$$x = 28(4,321,000) = \underline{120,988,000 \text{ babies}}$$

Approximately 121 million babies (rounded off number) were born in 1979.

In a similar manner, we can find the approximate number of deaths in 1979. This equals

$$\frac{11 \text{ deaths}}{1,000 \text{ people}} \cdot 4,321,000,000 \text{ people} = \underline{47,531,000 \text{ people}}$$

Note that we used the cancellations of units method this time. Here, as in many cases, more than one method is correct. The death rate (11 deaths per 1,000 people) was found from the 1979 World Population Data Sheet.

We can now find the approximate population increase for 1979 by subtracting the deaths from the births.

$$\text{population increase} = 120,988,000 \text{ births} - 47,531,000 \text{ deaths}$$
$$= \underline{73,457,000 \text{ people}}$$

With 3 years of that increase, the world's population would increase an amount approximately equal to the 1979 population of the U. S.

Example 2.7

Solve example 2.1 using ratios and a proportion. Given that the average American has 50 times the impact on the environment (in terms of resource consumption and pollution) as does a person in India, how many Indians would have the same impact as 220.3 million Americans?

Solution

We are solving this problem again to show that there is sometimes more than one method that can be used to solve a problem. It is important to consider which method is best for a particular situation.

Since 50 Indians have the same impact as one American, one ratio we will use is

$$\frac{50 \text{ Indians}}{1 \text{ American}}$$

Since we wish to know how many Indians will have the same effect as 220.3 million Americans, our second ratio is

$$\frac{x \text{ Indians}}{220{,}300{,}000 \text{ Americans}}$$

We equate the two ratios to form the proportion

$$\frac{50 \text{ Indians}}{1 \text{ American}} = \frac{x \text{ Indians}}{220{,}300{,}000 \text{ Americans}}$$

Cross multiplying

$$50 \cdot 220{,}300{,}000 = 1 \cdot (x) = x$$

$x = \underline{11{,}015{,}000{,}000 \text{ Indians}}$, as in Example 2.1

Exercises

Hunger and Food Related Problems

H1. In the U. S. we eat the equivalent of 1900 pounds of grain per person annually (mostly in the form of meat products) while in the underdeveloped world they eat about 400 pounds per year.[31] What is the ratio of the grain consumed by an American compared to that consumed by a person in the underdeveloped world?[32]

H2. The world is feeding 4 billion people and 15 billion livestock. What is the ratio of livestock to people being fed?

H3. The meat and dairy-centered diet in the U. S. requires about 3.5 acres per person.[33] The diet of an average person in the underdeveloped world requires about 1/5th of an acre.[33] What is the ratio of acres required by a person in the U. S. compared to that required by a person in an underdeveloped country? (Note: At the present time (1978), there is about one available acre per person in the world[33] and population is growing rapidly.)

H4. The amount of grains that feeds about 220 million Americans could feed about 1.6 billion people if consumed directly (not through meat products).[34]

Calculate the ratio of the people that could be fed directly by grains with those that are fed by our diet.

H5. It requires 7 or 8 lbs. of grains to produce one lb. of edible meat, in a feedlot.[35] How many lbs. of grains are necessary to add 500 lb. of edible beef to a cow in a feedlot? Give a range of values (highest and lowest).

H6. The amount of grain needed to produce one pound of beef in a feedlot is 7 or 8 pounds.[35] Two or 3 pounds of grain are necessary to produce one pound of poultry.[36] What is the ratio of grain necessary to produce one pound of beef to that needed to produce one pound of poultry? Use average values for the grain needed in each case.

H7. About 3500 gallons of water per day are required to provide the food for each American (including irrigation water, water to produce pesticides and fertilizers, and water for processing), while 475 gallons of water per day provide the necessary food for an East Indian.[37] Compute the ratio of water required for an American compared to that required for an East Indian.

H8. Using the information in the preceding problem, compute the ratio of the water required for an average American's diet to that required for a strict vegetarian who requires only about 350 gallons of water per day.[37]

H9. The farm animals in the United States produce about ten times the amount of solid waste (manure) that the U. S. people do.[38] If the animals produce 2 billion tons of waste per year,[38] what is the annual production for the U. S. population?

H10. Using the data in the chart below,[39] compute ratios for (a) calories per capita per day, and (b) protein-grams per capita per day for people in developed (wealthy) countries compared to people in developing countries.

1969 - 1971	Energy Calories per capita per day	Protein Grams per capita per day
Developed countries	3,100	96
Developing countries	2,200	58

Development Problems

D.1 During the decade of the sixties, the developed nations increased their income by $640 per capita, while the undeveloped nations increased their income by $40 per capita. Find the ratio of these increases.

D2. The U. S. Agency for International Development (Statistics and Reports Division) has reported the following figures:

	(1) Less Developed Countries	(2) Developed Countries	(3) (2) ÷ (1)
Income	$245	$3,510	
Percent literate	40	97	
Births per 1,000 population	40	18	
Life expectancy (years)	52	71	
Infant deaths per 1,000 live births	110	22	
Annual KWH output per person	240	5360	

Complete column (3) by computing the ratios for each item.

D3. Compute ratios for the following factors that compare health and education resources available in developed and "developing" countries in 1974.[40]

	(1) Developed Country (average)	(2) Developing Country (average)	(3) (1) ÷ (2)
Public Education Expenditures ($ per capita)	$229	$13	
Teachers (per 1,000 school-age children)	37	16	
Adult literacy (percents)	98	49	
Public Health Expenditures ($ per capita)	144	4	
Physicians (Per 10,000 population)	19	3	
Life expectancy at birth (years)	72	56	

Population Problems

P 1. Japan has over 110 million people in a land area a little less than that of California which has about 20 million people. Assuming that the land areas are roughly equal, estimate the ratio of the population density of Japan to that of California.

P 2. In 1975, Europe had an average of 97 inhabitants per square kilometer while Latin America had 16 inhabitants per square kilometer. Compute the ratio of the population density of Europe to that of Latin America.

P 3. About one pregnancy in every three ends in abortion. If there were 120 million births in 1976, how many births would there have been if there were no abortions? How many additional births does this represent?

2.3 Percent

Many problems related to human survival involve the computation of percents. For example, we may wish to find

1. the percent increase in the world's population during a given period,

2. the percent increase in resource usage or solid waste disposal during a given time,

3. the percent of the world's people who are poor, hungry, or lack proper educational or medical facilities.

In this section, we define percent and show applications to a variety of problems related to global survival.

Basically, percent is a ratio: it means parts per 100. For example, 20 percent (often written as 20%) means 20 parts per 100 parts. It may be written as 20/100 or in decimal form as 0.20.

More practice in using percents is given in the mathematical review in the appendix.

Example 2.8

In 1977, the world's developing countries contained 72 percent of the world's population.[41] If the world's population in 1977 was 4.1 billion people, how many of these people were in developing countries?

Solution

We first rewrite the question in a more direct way.

No. of people in developing countries = 72% of 4.1 billion people.

Let x = the number of people in developing countries. Again, we use x to denote an unknown quantity.

$$x = \frac{72}{100} \cdot 4.1 \text{ billion} \qquad (\% \text{ means per } 100)$$

$$x = (0.72)(4.1 \text{ billion}) \qquad (72/100 = 0.72\text{; considered in math review in appendix})$$

$$x = 2.95 \text{ billion}$$

$$\text{or} \quad x = 2{,}950{,}000{,}000 \text{ people in developing countries}$$

We could have obtained the same result by using a proportion:

$$\frac{72 \text{ people}}{100 \text{ people}} = \frac{x \text{ people}}{4.1 \text{ billion people}}$$

This result is very significant in that many people in the developing world are poor, illiterate, lack proper nutrition, housing, jobs, and health facilities. And the number of people in these countries is growing both in absolute and relative terms.

Example 2.9

In 1973, Pakistan spent \$405 million out of a total budget of \$607 million on military expenditures.[42] What percent of its budget was spent on the military?

Solution

Again, we can restate the question in a more concise form:

\$405 million is what percent of 607 million?

$$\$405 \text{ million} = \frac{x}{100} \cdot \$607 \text{ million}$$

Note the conversion of "what percent" to x/100 since "what" implies an unknown, or x, and percent means parts per 100, or 1/100.

The millions on both sides of the equation can be cancelled. Multiplying both sides of the equation by 100 and dividing by 607, we get

$$x = \frac{405 \cdot 100}{607} = \underline{66.7\%} \quad \text{(66.7\% of the budget is spent on the military)}$$

Here too, we could have obtained the result by means of a proportion.

$$\frac{\$405 \text{ million}}{\$607 \text{ million}} = \frac{x\%}{100\%} \quad \text{(\$607 million, the entire budget corresponds to 100\%)}$$

At a time when developing countries such as Pakistan have tremendous problems with regard to poverty, hunger, illiteracy, lack of jobs, inadequate water supply, education, and housing, Pakistan has been using the major portion of its budget for military expenditures.

Example 2.10

The annual consumption of meat in the United States has increased from about 50 pounds per person in 1945 to 115 pounds per person in 1975.[43] What percent increase does this represent?

Solution

Since we require a percent increase, first we compute the increase in pounds. This is 115 - 50, or 65 pounds per person.

Now, the basic question is 65 pounds per person is what percent of 50 pounds per person? (Always use the original value (here 50 pounds per person) as the reference value, when computing a percent.)

$$65 = \frac{x}{100} \cdot 50 \quad \text{(65 lb per person is what percent of 50 lb per person?)}$$

(Again, "what percent?" is replaced by x/100.)

To solve for the unknown, x, we multiply both sides of the equation by 100 and divide both sides by 50.

$$x = \frac{65 \cdot 100}{50} = 130\%$$

Note this tremendous increase in just 30 years. Our diet rich in meat and dairy products is a major factor with regard to global survival. Our diet requires about 3.5 acres per person while the diet of a person in an undeveloped country only requires about 1/5th of an acre. While much of the world suffers from malnutrition due to lack of food, our excessive consumption of food causes many diet-related diseases such as heart attack, stroke, and various forms of cancer.

Example 2.11

From 1946 to 1968, the percent increase in consumption of mercury per capita (per person) was 2150 percent.[44] For every pound of mercury used in 1946, how many were used in 1968?

Solution

Let x equal the number of pounds of mercury used in 1968 for every pound used in 1946. Recall that whenever we are looking for an unknown quantity, we call it x.

The increase in mercury usage from 1946 to 1968 is (x - 1) pounds, the amount used in 1968 (x pounds) minus the amount used in 1946 (1 pound).

The fractional increase is (x - 1) pounds/1 pound. To get a percent, we multiply by 100, since percent means per 100.

Hence,

$$\% \text{ increase} = \left(\frac{x-1}{1}\right) \cdot 100$$

$$2150\% = \frac{x-1}{1} \cdot 100 \quad \text{(the given \% increase is 2150\%)}$$

Dividing both sides by 100, we get

$$\frac{x-1}{1} = 21.5$$

$$x - 1 = 21.5 \qquad \left(\frac{x-1}{1} = x - 1\right)$$

Adding 1 to both sides of the equation, we get

$$x = \underline{22.5 \text{ pounds}}$$

For every one pound of mercury used in 1946, 22.5 pounds of mercury were used in 1968.

The tremendous increase in mercury and other chemicals added to the environment since World War II, largely due to changes in agricultural and industrial production, and transportation, have been major factors behind the environmental crisis.

Note that we could have used a proportion here also to get our result.

Since percent means "per 100", 2150% can be written as the ratio 2150/100.

Hence, $\frac{2150}{100} = \frac{(x-1)\text{ lb}}{1\text{ lb}} \qquad x - 1 = \frac{2150}{100} = 21.5 \text{ pounds}$

and x = 21.5 + 1 = 22.5 pounds, as before.

Exercises

Hunger and Food Related Problems

H1. If 10 million to 20 million people die annually from hunger and its effects,[11] and about 50 million people die annually from all causes, what percent of all deaths (give a range) are due to hunger and its effects?

H2. About 40 percent of 220 million Americans are considered to be overweight. How many Americans are overweight?

H3. About 4/5th of all grain produced in the U. S. is fed to animals.[45] What percent does this represent?

H4. Estimates are that about 500 million of the world's people face severe malnutrition.[46] What percentage of the world's approximately 4.2 billion people does this represent? (1978 figures)

H5. It has been estimated that from 10 to 20 million people die yearly due to starvation or its effects.[47] What percent of the world's people die from this cause each year if the world's population is about 4.3 billion? Give a range for your answer (that is, a maximum and a minimum value).

H6. From 1/3 to 1/2 of the world's people are malnourished (not enough protein) or undernourished.[48] What percentages do these numbers represent?

H7. It has been estimated that at least 40 million people in the United States lack an adequate diet.[49] If the U. S. population is about 220 million, what percent of our people lack a proper diet?

H8. According to agronomist Georg Borgstrom, of about 1 billion children in the world, 650 million won't reach adulthood due largely to direct or indirect effects of malnutrition.[50] What percent does this represent?

Development Problems

D1. In 1975, the U. S. and Canada provided 98.7 million metric tons (1 metric ton = 2200 lb.) of the world's total grain exports of 149.1 million metric tons. What percent did these two countries provide?

D2. In Africa out of a total population of about 345 million in 1972, 239 million were seriously poor, of which 134 million were destitute.[52] What percent of the population were (a) seriously poor and (b) destitute?

D3. Repeat exercise D2 for Asia where (excluding the People's Republic of China) out of 1,196 million people in 1972, 853 million were seriously poor, of which 499 million were destitute, and Latin America where out of 274 million people, 118 million were seriously poor of which 73 million were destitute.[53]

D4. According to Georg Borgstrom's paper, "War on Hunger: The Need for a Strategy,"[54] out of about 3.8 billion people in 1972,

a. 2.5 billion people were hungry or malnourished;

b. 2700 million people were short of water;

c. 750 million adults were illiterate;

d. 900 million people carry life-sapping hookworms;

e. over 500 million suffer from vision-depriving trachoma;

f. 250 to 300 million suffer from malaria;

g. over 2000 million people receive water which is both inadequate in supply and unacceptable in quality;

h. 500 million people yearly fall victim to water borne disease

Using these statistics, compute the percent of people in each category. What do your results indicate about the world today?

D5. The total number of persons in the developing countries unable to read or write increased from 701 million in 1960 to 756 million in 1970.[55] What was the percent increase in people unable to read or write? Does this mean that the percent of illiterate people in these countries increased?

D6. Comparison of U. S. and the rest of the world. The following is an excerpt from an issue of the Eli Lilly Company News Letter (approximately 1970):

> "If all the people in the world could be reduced proportionately into a theoretical town of 1,000 people, the picture would look something like this: In this town there would be 60 Americans, with the remainder of the world represented by 940 persons. This is the proportions of the population of the United States to the population of the world, 60 to 940. The 60 Americans would have half the income of the entire town with the other 940 dividing the other half. About 350 of these would be practicing Communists, and 370 others would be under Communistic domination. White people would total 303, with 697 being non-white. The 60 Americans would have 15 times as many possessions per person as all the rest of the world. The Americans would produce 60% of the town's food supply although they eat 72% above the maximum food requirements. They would either eat most of what they grow or store it for their own future use at enormous cost. Since most of the 940 non-Americans in the town would be hungry most of the time, it would create ill

feelings toward the Americans, who would appear to be enormously rich and fed to the point of sheer disbelief by the great majority of the townspeople. The Americans would also have a disproportionate share of the electric power, fuel, steel, and general equipment. Of the 940 non-Americans, 200 would have malaria, cholera, typhus, and malnutrition. None of the Americans would get these diseases or probably ever be worried about them."

Using these statistics, answer the following:

a. What percent of the world's people were Americans?

b. What is the ratio of the income of an average American to an average non-American?

c. What percent of the world's people were living in Communist or Communist-dominated countries?

d. What percent of non-Americans would have malaria, cholera, typhus, and malnutrition?

e. What was the ratio of non-white people to white people?

f. Comment on your results.

D7. In 1971, for every 100 deaths in Egypt, 30 occurred to children under 1 year of age and 23 occurred to children between 1 and 5 years of age.[56] What percent of Egyptian deaths occurred to children under 5 years of age?

D8. In 1960, 95% of Chilean mothers breast fed their babies until 13 months of age. By 1968, this percent had decreased to 5%.[57] What was the decrease in the percent of Chilean mothers who breast fed for 13 months?

Population Problems

P1. The population of the world doubled from 2 billion people in 1930 to 4 billion in 1976.[58] What was the percent increase in population from 1930 to 1976?

P2. The population of Lima, Peru grew from 175,000 to 3 million in about 50 years.[59] What percent increase does this represent?

Resource Problems

R1. From 1946 to 1968, the percent increase in consumption of plastic per capita (per person) increased by 1024 percent.[60] For every pound of plastics used in 1946, how many were used in 1968?

R2. Repeat problem 1 for the following items:

Item	Percent increase in consumption per capita (1946 to 1968)[61]
synthetic fibers	1792
nitrogen fertilizer	534
synthetic organic chemicals	495
aluminum	317
detergents	300
electrical power	276

Water Resources Problems

WR1. In 1969, according to one study, 95 million Americans drank water not meeting federal standards or of unknown quality.[62] What percent of the 1969 population of about 202 million people was this?

Waste Problems

W1. In 1920, the average American created 2.75 pounds of solid waste per day; in 1970 the figure was 5 pounds per day.[63] What was the percent increase?

Pollution Problems

PO1. How many pounds of sulfur are there in one ton of coal having a sulfur content of 4 percent?

PO2. When Thor Hyerdahl recently sailed around the world, he found that on 43 out of 57 days he was floating in oil, garbage, and various forms of debris. What percent of the days was he floating in the various types of pollution?

Notes

1. Pollution: Problems, Projects, and Mathematics Exercizes, a publication of the Wisconsin Dept. of Public Instruction, Bulletin No. 1082, State of Wisconsin, Dept of Public Instruction, Wisconsin Hall, 126 Langdon Street, Madison, Wisconsin 53702.

2. Some feel that these countries should be called "never-to-be-developed" countries since so much of the world's resources are being used up by the wealthier countries that there won't be enough left for the development of these poorer countries.

3. Unless otherwise indicated, all populations are taken from the 1979 World Population Data Sheet of the Population Reference Bureau, 1337 Connecticut Ave., N.W., Washington, D. C. 20036 (see pages 3-2 to 3-5.)

4. Ruth L. Sivard, World Military and Social Expenditures, 1978, WMSE Publications, Box 1003, Leesburg, Virginia 22075, 1978, p. 5.

5. Arthur Simon, Bread For the World, New York, Paulist Press, 1975, p. 123. In 1974, the world spent $250 billion, which exceeded the total income of the poorer half of the world, with billions of dollars to spare.

6. Statistical Abstracts of the U. S., 1978, Bureau of the Census, Department of Commerce, p. 442, states that per capita U. S. income in 1977 was $7,086.

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Chapter 3 Population Calculations

As indicated, recent world population growth has been very rapid and has had great impact on many problems. A graph of world population vs. time is discussed in the next chapter. In this section we shall consider several factors related to population growth.

We will utilize the 1979 World Population Data Sheet of the Population Reference Bureau, Inc. (see next 4 pages). This data sheet is updated annually and provides a valuable, current source of information on world population, since its figures are based on annual projections from censuses. Information is given for all United Nations members, and geopolitical entities with populations of 200,000 or more. Note that the data sheet is divided into continents (Africa, Asia, etc.), further divided into regions (Northern Africa, Western Africa, etc.) and finally subdivided into countries (Algeria, Egypt, Libya, etc.). There are separate listings for the USSR, which spans two continents, and Oceania, which includes Australia, New Zealand, and several small Pacific Islands. There is also a separate row for the more developed countries (U. S., U.S.S.R., Sweden, etc.) and also for the less developed countries (Angola, India, Nigeria, Egypt, etc.).

We will now discuss each column on the data sheet. Calculations that can be made based on the given data will be shown. More examples based on values in the Data Sheet will be given in other sections of the book. It will be very valuable for you to refer to appropriate places on the Data Sheet as we discuss it.

(1) Population Estimate, Mid-1979 (millions)

These figures in column 1 (columns are numbered from the left) are based on a population total from a very recent census or on the most recent official country or UN estimate, usually for mid-1975. These estimates were updated using population growth rates for each country during the 1970's. Note that these figures are estimates; it is extremely difficult to get exact values.

WORLD POPULATION DATA SHEET of the POPULATION REFERENCE BUREAU, INC., 1979

Region or Country[1]	(1) Population Estimate Mid-1979 (millions)[2]	(2) Birth Rate[3]	(3) Death Rate[3]	(4) Rate of Natural Increase (annual, percent)[4]	(5) Number of Years to Double Population[5]	(6) Population Projection to 2000 (millions)[6]	(7) Infant Mortality Rate[7]	(8) Population under 15 Years (percent)	(9) Population over 64 Years (percent)	(10) Life Expectancy at Birth (years)[8]	(11) Urban Population (percent)[9]	(12) Physical Quality of Life Index (PQLI)[10]	(13) Per Capita Gross National Product (US$)[11]
WORLD	4,321	28	11	1.7	41	6,168	95	36	6	60	39	65	1,800
MORE DEVELOPED[1]	1,173	16	9	0.7	99	1,349	28	25	10	71	68	92	5,210
LESS DEVELOPED[1]	3,148	33	12	2.1	33	4,819	107	40	4	56	28	55	490
AFRICA	457	46	17	2.9	24	831	143	44	3	47	25	33	450
NORTHERN AFRICA	106	43	14	2.9	24	185	127	44	3	53	41	41	700
Algeria	19.1	48	14	3.3	21	38.4	142	48	4	54	52	41	1,110
Egypt	40.6	38	12	2.6	27	63.5	108	41	3	55	44	44	310
Libya	2.8	47	13	3.5	20	5.7	130	49	4	53	60	43	6,680
Morocco	19.4	46	14	3.2	22	35.6	133	46	2	53	38	40	570
Sudan	17.9	46	18	2.7	26	32.3	141	44	3	46	20	34	300
Tunisia	6.4	36	10	2.6	27	9.8	125	45	4	55	50	46	860
WESTERN AFRICA	137	49	19	3.0	23	262	158	46	3	44	20	26	370
Benin	3.5	49	19	3.0	23	6.6	149	46	4	43	14	27	200
Cape Verde	0.3	28	9	1.8	38	0.4	105	41	3	58	20	45	140
Gambia	0.6	48	23	2.4	29	1.0	165	42	2	41	16	21	200
Ghana	11.3	48	17	3.1	22	21.2	115	47	4	49	31	39	380
Guinea	4.9	46	21	2.5	28	8.2	175	43	3	41	20	20	230
Guinea-Bissau	0.6	41	23	1.8	38	0.8	208	37	4	39	23	14	160
Ivory Coast	7.7	48	18	2.9	24	14.0	154	45	2	46	32	29	710
Liberia	1.8	50	17	3.2	22	3.5	148	47	2	48	29	26	430
Mali	6.5	49	22	2.7	26	11.6	190	48	1	40	17	14	110
Mauritania	1.6	50	22	2.8	25	2.9	187	42	6	40	23	18	270
Niger	5.1	51	22	2.9	24	9.6	200	46	3	40	9	14	160
Nigeria	74.6	50	18	3.2	22	148.8	157	47	2	45	18	27	420
Senegal	5.5	48	22	2.6	27	9.7	159	44	3	42	32	21	420
Sierra Leone	3.7	46	19	2.6	27	6.5	136	43	3	46	15	28	200
Togo	2.5	49	19	3.0	23	4.8	163	46	3	43	15	24	300
Upper Volta	6.7	48	22	2.6	27	11.8	182	44	3	40	4	16	110
EASTERN AFRICA	130	48	18	2.9	24	243	138	43	3	46	12	32	220
Burundi	4.4	47	20	2.7	26	7.8	140	44	2	42	5	25	130
Comoros	0.4	40	18	2.2	32	0.5	160	44	3	46	10	43	180
Djibouti	0.1	48	24	2.5	28	0.2	—	—	—	—	70	—	580
Ethiopia	31.8	50	25	2.5	28	55.4	162	45	3	40	12	22	110
Kenya	15.4	51	12	3.8	18	32.8	119	49	2	53	10	39	270
Madagascar	8.5	45	19	2.6	27	15.1	102	43	3	46	16	41	210
Malawi	5.9	51	19	3.2	22	11.8	142	44	4	43	9	30	140
Mauritius	0.9	27	8	1.9	36	1.3	46	37	4	63	44	72	760
Mozambique	10.2	45	19	2.6	27	18.1	140	43	3	44	6	27	150
Reunion	0.5	26	6	1.9	36	0.7	41	40	3	65	51	72	2,680
Rhodesia	7.2	47	14	3.4	20	14.6	121	47	3	52	19	46	500
Rwanda	4.9	50	19	3.0	23	9.6	127	46	3	43	4	27	130
Seychelles	0.1	26	8	1.8	38	0.1	43	41	6	65	37	73	650
Somalia	3.5	48	20	2.8	25	6.3	177	45	2	41	28	19	110
Tanzania, United Rep. of	17.0	46	16	3.0	23	33.1	125	45	3	47	7	30	200
Uganda	13.2	45	14	3.0	23	25.4	136	45	3	50	7	40	260
Zambia	5.6	49	17	3.2	22	10.8	144	46	3	46	36	38	450
MIDDLE AFRICA	52	45	20	2.6	27	88	165	43	3	43	28	27	250
Angola	6.9	48	23	2.4	29	11.9	203	43	3	39	18	16	330
Cameroon, United Rep. of	8.3	42	19	2.3	30	13.1	137	41	4	44	29	27	340
Central African Empire	2.4	43	19	2.3	30	3.8	190	41	4	44	36	18	250
Chad	4.4	44	21	2.3	30	7.6	165	42	4	41	14	18	130
Congo, People's Rep. of	1.5	45	19	2.6	27	2,5	180	43	3	44	40	27	500
Equatorial Guinea	0.3	42	19	2.3	30	0,5	165	41	4	44	45	28	340
Gabon	0.5	33	22	1.1	63	0.8	178	32	6	41	32	21	3,730
Sao Tome and Principe	0.1	38	12	2.6	27	0,1	75	—	—	—	24	—	420
Zaire	28,0	46	19	2,8	25	47,2	160	44	3	44	30	32	130

WORLD POPULATION DATA SHEET of the POPULATION REFERENCE BUREAU, INC., 1979

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
SOUTHERN AFRICA	32	39	11	2.8	25	53	96	42	4	56	44	52	1.250
Botswana	0.7	51	17	3.4	20	1.3	97	48	6	56	12	51	440
Lesotho	1.3	40	16	2.4	29	2.1	111	39	4	50	3	48	290
Namibia	1.0	44	15	2.9	25	1.7	142	44	3	49	32	38	1,000
South Africa	28.2	38	10	2.8	25	47.4	92	42	4	57	48	53	1,340
Swaziland	0.5	47	19	2.8	25	0.9	168	48	3	44	8	33	530
ASIA	2.498	29	11	1.8	38	3.588	99	38	4	57	27	58	650
SOUTHWEST ASIA	96	40	13	2.7	26	165	117	43	4	56	46	52	1.800
Bahrain	0.3	44	9	3.6	19	0.5	78	44	3	63	78	61	3.790
Cyprus	0.6	19	9	1.0	69	0.7	27	26	10	71	42	85	1.740
Gaza	0.4	53	18	3.5	20	0.7	125	51	5	52	87	—	—
Iraq	12.9	47	13	3.4	20	24.8	104	48	3	53	66	45	1,530
Israel	3.8	26	7	1.9	38	5.5	20	33	8	73	86	89	2,920
Jordan	3.0	46	13	3.3	21	5.8	97	47	3	53	42	47	710
Kuwait	1.3	42	5	3.7	19	3.1	39	44	2	69	56	75	12,700
Lebanon	3.1	36	11	2.4	29	4.9	59	42	5	63	60	79	—
Oman	0.9	49	19	3.0	23	1.7	142	45	3	47	5	—	2,520
Qatar	0.2	44	14	3.0	23	0.4	138	45	3	48	69	31	11,670
Saudi Arabia	8.1	49	18	3.0	23	15.6	150	45	3	45	21	29	3,980
Syria	8.4	44	13	3.2	22	16.3	114	49	4	57	48	52	900
Turkey	44.3	35	10	2.5	28	69.6	119	40	4	59	45	56	1,110
United Arab Emirates	0.9	44	14	3.0	23	1.9	138	34	3	48	65	34	14,420
Yemen	5.8	48	25	2.3	30	10.0	160	46	3	45	9	27	390
Yemen, Democratic	1.9	48	21	2.7	26	3.5	155	48	3	45	33	32	320
MIDDLE SOUTH ASIA	912	37	15	2.2	32	1.450	129	42	3	50	21	40	230
Afghanistan	18.3	50	27	2.3	30	31.2	226	44	3	39	15	17	190
Bangladesh	87.1	47	18	2.9	24	155.1	153	43	3	46	9	32	90
Bhutan	1.3	43	21	2.2	32	2.0	147	42	3	43	3	—	80
India	660.9	34	15	1.9	36	1.010.5	122	41	3	50	21	41	150
Iran	36.3	44	14	3.0	23	63.8	104	45	3	57	47	52	2,180
Maldives	0.1	50	23	2.7	26	0.2	—	44	2	—	11	—	80
Nepal	13.7	44	21	2.3	30	22.1	152	42	3	44	4	27	110
Pakistan	79.9	44	14	3.0	23	145.1	136	46	3	51	26	36	190
Sri Lanka	14.5	26	8	1.8	38	19.9	51	39	4	68	22	82	200
SOUTHEAST ASIA	346	36	13	2.2	32	539	97	43	3	50	21	56	340
Brunei	0.2	28	4	2.4	29	0.4	23	43	3	66	64	—	6.620
Burma	32.9	39	14	2.4	29	52.7	140	40	4	51	22	50	140
Dem. Kampuchea (Cambodia)	8.9	43	17	2.6	27	13.7	150	45	3	45	12	38	—
East Timor	0.7	44	21	2.3	30	1.1	175	42	3	40	11	—	—
Indonesia	140.9	35	14	2.0	35	210.5	91	44	2	50	18	48	300
Lao People's Dem. Rep.	3.7	44	20	2.4	29	5.8	175	42	3	40	15	26	90
Malaysia	13.3	31	6	2.5	28	20.1	44	41	4	66	27	73	930
Philippines	46.2	34	10	2.4	29	77.9	80	43	3	58	32	71	450
Singapore	2.4	17	5	1.1	63	3.1	12	31	4	71	100	86	2.890
Thailand	46.2	32	9	2.3	30	75.9	68	43	3	61	13	71	410
Vietnam	50.1	41	18	2.3	30	77.8	115	41	4	48	22	52	170
EAST ASIA	1,145	20	8	1.2	58	1,434	51	34	5	63	31	74	980
China, People's Rep. of[12]	950	20	8	1.2	58	1.193	56	35	5	64	24	71	410
Hong Kong	4.7	18	5	1.2	58	6.2	14	29	6	72	92	87	2,590
Japan	115.9	15	6	0.9	77	128.9	9	24	8	75	76	96	5,640
Korea, Dem. People's Rep. of	17.5	33	8	2.4	29	27.3	70	42	4	61	43	75	700
Korea, Republic of	37.6	24	7	1.7	41	51.2	38	38	4	65	48	82	810
Macao	0.3	30	8	2.2	32	0.3	78	38	5	—	97	69	940
Mongolia	1.6	37	8	2.9	24	2.7	70	44	3	61	47	78	830
Taiwan[13]	17.3	24	5	1.9	36	24.5	25	35	4	71	77	87	1.180
NORTH AMERICA	244	15	9	0.7	99	289	14	24	11	73	74	95	8,620
Canada	23.7	16	7	0.8	87	29.0	14	26	8	73	76	95	8,450
United States	220.3	15	9	0.6	116	260.4	14	24	11	73	74	95	8,640
	Pop. 1979 (1)	Birth Rate (2)	Death Rate (3)	Nat. Inc. (4)	Doub. Time (5)	Pop. 2000 (6)	Inf. Mort. (7)	Pop. < 15 (8)	Pop. > 64 (9)	Life Expec. (9)	Urb. Pop. (11)	PQLI (12)	GNP p.c. (13)

Region or Country[1]	(1) Population Estimate Mid-1979 (millions)[2]	(2) Birth Rate[3]	(3) Death Rate[3]	(4) Rate of Natural Increase (annual, percent)[4]	(5) Number of Years to Double Population[5]	(6) Population Projection to 2000 (millions)[6]	(7) Infant Mortality Rate[7]	(8) Population under 15 Years (percent)	(9) Population over 64 Years (percent)	(10) Life Expectancy at Birth (years)[8]	(11) Urban Population (percent)[9]	(12) Physical Quality of Life Index (PQLI)[8]	(13) Per Capita Gross National Product (US$)[10]
LATIN AMERICA	352	35	8	2.7	26	597	86	42	4	62	61	71	1,240
MIDDLE AMERICA	89	41	7	3.4	20	171	72	46	3	63	58	72	1,030
Costa Rica	2.2	30	5	2.5	28	3.4	34	44	4	68	41	85	1,240
El Salvador	4.5	42	8	3.4	20	8.5	60	46	3	59	39	64	570
Guatemala	6.8	43	10	3.3	21	12.3	76	45	3	58	36	54	790
Honduras	3.1	47	12	3.5	20	6.2	103	48	3	55	31	53	450
Mexico	67.7	41	7	3.4	20	132.3	70	46	3	65	64	75	1,110
Nicaragua	2.5	47	12	3.4	20	4.8	122	48	3	53	49	55	830
Panama	1.9	29	6	2.3	30	2.9	47	43	4	68	50	79	1,220
CARIBBEAN	30	29	8	2.0	35	43	70	40	5	63	49	73	1,100
Bahamas	0.2	25	5	2.0	35	0.3	25	44	3	69	58	84	3,450
Barbados	0.3	19	9	0.9	77	0.3	28	31	10	70	44	90	1,760
Cuba	9.9	20	6	1.4	50	12.7	23	37	6	71	64	85	900
Dominica	0.1	22	7	1.5	46	0.1	24	—	—	58	27	—	370
Dominican Republic	5.3	37	9	2.8	25	8.5	96	48	3	58	47	64	840
Grenada	0.1	27	6	2.2	32	0.1	24	—	—	63	15	78	450
Guadeloupe	0.3	17	6	1.1	63	0.4	35	37	6	68	48	81	2,380
Haiti	5.7	42	16	2.6	27	9.9	130	41	4	49	23	40	230
Jamaica	2.2	30	7	2.3	30	2.9	20	46	6	69	41	85	1,150
Martinique	0.3	14	6	0.9	77	0.4*	32	37	6	68	50	83	3,340
Netherland Antilles	0.3	28	7	2.2	32	0.4	25	38	5	62	48	82	1,750
Puerto Rico	3.5	23	6	1.7	41	4.6	18	35	7	72	62	90	2,460
Trinidad and Tobago	1.1	25	7	1.8	38	1.4	30	38	4	67	49	85	2,380
TROPICAL SOUTH AMERICA	193	36	9	2.8	25	332	99	42	3	60	60	67	1,280
Bolivia	5.2	44	16	2.8	25	8.9	157	42	4	48	34	39	540
Brazil	118.7	36	8	2.8	25	205.2	109	41	3	61	61	66	1,390
Colombia	26.1	31	9	2.2	32	42.3	98	43	3	59	60	72	710
Ecuador	8.0	42	10	3.1	22	15.1	70	44	4	60	42	69	770
Guyana	0.8	27	7	2.0	35	1.2	50	44	4	68	40	84	560
Paraguay	3.0	39	8	3.1	22	5.3	64	45	3	62	40	75	760
Peru	17.3	40	12	2.8	25	29.5	80	44	3	55	62	65	830
Suriname	0.4	30	7	3.5	20	0.7	30	50	4	66	66	83	1,500
Venezuela	13.5	36	6	3.0	23	24.1	49	43	3	65	75	79	2,820
TEMPERATE SOUTH AMERICA	41	23	9	1.4	50	51	57	30	7	67	80	83	1,560
Argentina	26.7	22	9	1.3	53	32.9	59	28	8	68	80	85	1,730
Chile	11.0	24	8	1.6	43	15.1	56	35	5	63	79	79	1,170
Uruguay	2.9	21	10	1.1	63	3.5	46	27	10	69	83	86	1,450
EUROPE	483	14	10	0.4	173	521	20	24	12	72	68	93	4,910
NORTHERN EUROPE	82	12	11	0.1	693	85	13	23	14	73	73	95	5,380
Denmark	5.1	12	10	0.2	347	5.3	9	22	14	74	67	96	8,050
Finland	4.8	14	9	0.4	173	5.1	12	22	11	72	59	94	6,150
Iceland	0.2	18	6	1.2	58	0.3	10	30	9	76	87	97	4,570
Ireland	3.3	21	10	1.1	63	4.0	16	31	11	71	52	93	2,880
Norway	4.1	12	10	0.3	231	4.4	10	24	14	75	44	97	8,540
Sweden	8.3	12	11	0.1	693	8.6	8	21	15	75	83	97	9,250
United Kingdom	55.8	12	12	0.0	—	56.6	14	23	14	72	76	94	4,430
WESTERN EUROPE	153	12	11	0.1	693	158	13	22	14	72	80	95	7,700
Austria	7.5	11	12	-0.1	—	7.6	17	23	15	72	52	93	6,140
Belgium	9.8	12	11	0.1	693	10.7	12	22	14	71	95	93	7,580
France	53.4	14	10	0.4	173	57.2	11	24	14	73	71	95	7,290
Germany, Fed. Rep. of	61.2	10	12	-0.2	—	59.7	16	21	15	72	92	94	8,160
Luxembourg	0.4	11	12	0.0	—	0.4	11	20	13	71	68	92	7,150
Netherlands	14.0	12	8	0.5	139	15.5	10	25	11	75	77	96	7,160
Switzerland	6.3	12	9	0.3	231	6.4	11	22	13	73	55	95	9,970

WORLD POPULATION DATA SHEET of the POPULATION REFERENCE BUREAU, INC., 1979

	(1) Pop. 1979	(2) Birth Rate	(3) Death Rate	(4) Nat. Inc.	(5) Doub. Time	(6) Pop. 2000	(7) Inf. Mort.	(8) Pop. < 15	(9) Pop. > 64	(10) Life Expec.	(11) Urb. Pop.	(12) PQLI	(13) GNP p.c.
EASTERN EUROPE	109	18	11	0.7	99	121.7	24	23	11	71	58	92	3,140
Bulgaria	8.9	16	11	0.5	139	9.6	24	22	11	71	59	92	2,590
Czechoslovakia	15.2	19	12	0.7	99	17.2	20	23	12	70	67	93	4,090
German Democratic Rep	16.7	13	13	0.0	–	16.6	13	21	16	72	76	94	4,940
Hungary	10.7	17	12	0.4	173	11.3	26	21	13	70	51	90	2,570
Poland	35.4	19	9	1.0	69	41.2	24	24	10	71	56	92	3,150
Romania	22.1	20	10	1.0	69	25.8	31	25	10	70	48	91	1,580
SOUTHERN EUROPE	139	16	9	0.7	99	158	24	26	11	71	60	89	2,930
Albania	2.6	30	8	2.2	32	3.8	87	40	4	68	34	76	610
Greece	9.5	15	9	0.6	116	10.6	20	24	12	72	65	89	2,810
Italy	56.9	13	10	0.4	173	61.2	18	24	12	72	67	92	3,450
Malta	0.3	18	9	0.9	77	0.4	14	25	9	71	94	87	1,680
Portugal	10.0	18	10	0.9	77	11.8	39	28	10	69	29	80	1,850
Spain	37.6	18	8	1.0	69	44.0	16	28	10	72	70	92	3,190
Yugoslavia	22.2	18	8	0.9	77	25.6	35	26	9	68	39	84	1,960
USSR	264	18	10	0.8	87	311	30	26	9	70	62	91	3,010
OCEANIA	23	22	9	1.3	53	30	43	31	8	69	71	86	5,560
Australia	14.4	16	8	0.8	87	17.9	14	27	9	72	86	94	7,340
Fiji	0.6	27	4	2.3	30	0.8	41	41	2	70	37	79	1,220
New Zealand	3.2	18	8	1.0	69	4.0	14	29	9	72	81	94	4,370
Papua-New Guinea	3.1	45	16	2.9	24	5.1	106	44	4	49	13	42	480
Samoa, Western	0.2	37	7	3.0	23	0.2	40	50	3	63	21	84	350
Solomon Islands	0.2	41	10	3.1	22	0.4	52	44	3	–	9	–	250

General Notes

World Population Data Sheets of various years should not be used as a time series. Because every attempt is made to use the most recent and most accurate information, data sources vary and changes in numbers and rates from year to year may reflect improved source material, revised data, or a later base year for computation, rather than yearly changes.

Sources of data: Aside from population estimates and projections (see footnotes 2 and 6), number of years to double population (footnote 5), the Physical Quality of Life Index (footnote 10), and per capita Gross National Product (footnote 11), most of the data in this *Data Sheet* were reported in the following United Nations (UN) publications: *Demographic Yearbook*, 1976 and 1977 editions; *Population and Vital Statistics Report, Data Available as of 1 January 1979*, Statistical Papers, Series A, Vol. XXXI, No. 1; "Demographic Estimates and Projections for the World, Regions, and Countries as Assessed in 1978," provisional report prepared by the UN Population Division, January 25, 1979. Some of the figures shown were from other sources, including: U.S. Bureau of the Census, *World Population: 1977*, October 1978; unpublished materials and reports of the Census Bureau's Foreign Demographic Analysis Division; Dorothy L. Nortman and Ellen Hofstatter, *Population and Family Planning Programs*, 9th edition. A Population Council Fact Book, 1978; World Bank, "Population Projections: 1975-2000," prepared by K. C. Zachariah and My Th Vu of the World Bank's Population and Human Resources Division, April 1978; various analytical studies; and official data reported by individual countries. The source for any figure shown on the *Data Sheet* may be obtained by writing or telephoning the Population Reference Bureau.

Figures for the regions and the world: Population totals (columns 1 and 6) take into account small areas not listed on the *Data Sheet*. These totals may also not equal the sums of their parts because of independent rounding. All other regional and world figures are weighted averages for countries for which data are available.

Dashes indicate that data are unavailable.

Footnotes

[1] The *Data Sheet* lists all geopolitical entities with a population larger than 200,000 and all members of the United Nations (UN) regardless of population size. Classification of "more developed" and "less developed" regions follows the latest (1978) practice of the UN. The "more developed" regions comprise all of Europe, North America (Canada and the United States), Australia and New Zealand, Japan, and the USSR. The UN now includes Cyprus, Israel, and Turkey in the Southern Europe subregion and considers them "more developed." In this *Data Sheet*, Cyprus, Israel, and Turkey are [illegible] in the overall count on "more developed" regions, but are retained in their original subregion of Southwest Asia in the detailed regional breakdown. "Less developed" regions comprise all other countries for which data are available. Argentina, Chile, and Uruguay (Temperate South America), formerly classified as "more developed," are now included in the overall "less developed" category.

[2] Based on a population total from a very recent census or on the most recent official country or UN estimate; for almost all countries the estimate was for mid-1977. Each estimate was updated by the Population Reference Bureau to mid-1979 by applying the same rate of growth as indicated by population change during part or all of the period since 1970.

[3] Annual number of births or deaths per 1,000 population. For the more developed countries with complete or nearly complete registration of births and deaths, nearly all the rates shown pertain to 1976 or 1977. For most less developed countries with incomplete registration, the rates refer to the 1975-80 period. These rates were used in the medium variant estimates and projections as assessed by the UN in 1978 (UN, Demographic Estimates and Projections ...). These figures should be considered as rough approximations only. For some countries, estimates were obtained from published reports of the U.S. Bureau of the Census, the World Bank, the World Fertility Survey, government statistical offices, and special studies.

[4] Birth rate minus the death rate. Since the rates were based on unrounded birth and death rates, some rates do not exactly equal the difference between the birth and death rates shown because of rounding.

[5] Based on the rate of natural increase shown and assuming no change in the rate.

[6] Except for the United States and Taiwan, estimated by the Population Reference Bureau by applying the percentage increase in the population 1979-2000 implied by the UN medium variant projections to the population total as estimated for mid-1979. For the United States, the figure shown is the Series II projection given in the U.S. Bureau of the Census, *Projections of the Population of the United States: 1977 to 2050*, P-25, No. 704, July 1977. For Taiwan, the figure shown is from the World Bank projections, April 1978.

[7] Annual number of deaths to infants under one year of age per 1,000 live births. For countries with complete or nearly complete registration of births and deaths, nearly all rates pertain to 1976 or 1977. For many less developed countries with incomplete registration, rates are the latest available estimates generally obtained from the UN sources noted above; from World Health Organization, *World Health Statistics Report*, Vol. 29, No. 11, 1976; or from U.S. Bureau of the Census, *World Population: 1975*, and *World Population: 1977*.

[8] Average number of years a newborn child could be expected to live if current mortality conditions were to continue throughout his or her lifetime. For the more developed countries with reliable mortality data, nearly all estimates shown pertain to part or all of the 1970-77 period. For most of the less developed countries with unreliable mortality data, estimates refer to some part of the period since 1970 and were prepared by one of the following organizations: the UN Population Division; the International Demographic Statistics Branch of the Population Division of the U.S. Bureau of the Census; or the World Bank's Population and Human Resources Division. Estimates of life expectancy for most of the less developed countries should be considered as rough approximations only.

[9] The percentage of the total population living in areas defined as urban by each country.

[10] The Physical Quality of Life Index (PQLI) was developed by the Overseas Development Council, Washington, D.C., in response to the need for a non-income measure that summarizes the many aspects of human well-being. It combines three indicators—infant mortality, life expectancy at age one, and literacy—into a single composite index. The index runs from 0 to 100, 0 being the lowest level of well-being and 100 being the highest. For a more detailed description of the nature and uses of the PQLI, contact the Overseas Development Council, 1717 Massachusetts Avenue NW, Washington, D.C. 20036.

[11] Data refer to 1977 except for 14 of the smaller countries for which the data refer to 1976. All data for the individual countries are from World Bank, *World Bank Atlas: Population, Per Capita Product, and Growth Rates*, 1978.

[12] Although most observers agree that the population of the People's Republic of China is now over 900 million and that birth and death rates have dropped significantly in recent years, there is a substantial range of opinion regarding China's current total population and levels of fertility and mortality. After evaluating estimates from a variety of sources, the Population Reference Bureau considers the estimates shown in this *Data Sheet* to be reasonable.

[13] The UN does not show figures for Taiwan. These figures were estimated separately on the basis of official Taiwan data.

April 1979

Observe that the world population for mid-1979 was about 4,321 million people and that the most populous country was China with a population of about 950 million. (Please find these numbers in column 1.) Hence, China has (950/4321) • 100 = 22.0 percent of the world's people.

(2) Birth Rate

The second column gives the number of births per 1,000 population for the various countries and regions. This figure is obtained by dividing the number of births in a given year by the estimated population of the country in the middle of the year. For example, if a country had 25 million people in the middle of 1979 and 600,000 births that year, its birth rate (BR) can be found from the proportion

$$\boxed{\frac{\text{Total birth rate}}{\text{Total population}} = \frac{\text{Birth rate (per 1,000 people)}}{\text{1,000 people}}} \qquad \text{Eq. 3.1}$$

$$\frac{\text{600,000 births}}{\text{25,000,000 people}} = \frac{\text{BR Births}}{\text{1,000 people}}$$

Multiplying both sides by 1,000 to solve for the birth rate, BR,

$$\text{BR} = \frac{600{,}000}{25{,}000{,}000} \times 1{,}000 = 24, \text{ or 24 births per 1000 people}$$

This figure is considered to be a crude birth rate since it does not consider age, ratio of males to females, and other data relevant to the composition of the population. The birth rate may be high because the average number of children per couple is high, or because a large percentage of the population is in the reproductive age period.

Note that the birth rate for the world is 28 and that it varies from a high of 53 for Gaza to a low of 10 for the Federal Republic of Germany. Hence the range (highest - lowest value) is 53 - 10 or 43 births per thousand. (The range is discussed in more detail in Chapter 11.)

(3) Death Rate

The death rate is computed in the same way as the birth rate; it represents the number of deaths per 1,000 population. It also is based on the mid year population and is considered the crude death rate because it doesn't take into account factors related to the composition of the population. Thus death rates, like birth rates, require more information before their real significance can be appreciated. For example, a high death rate may not mean poor conditions in a country, but may reflect an older population.

The main factor behind the recent rapid growth of population has not been an increase in the birth rate (it has actually declined), but a sharp drop in death rates (especially in developing countries) due to better medical techniques.

(4) Rate of Natural Increase (annual percent)

This rate is found by subtracting the death rate from the birth rate and converting to a percent. For example, for the world, (see columns 2 and 3 of the Data Sheet):

birth rate	=	28	(births per 1,000 people)
death rate	=	11	(deaths per 1,000 people)
difference	=	17	(births - deaths, per 1,000 people)

To convert to a percent we have to find the net increase in population (births - deaths) per 100 people.

$$\frac{17 \text{ people}}{1000 \text{ people}} = \frac{x \text{ people}}{100 \text{ people}} \qquad x = \frac{17(100)}{1000} = 1.7$$

The rate of natural increase in 1.7 percent.

A short-cut equation would be,

$$\boxed{\text{Rate of natural increase} = \frac{\text{birth rate} - \text{death rate}}{10}} \qquad \text{Eq. 3.2}$$

For example, for the United States:

birth rate = 15 (births per 1,000 people)

death rate = 9 (deaths per 1,000 people)

$$\text{Rate of natural increase} = \frac{15 - 9}{10} = \frac{6}{10} = 0.6 \text{ percent}$$

Note that this figure does not take migration (immigration - emigration (outflow of people)) into account; hence the term natural increase is used by demographers (population analysts).

(5) Number of years to double population

This represents another measure of population increase and one that is easier to understand. It indicates the number of years for a population to be twice what it is in 1979, based on its current rate of population growth.

This figure can be approximated from the following equation, which will be verified later (using logarithms, in chapter 8):

$$\boxed{\text{doubling time} = \frac{70}{\text{Growth rate (\%)}}} \qquad \text{Eq. 3.3}$$

This equation is based on the fact that people, like dollars left in a bank, grow according to a compound interest law. More details will be given in Chapter 5 on sequences. Thus a population growing at 2% per year, such as Guyana, will not double in 50 years (100/2), but in about 35 years (70/2).

It will be instructive to check some of the values in column 5 against the values in column 4 for some countries. For Honduras, for example, the rate of natural increase (col. 4) is 3.5%.

$$\text{doubling time} = \frac{70}{3.5\%} = 20 \text{ years}$$

This checks the value in column 5 for Honduras. In some cases, there may be slight discrepancies due to round off.

The growth rates used in the data sheets are the natural growth rates; they do not take migration into account. Hence they will not be true doubling times, if there is significant migration (immigration - emigration). The United States, for example, has a much smaller doubling time than the 116 years indicated on the Data Sheet, due to the large immigration (legal and illegal) to the U. S.

The very small doubling times indicated on the data sheet are very serious. At current rates, the world's population will double in 41 years. Consider this in relation to the many current global problems such as hunger, poverty, unemployment, the rich-poor gap, lack of medical facilities, resource depletion, etc., with the 1979 populations.

Consider a country like Mexico whose population is projected to double in 20 years. At that rate, in a century Mexico will have 5 doublings which means 32 Mexicans for every one there today. Clearly, such growth cannot continue. But when and under what conditions will it stabilize?

Also consider that a doubling of population means the need for a country to double its food supply, schools, roads, houses, medical facilities, etc. just to maintain present standards, which are inadequate today for most of the world's people. Can countries, especially poorer ones, double their services in the 20 or 30 years in which their populations are doubling?

More information and problems related to doubling times are presented in the chapter on sequences (chapter 5).

(6) Population Projection to 2000 (millions)

These figures are based (except for the U. S.) on the population estimates for mid-1979 and on United Nations estimates for median (middle of range of projections - see chapter 10) growth rates from 1978 to 2000 (see note 6 on the data sheet). Note that the projected population for the world in the year 2000 is 6168 million people. This represents a percent increase of

$$\frac{\text{population projection for 2000} - (\text{1979 population estimate})}{\text{1979 population estimate}} \times 100$$

$$\frac{(6{,}168 - 4{,}321)\text{ million people}}{4{,}321\text{ million people}} \times 100 = \frac{1{,}847}{4{,}321} \times 100 = 42.7\%$$

in just 22 years.

Also note the tremendous changes projected for certain countries; for example, Mexico's population, at current rates of change, will increase from 67.7 million people to 133.2 million people by the end of the century, an increase of nearly 100 percent.

(7) Infant Mortality Rate

Column 7 gives the annual number of deaths to infants under one year of age per 1,000 live births. It is the decrease in this figure in recent years due to better medical services that has been a major factor in recent explosive population growth. Yet many countries still have very high rates of infant mortality. In the world, 95 of every 1000 babies (9.5%) don't survive the first year; many countries have rates well over 100 infant deaths per 1000 live births.

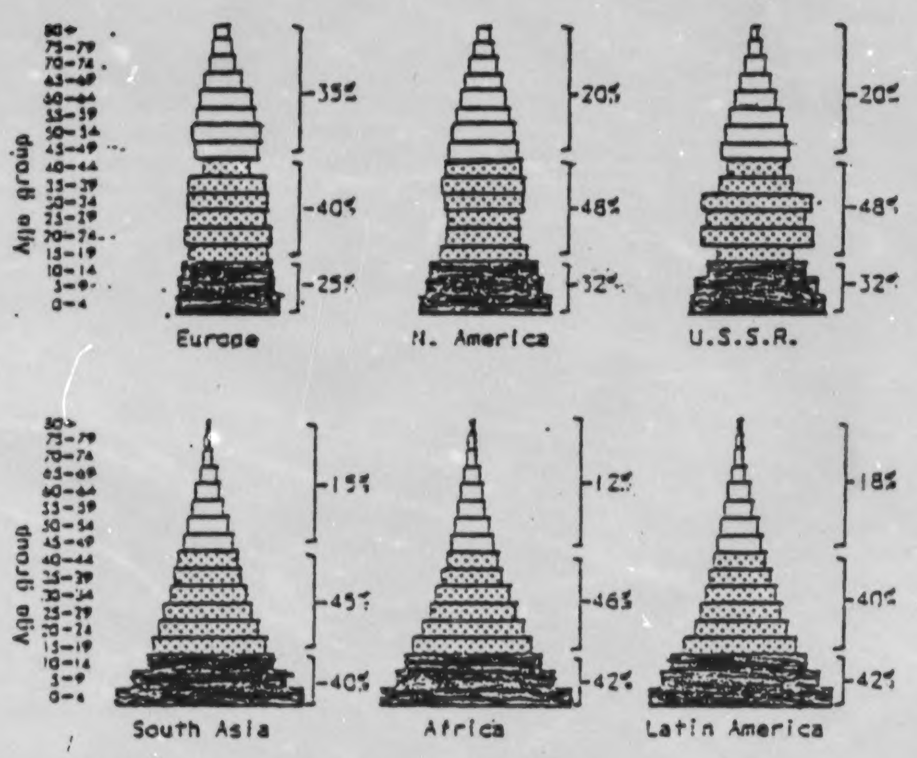
(8) Population under 15 years (percent)

Many population experts have pointed out the significant fact that about 40% of the people in the less developed countries are under 15 years of age. This means that there will be many people entering the reproductive ages in the near future which will give a momentum (forward thrust) to population growth that will be very hard to slow down. Note that Belgium which has reached a stabilization of population has only 22 percent of its population under 15 years of age, whereas Mexico which is growing very rapidly (doubling time = 20 years) has 46% of its population under 15 years of age.

The age structure of a country is extremely important with regard to future population growth. Figure 3.1 gives several age structures for human populations.

Note the very high percentages of the populations under 15 years of age in

AGE STRUCTURE IN HUMAN POPULATIONS IN MAJOR AREAS OF THE WORLD



Black = prereproductive years
Stippled = reproductive years
Open = postreproductive years

From Population Bulletin, October, 1965. With permission of the Population Reference Bureau, Inc.

Figure 3.1

the underdeveloped regions: South Asia (40%), Africa (42%), and Latin America (42%). Note, by contrast, that only 25% of the population of Europe is below 15 years of age; many European countries have reached or are very close to reaching a stabilization of population. (There will be a further discussion of these age structure pyramids in article 4.4.)

(9) Population over 64 years (percent)

Column 9 gives the percent of the population in the various countries and regions over 64 years of age. Note that 6% of the world's people in 1979 were over 64 years old. As expected this figure is higher (10) for more developed countries and lower (4) for less developed countries.

The dependency ratio is defined as the sum of the percents of people under 15 years of age and over 64 years of age divided by the percent of people between 15 and 64 years of age. In equation form,

$$\text{dependency ratio} = \frac{\%\text{ under 15} + \%\text{ over 64}}{\%\text{ between 15 and 64}}$$

For the United States, for example,

% under 15 = 24%

% over 64 = 11%

sum = 35%

% between 15 and 64 = 100% - 35% = 65%

dependency ratio = 35%/65% = 0.538

By contrast, Mexico has a dependency ratio just under one (check this). High dependency ratios mean that development is difficult, because a large part of the population is in non-productive or relatively non-productive ages.

(10) Life Expectancy at Birth (years)

A baby born in the world in 1979 can expect to live, on the average, 60 years (see column 10, for the world). This figure varies from 76 years for Iceland to 39 years for Afghanistan, Angola, and Guinea-Bissau (a range of 76 - 39 = 37 years), an indication of the wide discrepancies in conditions in various countries of the world.

Note that figures for life expectancy at birth are actually based on the average age at which people died in the previous year. In using these figures we should consider recent changes with regard to increases in poisons in our air, water, and food, threats of nuclear power plant accidents and nuclear conflict, as well as improvements in medical knowledge and technology.

(11) Urban Population (percent)

In recent years there have been great movements of people from rural areas into cities. Population growth in many cities has been phenomenal. For example, Mexico City's population is projected to increase from around 3 million in 1950 to 8 million in 1970, to close to 32 million by 2000.[1] Unfortunately, cities are unable to cope with these tremendous increases in terms of providing adequate jobs, housing, transportation, medical and educational facilities, etc.

(12) Physical Quality of Life Index (PQLI)

This column has been added to the World Population Data Sheet for the first time in 1979. The concept of a Physical Quality of Life Index was introduced by the Overseas Development Council in 1977 to meet the need for a measurement of the level of progress achieved by any country in meeting basic human needs.[2] It has received wide acceptance by international groups.

Per capita gross national product (GNP) (discussed in the next section) does not adqeuately reflect the conditions of the people in a country. For example, Sri Lanka has a per capita GNP of only $200 but has fairly good life expectancy and infant mortality rates. Some countries like Iran and Saudi Arabia have high per capita GNP's with very poor conditions for most of their people. Hence, the PQLI was developed to give a non-income measurement that would indicate many aspects of well being.

The Physical Quality of Life Index is a single composite index based on three indicators - infant mortality, life expectancy at age one, and literacy. These three indicators are used because they are readily available and they measure actual conditions in a country, rather than inputs.

Each of the three components is indexed on a scale from 0 (the most unfavorable performance in 1950) to 100 (the best performance expected in the year 2000). For life expectancy at age one, the most unfavorable performance in 1950 (38 years in Guinea-Bissau) is rated at 0, and the most favorable figure expected by the year 2000 (77 years) is rated as 100. Similarly, for infant mortality, the poorest performance in 1950 (229 per thousand in Gabon) is rated as 0 and the best performance expected by the year 2000 (7 per thousand) is rated as 100. Since literacy figures are generally given as percentages, they are already on a 0 to 100 scale. The PQLI is obtained by averaging (adding the 3 values and dividing by 3) the three indices (life expectancy, infant mortality, and literacy). Note that the low and high values of the indices are unchanging; hence, future improvement (or deterioration) in social conditions will show up as increases or decreases in the PQLI.

(13) Per Capita Gross National Product (US$)

This column gives the per capita (per person) gross national product, GNP, (the total value of goods and services in a country) in U. S. dollars.

These figures dramatically illustrate the tremendous gap between wealthy and poor nations. Note the many values over $5,000, and the many values below $500. This gap has been growing in recent years. Lester Brown has written in World Without Borders, "In effect, our world today is in reality two worlds, one rich, one poor; one literate, one largely illiterate; one industrial and urban, one agrarian and rural; one overfed and overweight, one hungry and malnourished; one affluent and consumption-oriented, one poverty-stricken and survival-oriented."[2]

Generally low values of gross national product are associated with high birth rates, rapid population growth, hunger, poverty, and illiteracy. The income gap has been growing in recent years. Recently, the poorer countries have been attempting to form alliances to demand a fairer share of the world's resources. A peaceful, equitable resolution of problems resulting from the present great disparities of the world's wealth will be one of the great challenges facing the world in the future.

It is important to note that poverty and hunger are prime causes of population growth. When many children die young, parents tend to have more children to insure that some will survive. Note that, although improvements have been made, nearly 10% of the world's children don't survive their 1st year, and in some poorer countries, 30 to 40% of the children die before they reach their fifth birthday. In the poor countries, children are an economic asset as they help the family even when very young and they provide the only form of economic security that the parents can count on when the parents become too old to work. The relationship between population growth and income is shown graphically in article 4.3.

Exercises

Use the 1979 Population Reference Bureau Data Sheet to answer the following, where appropriate.

1. What percent of the world's population are in

   (a) the United States (b) China (c) India (d) Asia (e) Brazil

2. What is the ratio of the population of China to the population of

   (a) the United States (b) Brazil (c) Mexico

3. Compute dependency ratios for (a) the U. S. (b) Sweden (c) Mexico (d) the Netherlands (e) Peoples Republic of China (f) India (g) Nigeria

   Do you observe any correlation when comparing these dependency ratios with per capita GNP values for these countries?

4. The number of years to double a population is approximately equal to 70/(annual rate of natural increase, in percent). Check the numbers given on the chart for (a) the U. S. (b) Sweden (c) Mexico (d) India (e) the World. Note that there may be some slight discrepancies due to round off.

5. What percent increases in population are projected from 1979 to 2000 for (a) the U. S. (b) Mexico (c) Sweden (d) India (e) the world?

6. What percent of children don't survive their first year in (a) the world (b) the U. S. (c) Africa (d) Mexico (e) Sweden (f) India?

7. What is the ratio of life expectancies of the country that has the greatest life expectancy for its people to the country that has the smallest?

8. What is the ratio for gross national products per capita, of the country that has the largest GNP per capita, to the country that has the smallest?

9. Divide the continents (consider the USSR and Oceania as continents for this problem) into 2 related categories (relatively high and relatively low) with regard to each of the following: (a) birth rate (b) death rate (c) growth rate (d) doubling time (e) infant mortality (f) percent of population under 15 years of age (g) percent of population over 64 years of age (h) life expectancy at birth (i) urban population--percent (j) PQLI (k) per capita gross national product. Comment on your findings.

10. Compute the range (highest value - lowest value) for the following variables: (a) birth rate (b) death rate (c) rate of natural increase (annual percent) (d) infant mortality (e) population under 15 years (%) (f) population over 64 years (%) (g) life expectancy at birth (years) (h) urban population (%) (i) PQLI (i) per capita GNP (US$)

11. Compute the ratios for each of the variables on the 1979 World Population Data Sheet (for example, birth rate, doubling time, infant mortality rate) for the

less developed countries (row 3) compared to the more developed countries.

12. In 1940 the world's crude birth rate was 36 births per thousand and the death rate was 30 deaths per thousand. Find the percent increase in population. Note that for the entire world, migration need not be considered.

    Repeat for the year 1971 when BR was 34 births per thousand and DR was 14 deaths per thousand.

    Comment on these results.

13. (a) By about how many people did the world's population increase in 1979? Hint: use values for the mid-1979 population estimate and the rate of natural increase (%).

    (b) By about how many people did the population increase per day? per hour? per minute? per second?

    (c) At this rate of population increase, how long would it take for the world's population to increase by an amount equal to the 1979 population of the U. S.?

    (d) About how long does it take for the world's population to increase enough to equal

    (1) the 600,000 Americans killed in all U. S. wars?

    (2) the 35,000,000 people killed in all wars fought in the past 500 years?

    (3) the 75,000,000 people killed due to the Bubonic Plague (1347-1351)?

14. What is the approximate increase in the population of India (a) in all of 1979 (b) each day of 1979?

15. Of every 5000 people in Asia, how many, on the average, will be under 15 years of age?

16. What is the ratio of the life expectancy of a person born in the U. S. compared to that of a person born in Afghanistan (in middle south Asia)?

17. According to the group, Zero Population Growth, in 1975, in the United States, the total population, as of July 1, was 213,540,000 people, there were 3,144,000 live births, 1,893,000 deaths, a net legal immigration of 450,000 people and an estimated illegal immigration of 800,000 people.

(a) What was the natural increase?

(b) What was the birth rate (births per 1,000 people)?

(c) What was the death rate (deaths per 1,000 people)?

(d) What was the population increase, excluding illegal immigration?

(e) What was the population increase, including illegal immigration?

(f) What was the percent increase in population, excluding illegal immigration?

(g) What was the percent increase in population, including illegal immigration?

(h) What was the ratio of illegal immigrants to legal immigrants?

Notes

1. Population Reference Bureau chart, "Growth of World Urban Centers."

2. Martin M. McLaughlin and the staff of the Overseas Development Council, The United States and World Development, Agenda, 1979, New York, Praeger, 1979, pp. 129-144.

The discussion of the PQLI is based largely on this source.

3. Lester R. Brown, World Without Borders, New York, Vantage Books, 1972, p. 41.

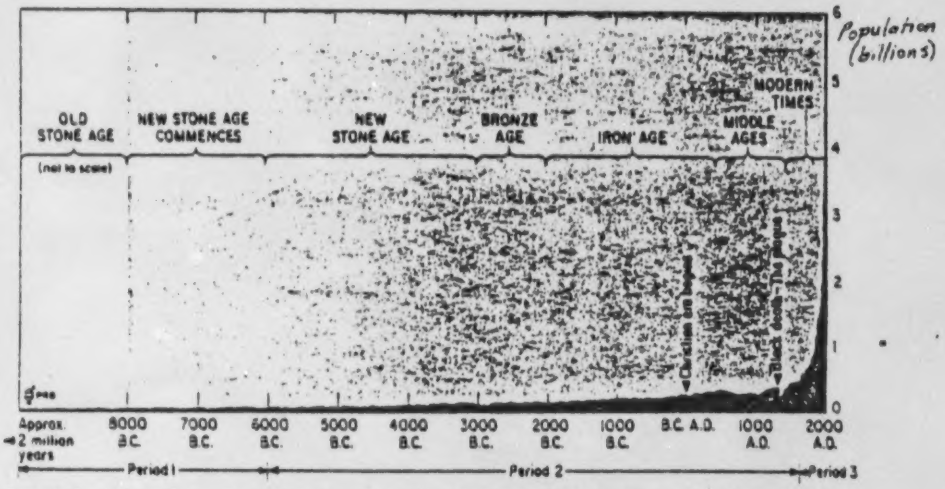
Chapter 4 Graphical Methods

Information related to issues of global survival can often be conveniently shown in graphical form. In this chapter we will study how to interpret and draw graphs, circle charts, bar charts, histograms, and frequency polygons, utilizing examples related to global issues.

4.1 Graphs

Perhaps the most important graph related to human survival is shown below. It indicates the explosive recent growth of population with time.

POPULATION THROUGH HISTORY



Source: Population Reference Bureau, Inc. "How Many People Have Ever Lived on Earth?", *Population Bulletin*, Vol. XVIII, No. 1.

Man's numbers reached a half billion by the year 1650 A.D.; at present rates population is increasing by one billion in a dozen years.

Figure 4.1

Along the horizontal line (axis) is plotted time in years. The years are uniformly spaced along the line and every 1000 years starting with 8000 B.C. is indicated. Along the vertical line is plotted population in billions.

Much useful information can be quickly obtained from the graph:

1. Population growth through most of the world's history has been relatively slow. Note the very gradual slope (incline) for most of the graph.

2. There was a sharp drop in population during the Black death plague.

3. In the last few hundred years (see the extreme right side of the graph), population growth has been extremely rapid.

To focus more closely on this very rapid population growth, we will plot a graph using the population figures shown below.[2]

year	population (billions)	point
1650	1/2	a
1850	1	b
1930	2	c
1976	4	d
2016	8 (projected)	e

Table 4.1

To facilitate plotting of this and other graphs, we use quadruilled (lined) paper.

As is the common practice, we will plot time along the horizontal axis. Note that a scale is chosen (often by trial and error) so that most of the graph sheet is used and we have convenient markers. Five boxes represent 100 years (from 1800 to 1900 for example); hence each box represents 20 years, each half box represents 10 years, etc. Population in billions of people is plotted along the vertical scale. Each box represents 1/2 billion people

To plot point d, for example, (4 billion people in 1976), we use the horizontal line corresponding to 4 on the population (vertical) scale. Three boxes to the right of 1900 represents 1960, 4 boxes to the right of 1900 represents 1980. Hence, 1976 is 3 boxes plus 4/5th of a box ((1976 - 1960)/20) to the right of the mark for 1900. Check this point and all other points (a, b, c, and e) plotted to get the graph to make sure that you know how to do it.

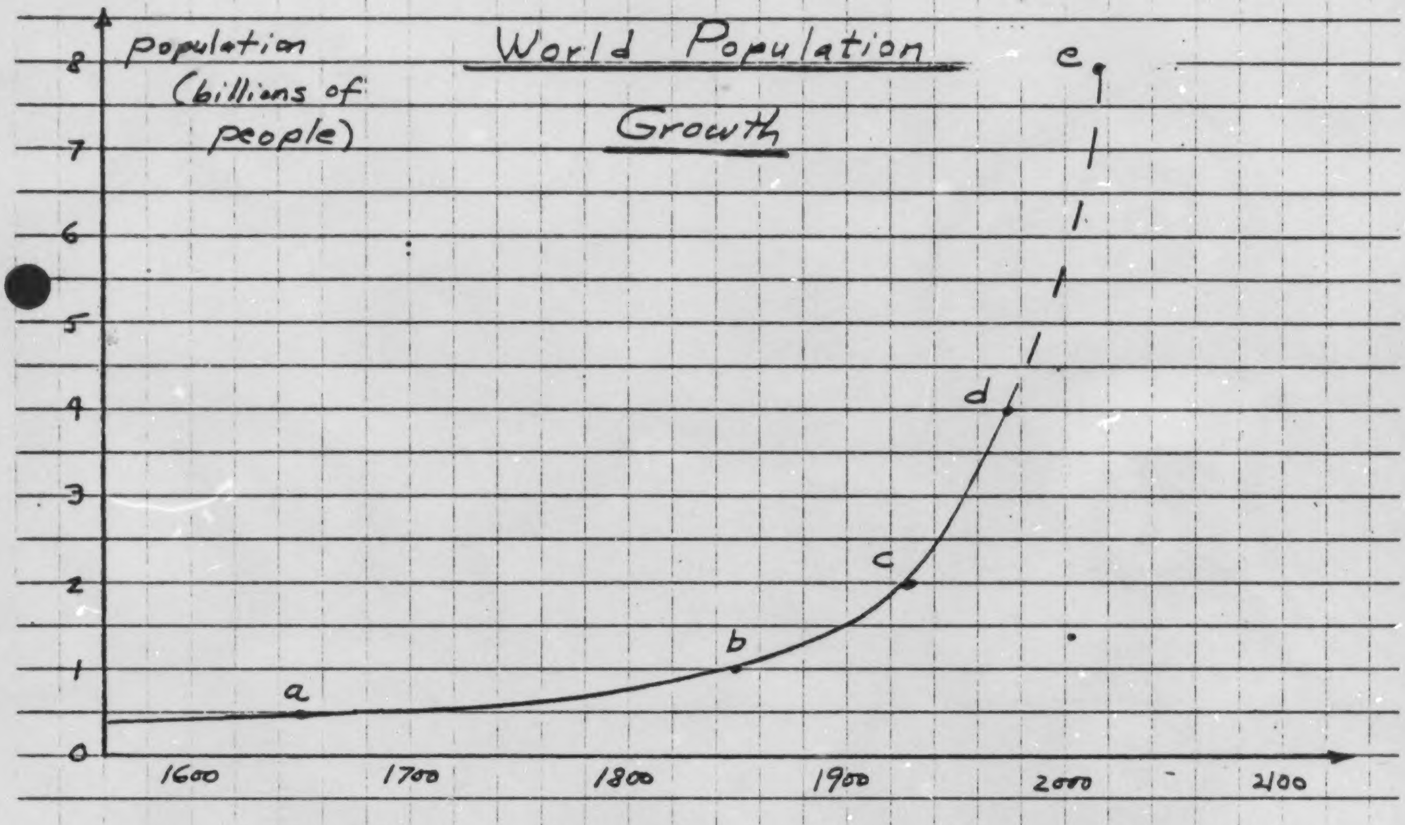


Figure 4.2

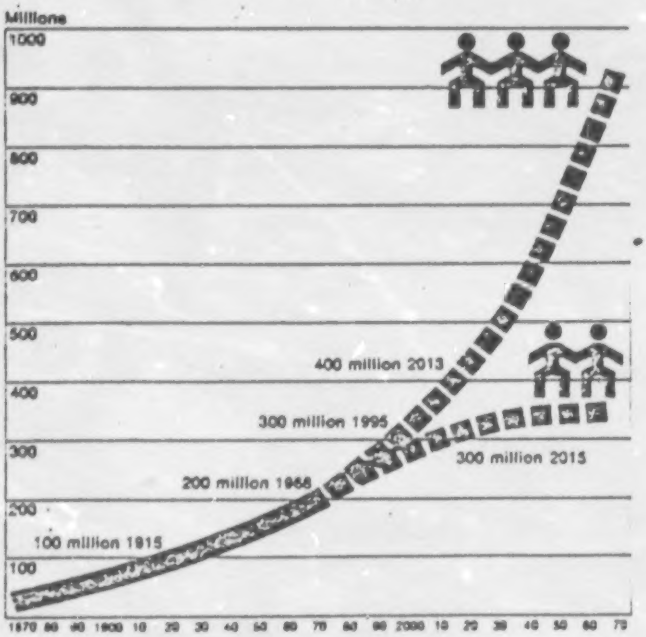
Once the points (a, b, c, d, e) have been plotted, they are connected as well as possible by a smooth curve. The use of a French Curve (a device with curves of many different shapes) makes the plotting easier.

Figure 4.2 also shows the tremendous population changes that have occurred in the last few centuries and especially in the more recent past. While the world's population didn't reach 1 billion people until 1850, at current rates, population increases by 1 billion people every 12 years.

The dashed line from point d to point e represents an extrapolation (continuation of a graph to predice a later value). Extrapolations are valuable in providing information about future conditions if present trends continue. However, they should always be treated with caution as conditions can abruptly change; consider the tremendous increases in prices for imported oil in 1973, for example.

What about U. S. population? Although our birth rate is below the replacement level (discussed in Chapter 6) of 2.1 children per couple, our population continues to rise due to immigration, our relatively young population with many women of reproductive age, and the increase in life expectancy. Two projections for the future, starting with a population of 200 million people in 1968 are shown in figure 4.3.

U.S. Population: 2- vs. 3-Child Family [3]



Source: Commission Report, p. 23.
Note: Assuming small future reductions in mortality and future immigration at present levels.

Fig. 4.3

If there is an average of 2 children per American family in the future, U. S. population is projected to level off at about 340 million people in 2060. If the average American family had an average of 3 children per couple, population would rise very rapidly and exceed 900 million people by the year 2,070. U. S. population growth has slowed in recent years as families have had an average of about 1.7 children per couple. Exercise 3 at the end of this article gives data on U. S. population from 1790 to 1979. More concepts related to population growth are considered in Chapter 5.

Issues related to energy have been of much importance in the 1970's and will become increasingly so in the next decades. There have been large increases in the price of oil, long lines at gasoline stations, much discussion of the future of nuclear power, especially after the March, 1979 accident at the Three Mile

Island nuclear power plant, and increased consideration of renewable energy sources, such as the sun and wind. The following example provides background for consideration of these issues.

Example 4.1

Plot a graph using the data below for the total production of electrical energy for various years.[4]

year	Electrical Energy (trillion kilowatt-hours	point
1902	6	a
1912	25	b
1920	57	c
1930	114	d
1940	180	e
1950	389	f
1960	844	g
1970	1640	h
1975	2003	i
1977	2211	j

Table 4.2

Solution

The graph is plotted below. The horizontal scale is years (4 boxes represents 10 years) and the vertical scale is electrical energy production (each box represents 100 trillion kilowatt-hours). Check the plotted points a through j to verify that you know how to plot them.

Note: In many real-world situations, plotted points will be such that a smooth curve cannot connect all of them. In such cases, draw a "curve of best fit," that averages out all the points as well as possible. However, show the plotted points so that others will realize that an average curve is being drawn and they can interpret it accordingly.

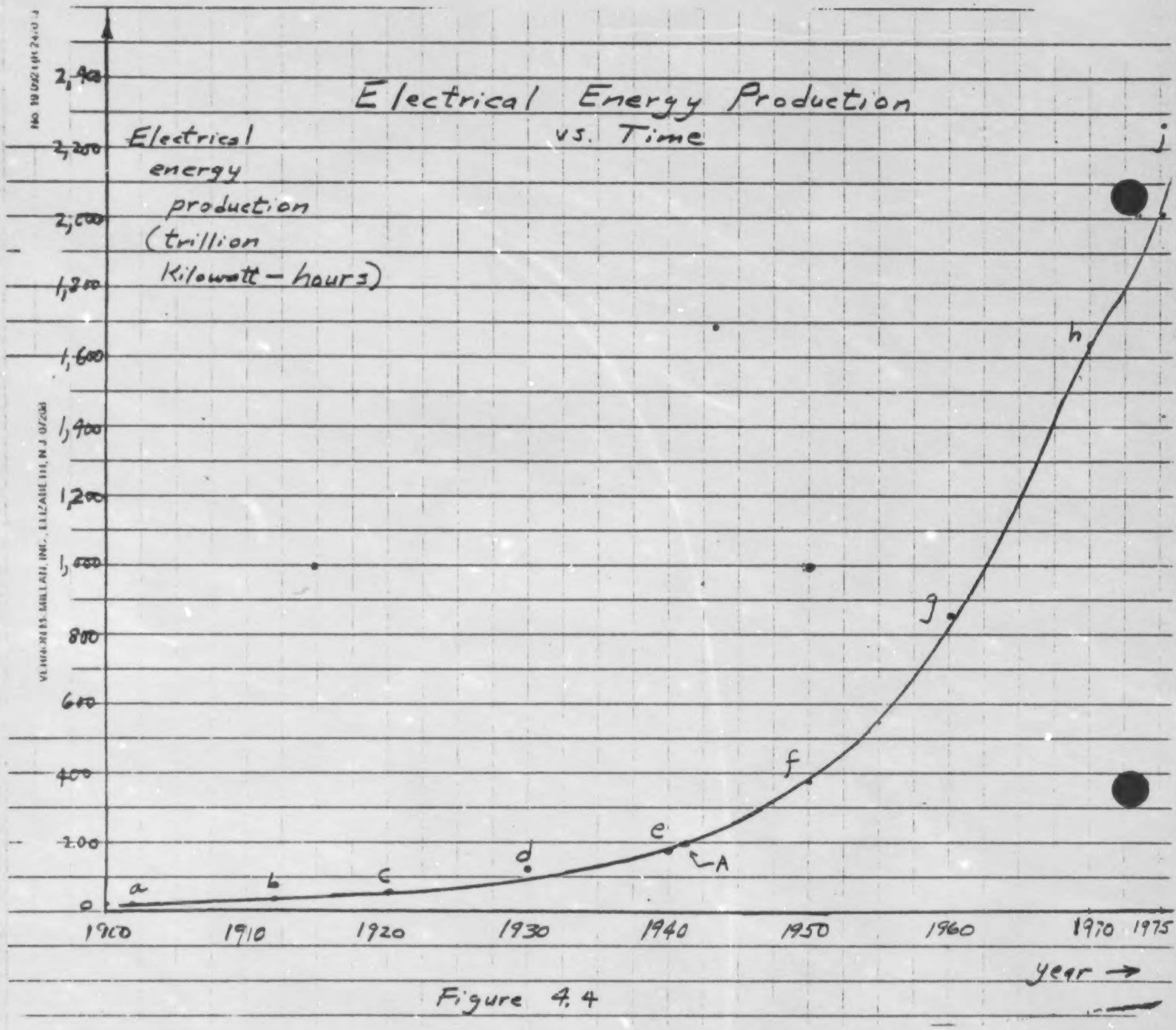


Figure 4.4

Many important observations can be made from the graph:

1. The electrical energy production growth has been extremely rapid. The curve is of the J type which shows exponential growth (this is explained in more detail in chapters 5 and 7).

2. The doubling time can be estimated. One way is to find the years on the graph when the electrical energy production was 100, 200, 400, 800, and 1600 (trillion Kw-hrs) as shown in the table below:

Electrical Energy Production (trillion Kw-hrs)	Year	
100	1930	
		} 11 years
200	1941	
		} 10 years
400	1951	
		} 8 years
800	1959	
		} 10 years
1600	1969	

Table 4.3

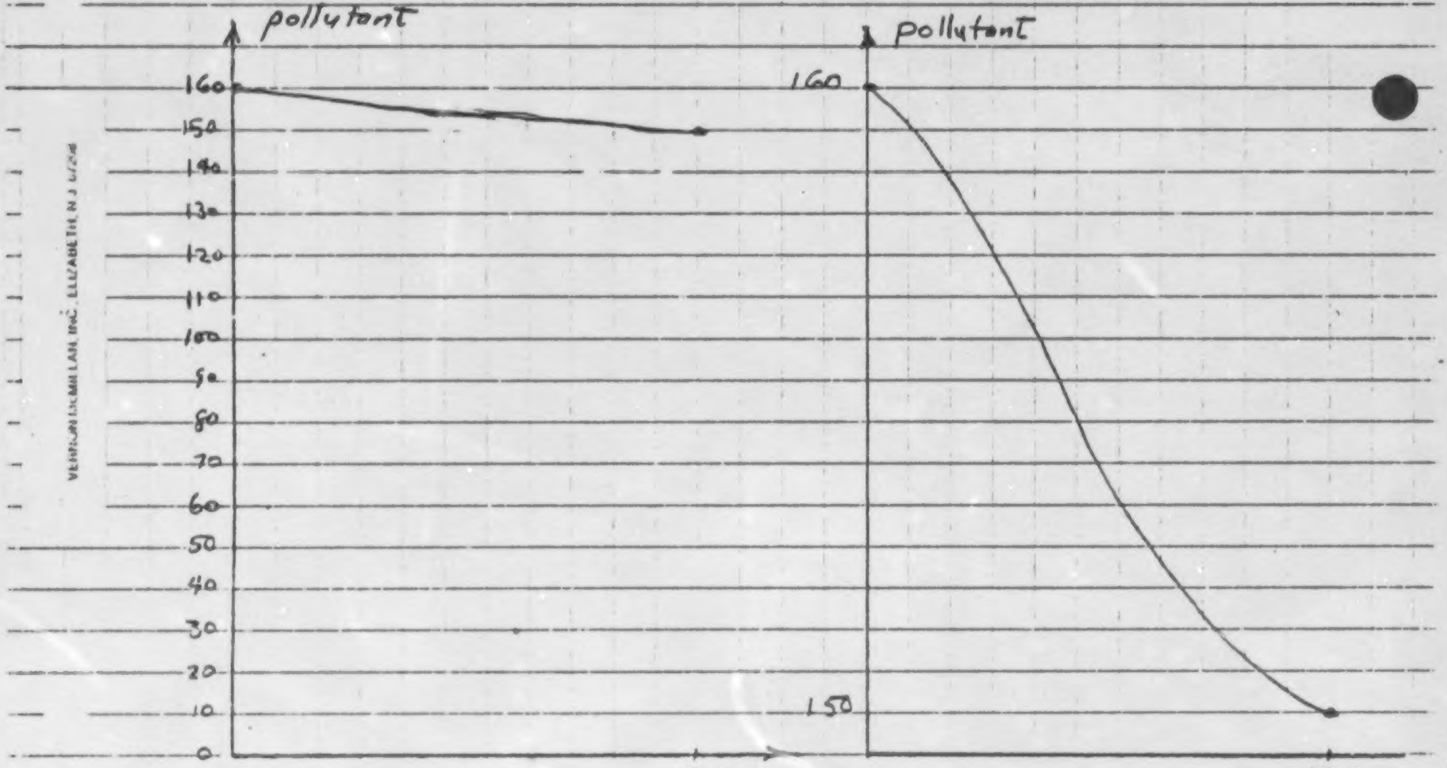
Note that the average doubling time is about 10 years. This short doubling time is an important factor behind the current concern about sufficient energy.

The values in table 4.3 were obtained by interpolation (reading a value from a graph between plotted points). For example, to obtain the year when the electrical energy production was 200 trillion kilowatt-hours, we see where the horizontal line through 200 on the vertical scale hits the curve (point A). We note that this point is almost half of the way from the vertical lines for 1940 toward that for 1942½, hence about 1941. Check the other values in table 4.3 to make sure that you know how to interpolate. Note that we have given values to the nearest year, as extreme accuracy is not necessary in getting the average doubling time.

3. If the doubling time continues at about ten years, 3200 trillion Kw-hrs (twice 1600 trillion Kw-hrs) should be reached in 1979, ten years beyond 1969.

As indicated before, extrapolation should always be viewed with much caution, especially in today's rapidly changing world. With the large increases in prices for energy, especially electrical energy, the rate of growth in electrical energy use has decreased in the last few years.

Sometimes graphs can mislead. Consider Figure 4.5(a) which shows a decrease in some pollutant from 160 units to 150 units (about a 6% decrease) in a one year period.



(a)

(b)

Fig 4.5

By changing the vertical scale, the impression of a major decrease in pollution can be given (Fig. 4.5(b)).

Vertical scales should always start from zero, or a clear indication of a break in the scale should be shown.

Exercises

Plot graphs for each of the following sets of data (exercises 1 through 10). Estimate doubling times, where appropriate. Indicate the significance of trends shown on the graph with regard to global survival issues.

1. World crop land vs time[5]

year	1800	1900	1950	1970	2000
Crop land (hectares/person)	1.42	0.82	0.50	0.34	0.18 (est.)

A hectare is a unit of area in the metric system, equal to 2.471 acres.

2. U. S. urban population (percent) vs. time

Year	1800	1830	1860	1890	1920	1950	1968
Percent	6	9	20	35	51	64	73

3. U. S. population growth vs. time[6]

Year	U. S. population (thousands)	Year	U. S. population (thousands)
1790	3,929	1890	63,056
1800	5,297	1900	76,094
1810	7,224	1910	92,407
1820	9,618	1920	106,461
1830	12,901	1930	123,188
1840	17,120	1940	132,122
1850	23,261	1950	151,684
1860	31,513	1960	180,671
1870	39,905	1970	204,879
1880	50,262	1979	220,300

4. Energy Use in the U. S. Food System vs. time[7]

Year	1940	1947	1950	1954	1958	1960	1964	1968	1970
Total energy (10^{12} Kcals)	686	1029	1134	1252	1361	1440	1691	2022	2172

5. Projected U. S. Water requirements vs. time[8]

Year	1965	1980	2000	2020
Projected water requirements (billion gallons/day)	269.6	442.6	804.6	1368.1

6. Annual sales of soft drink bottles (returnable and nonreturnable) (in millions of bottles) vs. time[9] (2 graphs)

Year	returnable	nonreturnable	Year	returnable	nonreturnable
1955	1233	169	1963	1772	560
1956	1330	171	1964	1912	632
1957	1217	183	1965	1914	1,006
1958	1240	192	1966	1922	1,980
1959	1415	211	1970	1600	6,000
1960	1407	249	1973 (est.)	1400	9,000
1961	1338	402			
1962	1574	477	1976 (est.)	1200	13,500

7. Imports of manufactured fertilizer by "developing" countries in terms of (a) millions of $ and (b) million metric tons, vs time[10] (2 graphs on one sheet)

Year	1971	1972	1973	1974
Millions of $	533	625	875	1450
Million metric tons	4.6	5.2	5.8	4.9

8. Abandoned cars towed away in N. Y. City vs. time[11]

Year	Abandoned cars	Year	Abandoned cars	Year	Abandoned cars
1960	2,500	1965	21,943	1970	72,961
1961	5,117	1966	23,795	1972	82,300
1962	6,299	1967	25,842	1973	73,300
1963	13,579	1968	31,578		
1964	23,386	1969	57,742		

9. Based on the U. S. population in 1972, the U. S. Census Bureau has made the following projection:[12]

Average Children per woman	1972	1980	1990	2000	2020
		(in millions)			
3.10 (with immigration)	209	237	279	322	447
2.78 "	209	234	276	305	397
2.45 "	209	231	261	288	351
2.11 "	209	228	251	271	307
2.11 (no immigration)	209	223	242	256	280

Plot 5 graphs on one sheet using this data.

10. World prices of many of the raw materials of which the "developing" countries are the major exporters fluctuate widely and severely affect the earnings of these countries from year to year. Plot graphs for price fluctuations for the items indicated below:[13]

Item	1972	1973	1974	1975	1976	1977
Ghanan Cocoa (c/lb)	32	65	98	75	110	220
Copper (London metal exchange c/lb)	49	81	93	56	64	59
Saudi Arabia Petroleum ($/barrell)	1.9	2.7	9.8	10.7	11.5	12.4
Rice ($/metric ton)	116	350	542	363	254	272

11. Plot a graph of the data below which reflects world food security from 1960/61 to 1977/78.

Year	Reserves as days of annual grain consumption	Year	Reserves as days of annual grain consumption
1960/61	103	1969/70	93
1961/62	112	1970/71	76
1962/63	93	1971/72	73
1963/64	94	1972/73	60
1964/65	88	1973/74	37
1965/66	88	1974/75	40
1966/67	68	1975/76	37
1967/68	79	1976/77	36
1968/69	87	1977/78	43

12. For the graph shown below,[15] calculate the ratio of per capita income for the rich countries compared to the poor countries in a) 1960; b) 1970; c) 1974.

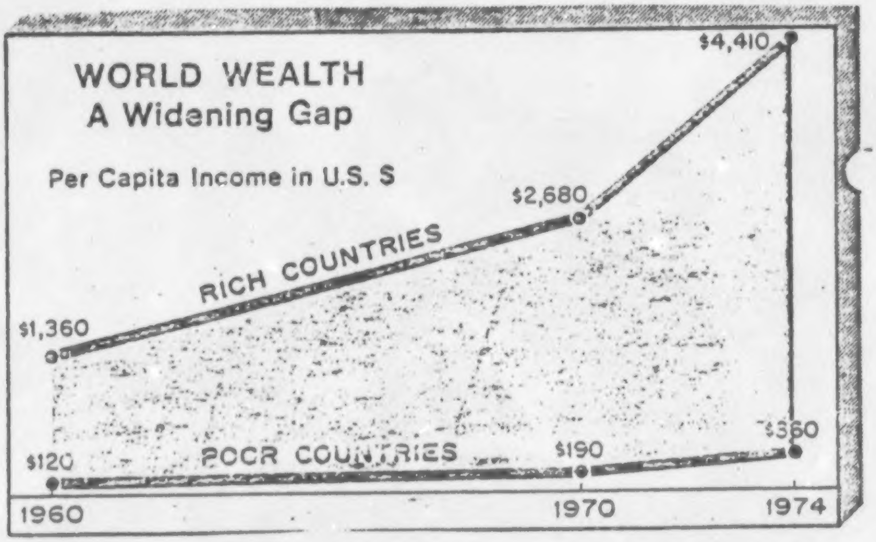
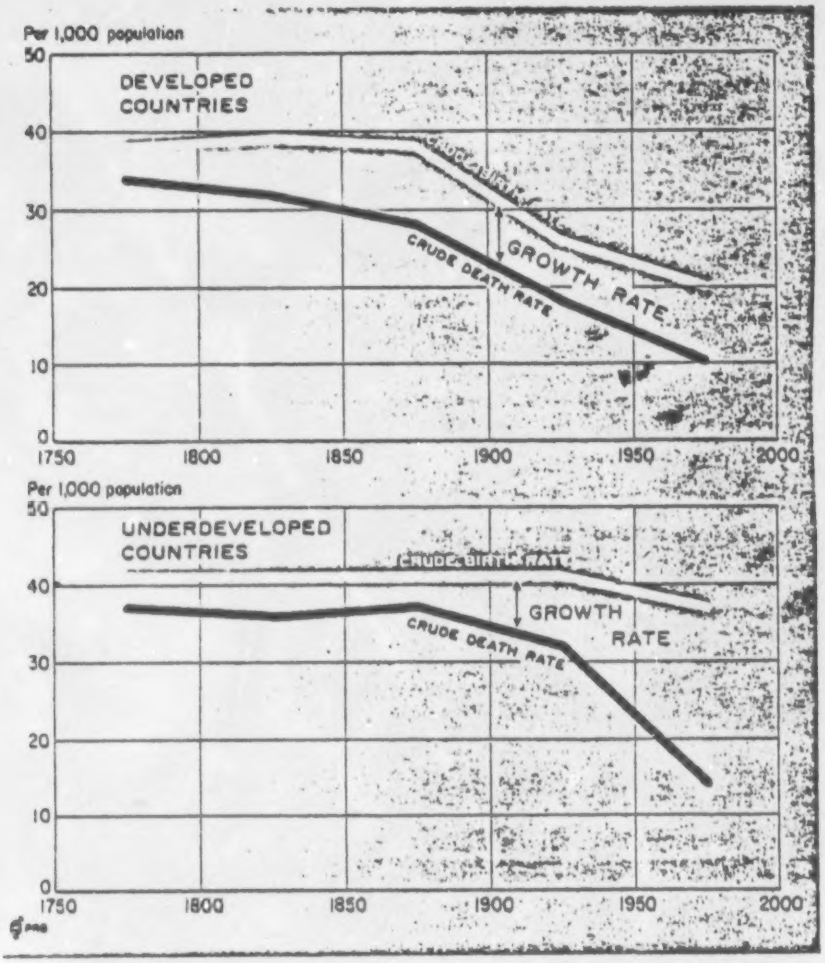


Fig. 14.6

13. Using the graphs below, estimate the rate of natural population growth (birth rate - death rate) in percent, for developed countries and for underdeveloped countries in a) 1850; b) 1900; c) 1950; and d) 1970. Which has been more important (the birth rate or the death rate) in causing the recent population explosion?

ESTIMATED BIRTH AND DEATH RATES, 1770-1970 [16]

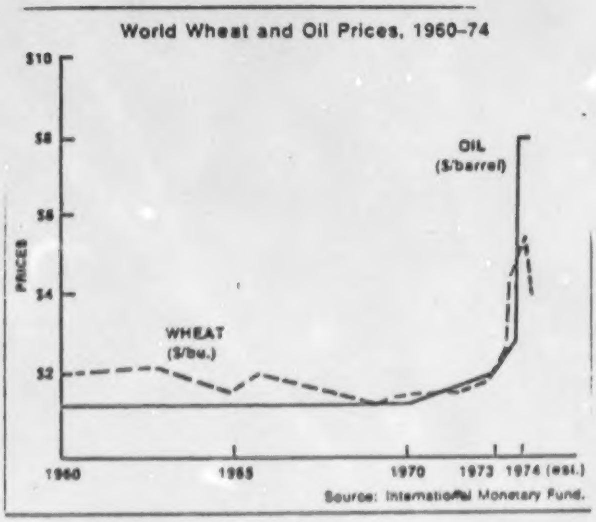


Source: United Nations. *A Concise Summary of the World Population Situation in 1970.* New York: United Nations, 1971.

In developed countries, decreased mortality was accompanied by dropping fertility; mortality drop was abrupt in underdeveloped countries.

Fig. 4-7

14. Figure 4.8 shows the very large increases in price for oil and wheat from 1973 to 1974. Compute the ratios of 1974 prices to 1973 prices for both oil and wheat.


World Wheat and Oil Prices, 1960–74

Source: International Monetary Fund.

Fig. 4.8

15. Using figure 4.9[17] below, estimate the ratio of total energy used in the food system to calories produced for a) 1940; b) 1960; and c) 1970.

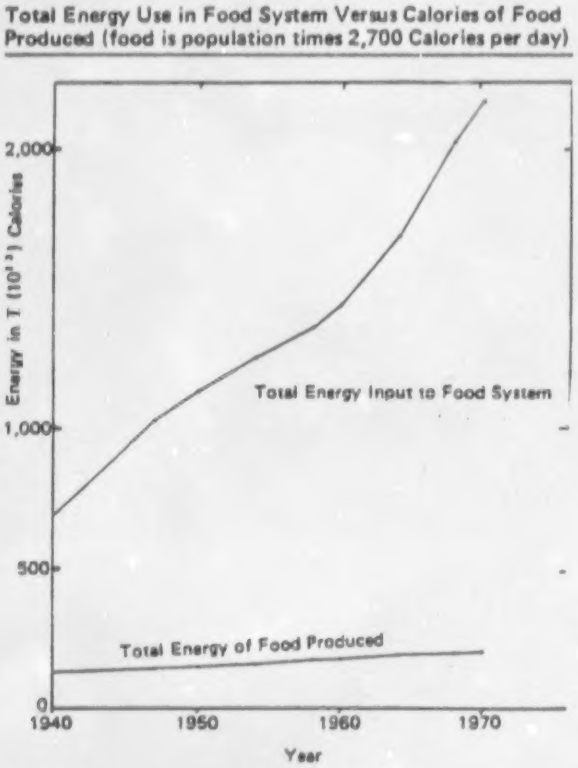

Total Energy Use in Food System Versus Calories of Food Produced (food is population times 2,700 Calories per day)

Fig. 4.9

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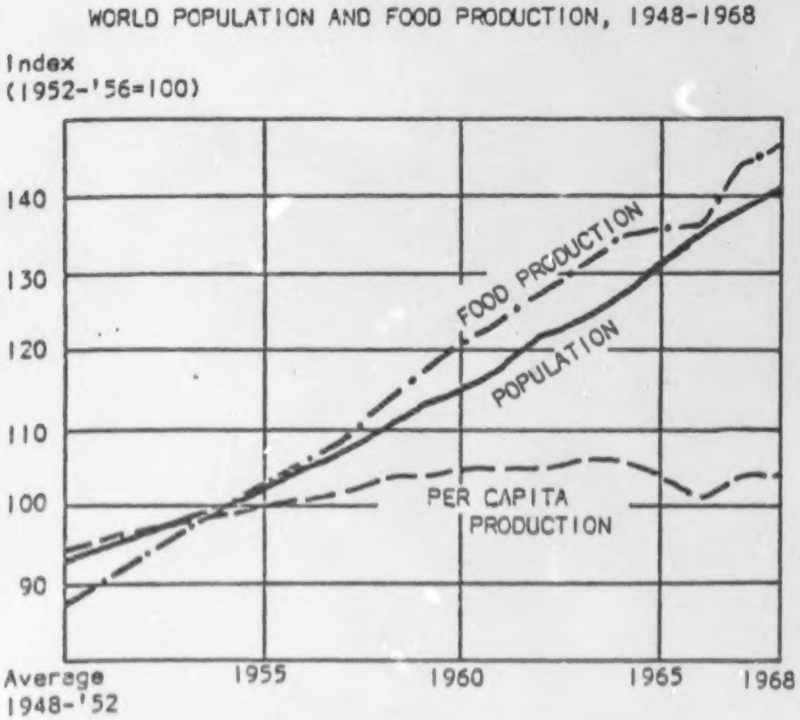
16. The graph below compares the growth in food production and in population until 1968. Note that the index (vertical scale) is considered to be 100 in 1954. Calculate the index (food production/population) using the two upper curves and compare with the results shown on the lower (dashed) curve for a) 1960; b) 1965; and c) 1968.



From Man's Population Predicament, Population Bulletin, April, 1971. With permission of the Population Reference Bureau, Inc. Source: The State of Food and Agriculture, Food and Agriculture Organization of the United Nations, Rome, 1968

Fig. 4-9(a)

4.2 Circle Diagrams

A circle diagram, also known as a circle chart or a pie chart, is a graphical method to illustrate conditions when several parts make up a whole.

The circle diagram below indicates the various components by weight of air pollution in a certain city. From this diagram it is easy to see the relative proportions of each component; for example, carbon monoxide makes up nearly half (47%) of the total pollution.

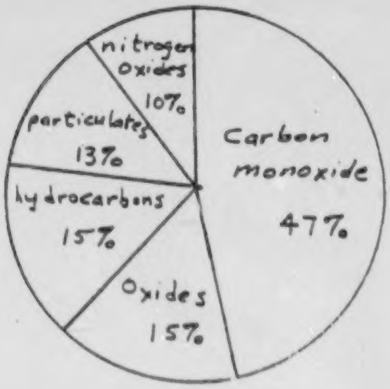


Fig. 4-10

Note that the various components add up to 100%. In some cases, there may be a slight discrepancy from 100%, due to round off.

There are 360° (360 degrees) in a circle. The number of degrees needed to plot the circle diagram shown above can be found as follows:

$$\frac{\text{degrees (of pollutant)}}{360^\circ} = \frac{\text{percent of pollutant}}{100\%}$$

$$\text{degrees (of pollutant)} = \frac{\text{\% of pollutant}}{100} \times 360^\circ$$

carbon monoxide:	47% of 360°	$\frac{47}{100}(360^\circ) = 169.2^\circ$
oxides	15% of 360°	$\frac{15}{100}(360^\circ) = 54.0^\circ$
hydrocarbons	15% of 360°	$\frac{15}{100}(360^\circ) = 54.0^\circ$
particulates	13% of 360°	$\frac{13}{100}(360^\circ) = 46.8^\circ$
nitrogen oxides	10% of 360°	$\frac{10}{100}(360^\circ) = \underline{36.0^\circ}$
		360.0

Note that the degrees add up to exactly 360°. In some cases, due to round-off, it may differ slightly from 360°.

Here is an example of how to draw a circle chart from given data about an important global issue.

Example 4.2

Draw a circle chart for the World's population distribution by regions in 1979. Obtain data from the 1979 World Population Data Sheet.

Solution

Region	Population (millions)
Africa	457
Asia	2498
North America	244
Latin America	352
Europe	483
USSR	264
Oceania	23
Total	4,321

First we compute the total population of the world in 1979 by adding up the various components. This gives 4,321 million people. This checks with the value given for the population of the world on the 1979 World Population Data Sheet.

Next, we will find the number of degrees for each area using the following proportion:

$$\frac{\text{population of region}}{\text{world population}} = \frac{\text{degrees for region (on the circle diagram)}}{360^\circ}$$

For Africa:

$$\frac{457 \text{ million people}}{4{,}321 \text{ million people}} = \frac{x^\circ}{360^\circ} \qquad x = \frac{457}{4{,}321}(360^\circ) = 38.1^\circ$$

In a similar manner,

For Asia $\frac{2,498}{4,321}\,(360^\circ) = 208.1^\circ$

For North America $\frac{244}{4,321}\,(360^\circ) = 20.3^\circ$

For Latin America $\frac{352}{4,321}\,(360^\circ) = 29.3^\circ$

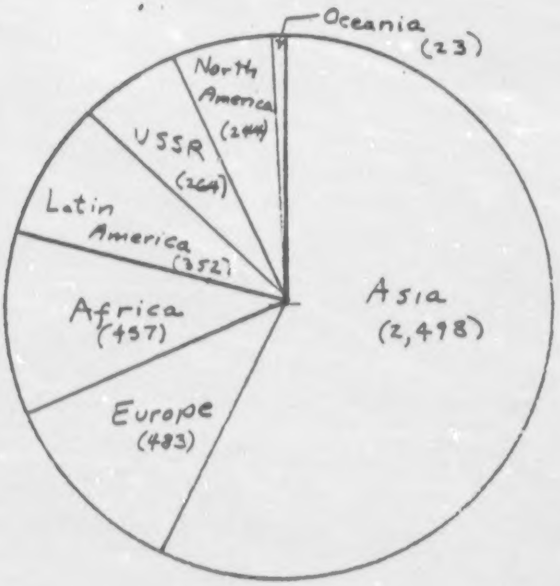
For Europe $\frac{483}{4,321}\,(360^\circ) = 40.2^\circ$

For the USSR $\frac{264}{4,321}\,(360^\circ) = 22.0^\circ$

For Oceania $\frac{23}{4,321}\,(360^\circ) = \underline{1.9^\circ}$

359.9°

359.9° is almost 360°, so the results check. Finally using a protractor (a device which is useful for plotting degrees), the following circle diagram is drawn.



numbers in parentheses indicate the population for a region in millions of people.

World Population Distribution

Fig. 4.11

Some general comments about drawing circle diagrams:

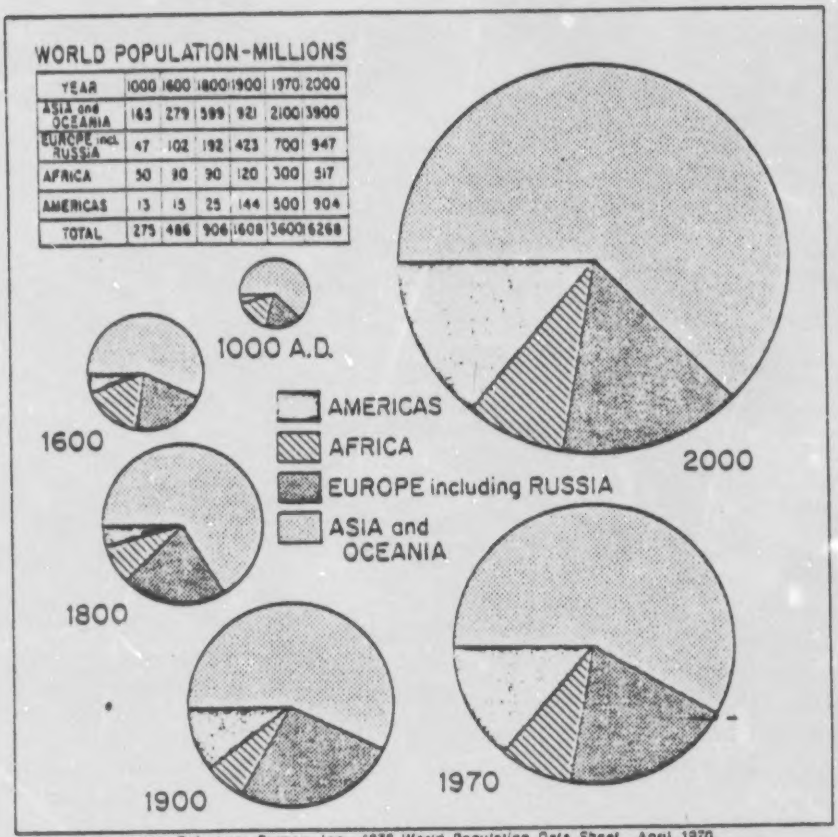
1. The size (diameter) of the circle is not significant in that relative proportions are shown - so use any practical diameter for the circle.

2. Check the size of the last piece of the circle to see that no errors have been made in laying off the previous pieces (in the circle diagram above, after laying off the number of degrees for North America, the number of degrees remaining for Oceania should agree (closely, since there will always be some error) with the computed number of degrees for Oceania.)

3. If a protractor is not available, portions of the circle can be approximated by eye. Often a rough sketch, to show relative sizes, will be sufficient. In such cases, take advantage of the fact that half a circle is 180°, a quarter of the circle is 90°, etc.

The circle diagrams shown below[18] indicate the changes in total world population from the year 1000 to 2000. The areas of the circles are proportional to the total population for each year shown. Note that throughout the period considered, more than half of the world's people lived in Asia and Oceania and the percentage of people living in the Americas increased (from about 3% to 14%). Also observe, from the table, that from 1600 to 1800, the population of Africa remained constant; these were the years when many slaves were removed from Africa.

ONE THOUSAND YEARS OF POPULATION GROWTH



WORLD POPULATION-MILLIONS

YEAR	1000	1600	1800	1900	1970	2000
ASIA and OCEANIA	165	279	599	921	2100	3900
EUROPE incl. RUSSIA	47	102	192	423	700	947
AFRICA	50	90	90	120	300	517
AMERICAS	13	15	25	144	500	904
TOTAL	275	486	906	1608	3600	6268

Sources: Population Reference Bureau, Inc. *1970 World Population Data Sheet.* April 1970.
Population Reference Bureau. *"A Thousand Years of World Population Growth"* (wall chart), 1960.

The Americas held 3 percent of the world's population before Columbus; by 1970 the area's share of a much bigger pie was 14 percent.

Fig. 4-12

To obtain the number of degrees for Africa in 1970 for example, we use the following method:

population of Africa in 1970 = 300 million people (see table in Fig. 4-12)

world population in 1970 = 3600 million people

$$\frac{\text{degrees for Africa}}{360^\circ} = \frac{\text{population of Africa in 1970}}{\text{total world population in 1970}}$$

$$\frac{x^\circ}{360^\circ} = \frac{300 \text{ million people}}{3600 \text{ million people}}$$

$$x^\circ = \frac{300}{3600} \times 360^\circ = 30^\circ$$

Example 4.3

The circle chart below shows the average percent of immigrants from various regions in the period 1971 - 1974.[19]

a) How many degrees on the circle represent Latin America?

b) What is the average number of people who came annually from Asia?

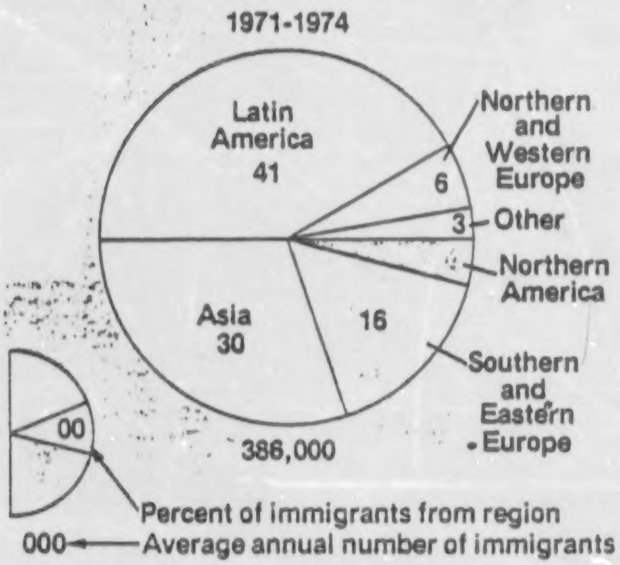


Fig. 4.13

Solution

a) We compute the degrees on the circle for Latin America from the following proportion

$$\frac{41\%}{100\%} = \frac{x^{\circ}}{360^{\circ}}$$

$$x = 360^{\circ} \cdot \left(\frac{41}{100}\right) = 360(0.41) = 147.6^{\circ}$$

b) The circle chart shows that 30% of the average annual number of immigrants, 386,000, come from Asia. Hence

$$\frac{\text{Immigrants from Asia}}{\text{total immigrants}} = \frac{30\%}{100\%}$$

$$\frac{\text{Immigrants from Asia}}{\text{386,000 people}} = \frac{30}{100} = 0.3$$

Immigrants from Asia = 0.3(386,000) = 115,800 people.

Exercises

Draw circle charts for the following sets of data (exercises 1-9).

1. Percent of world arms sold and traded by various countries in 1976[20]

Country	Percent
United States	38.8
U.S.S.R.	27.9
France	6.3
Federal Republic of Germany	4.9
United Kingdom	4.8
Other	17.3

2. Disposal of Solid Waste in the U. S.[21] (1969)

Methods of Disposal	Million Tons of Waste
Open dumps	146
Sanitary landfill	25
Incinerators	15
Uncollected	60
Other*	4

* salvaged from collected material, composted or dumped at sea.

3. Sources of Air Pollution in the U. S.[22] (1968)

Source	Percent
Transportation	42
Fuel combustion in stationary sources	21
Industrial processes	14

(continued)

Source	Percent
Forest fires	8
Solid waste disposal	5
Miscellaneous	10

4. Percent shares of energy consumption (1977), in the U. S.[23]

Use	Percent
Homes and businesses	37.3
Industry	37.0
Transportation	25.7

5. How energy demand is met (1977) in the U. S.[24]

Source	Percent
Imported oil	20.3
Natural gas	27.0
Domestic oil	26.7
Coal	19.2
Nuclear	2.7
Hydroelectric and geothermal	4.1

6. Income gap in Ecuador[25]

In 1970 the income in Ecuador was distributed as follows:

	% of national income
richest 20%	73.5%
2nd 20%	14.5
3rd 20%	5.6
4th 20%	3.9
poorest 20%	2.5

7. In 1977, the distribution of total energy consumption, in energy equivalent to one million barrels of petroleum a day for a year, was as follows:[26]

United States	33.5
Western Europe	23
Japan	6
Centrally Planned Economies (USSR, Eastern Europe, Cuba, China)	29
Rest of World	17

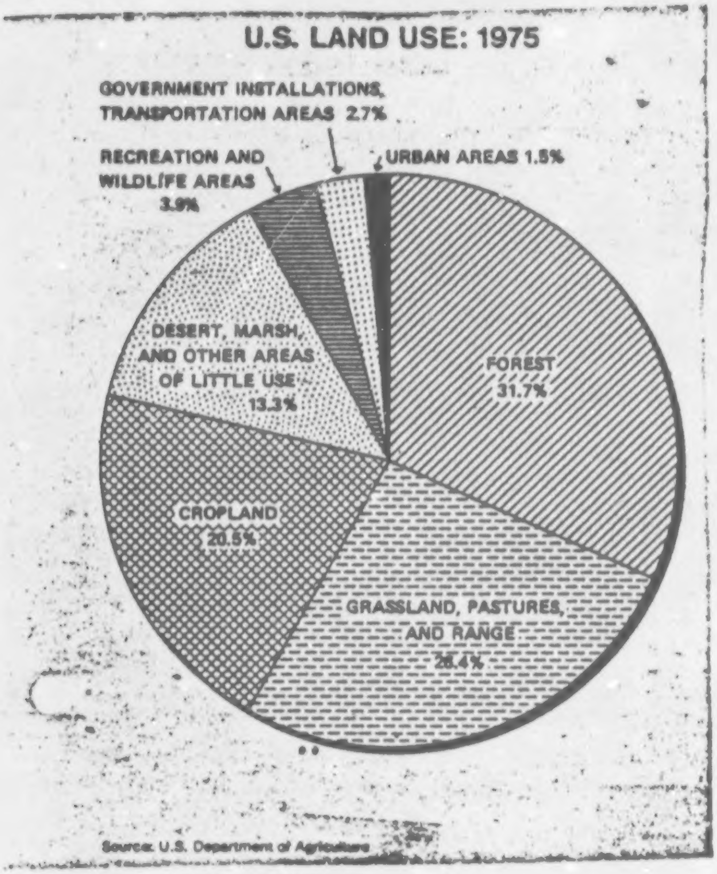
8. The quality classifications of tilled land for 1882 and 1952 is shown below.[27]

	1882 percentage	1952 percentage
Good	85.0	41.2
Half of original humus lost	9.9	38.5
Marginal soil	5.1	20.3

9. Income Gap in Brazil

After five years with one of the highest economic growth rates in the world, the economic minister of Brazil estimated in 1972 that in the previous five year period, 5% of the population were better off, 50% were in about the same condition, and 45% were worse off.[28]

10. For the circle diagram shown below:
    a) What was the main use of land in the U. S. in 1975?
    b) Approximately what fraction of U. S. land was used for crops?
    c) How many degrees of the circle are there in the section for forest land?



Source: U.S. Department of Agriculture

Fig. 4.14

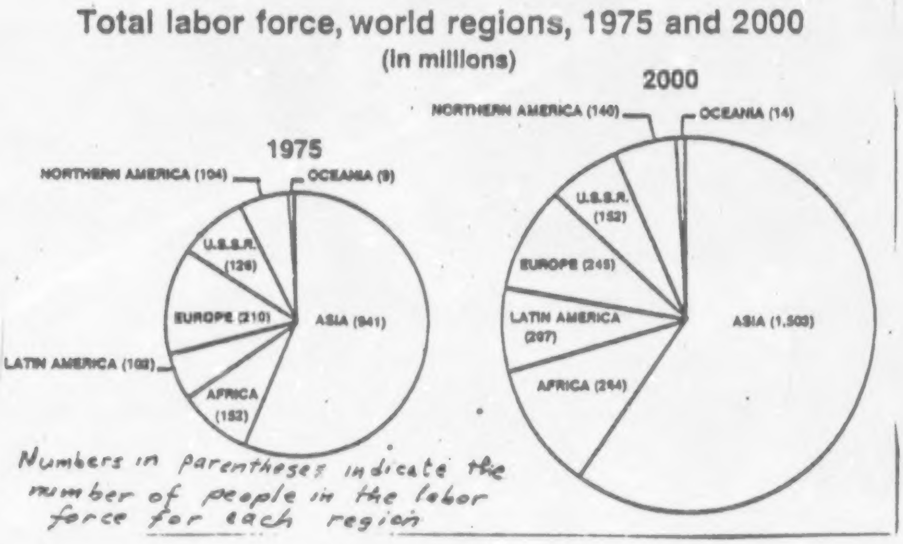
11. For the circle diagrams shown below:

    a) What was the total labor force in millions in 1975?

    b) What percent of this labor force was in Asia?

    c) How many degrees on the 1975 circle correspond to Latin America?

    d) Repeat parts (a), (b), and (c) for the year 2000.



Total labor force, world regions, 1975 and 2000
(in millions)

Numbers in parentheses indicate the number of people in the labor force for each region

Fig. 4.15

12. Use the circle diagrams shown below and populations from the 1979 World Population Data Sheet to answer the following:

    a) How many children are under 5 years of age in (1) the world, (2) the more developed countries, (3) the less developed countries?

    b) Repeat (a) for children under 15 years of age.

    c) How many degrees correspond to children under 15 years of age in the less developed countries?

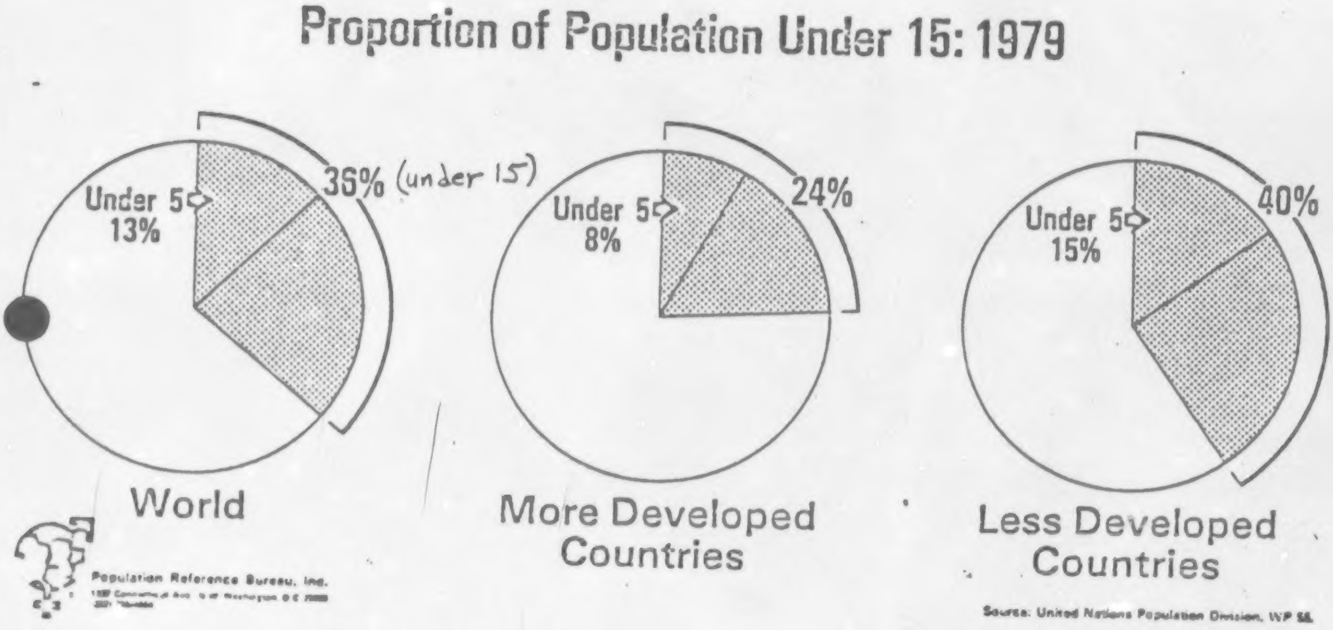


Fig. 4.16

13. For the circle diagram shown below:

    a) How many degrees correspond to Northern and Western Europe?

    b) How many immigrants came annually (on the average) from Southern and Eastern Europe?

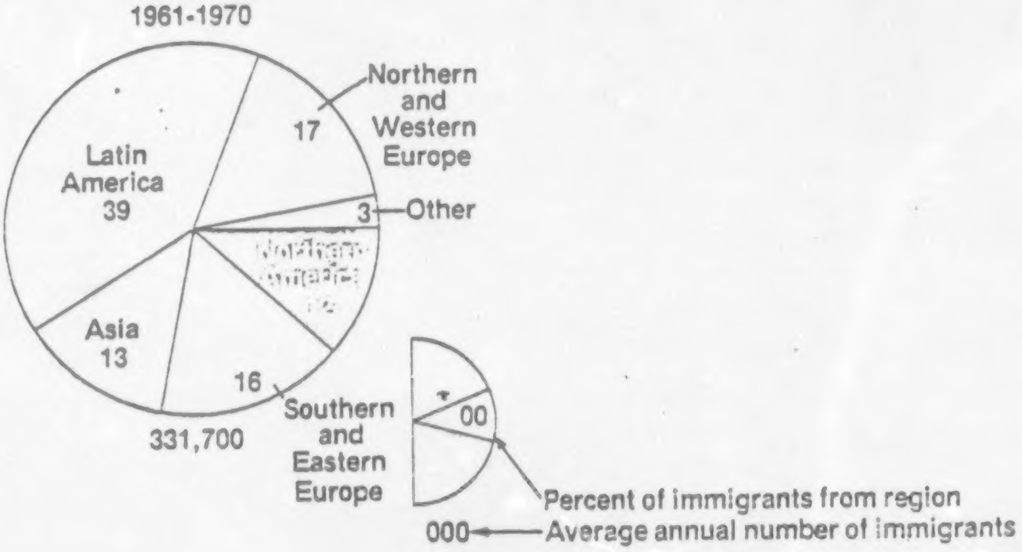


Fig. 4.17

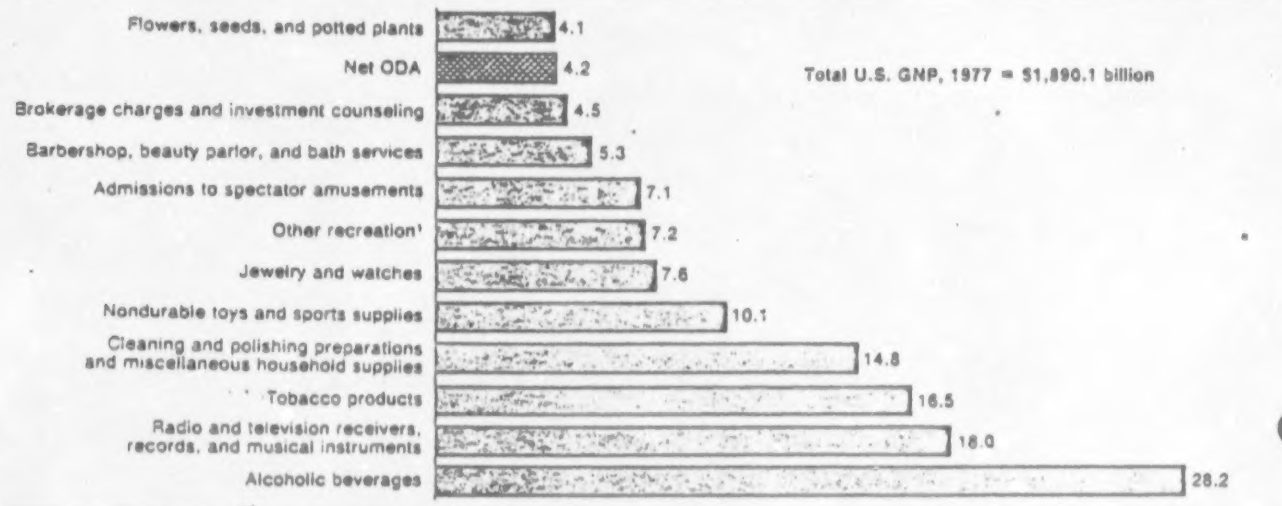
4.3 Bar Charts

A bar chart is another graphical technique that is very valuable in presenting data in an easy to visualize form. Several applications to global issues are illustrated in this section.

The bar chart below illustrates how much more Americans spent for personal consumption expenditures than for official development assistance (ODA) to the developing countries in 1977.

Selected U.S. Personal Consumption Expenditures and Net ODA Flows, 1977[32]
($ billions)

In 1977, the U.S. government provided $4.2 billion in official development assistance to the developing countries—a relatively small amount compared to expenditures on other goods. Individual Americans, for example, spent nearly as much ($4.1 billion) on flowers and plants, more than three and one-half times as much on cleaning and polishing preparations, and nearly seven times as much on alcoholic beverages.



¹Expenditures on pets and pet care, cable television, lotteries, camping, photo studios, and film processing.

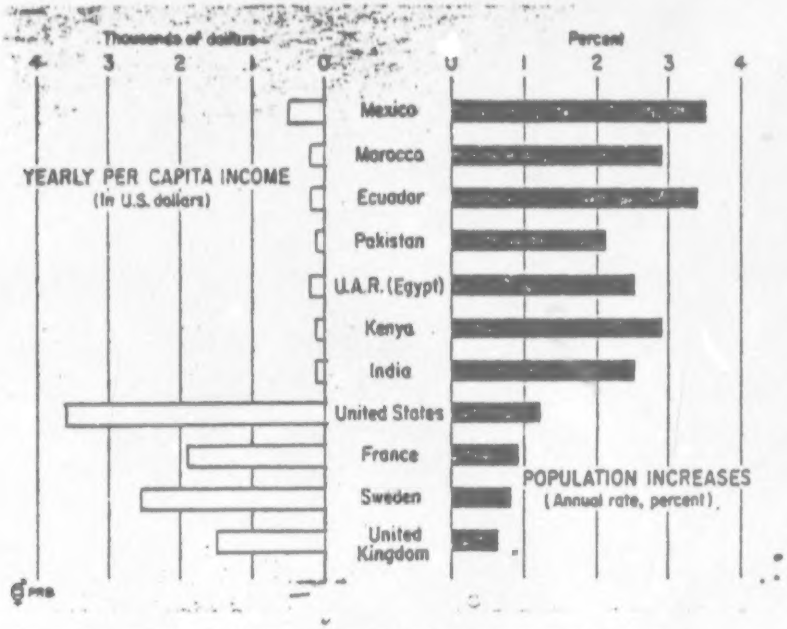
SOURCE: Based on data from U.S. Department of Commerce, *Survey of Current Business*, Vol. 58, No. 7, July 1978, p. 37.

Fig. 4.18

Americans spent $24 billion dollars ($28.2 billion - $4.2 billion) more for alcoholic beverages in 1977 than for official development assistance. We spent almost 4 times (16.5/4.2) as much for tobacco products as for official development assistance.

The two bar charts below show the relationship in 1970 between average per capita income in a country and population growth.[33]

PER CAPITA INCOME AND POPULATION GROWTH, SELECTED COUNTRIES



Sources: United Nations. *United Nations Statistical Yearbook.* New York: United Nations, 1970.
United Nations. *United Nations Demographic Yearbook.* New York: United Nations, 1970.
Population Reference Bureau, Inc. *1970 World Population Data Sheet.* April 1970.

Countries with low per capita income have high population growth.

Fig. 4.19

Note, first, the tremendous poverty in many countries of the world; for example, the average per capita income in Pakistan, Kenya, and India was only about $100 in 1970.

In general, the poor countries are those with the largest population increases (note the bars for the top six countries). In these countries there are no old age security programs and children are depended on for help in the fields as well as economic security when the parents become older. Also, infant mortality rates are still relatively high in these poor countries and parents have many children in the hope that at least a few will survive into the old age period of the parents.

The bar charts shown below[34] were used by SANE (A Citizen's Organization for a Sane World) to illustrate their view that both the United States and Russia have far more nuclear weapons than necessary for an "effective" deterrent and hence there is a need for disarmament. Note that, although then Secretary of Defense McNamara estimated in 1968 that 400 nuclear warheads would suffice for deterrent purposes, as they could inflict tremendous damage to an enemy, both the U. S. and the USSR have continued to produce many more nuclear weapons. In 1977, even under a SALT (Strategic Arms Limitation Agreement), the U. S. had 9800/400 or almost 25 times the "effective" deterrent force and the USSR had 3950/400 or almost ten times the "effective" deterrent force.

THE NUCLEAR ARMS RACE

Number of Nuclear Warheads Deliverable by Intercontinental Ballistic Missiles (ICBM's), Submarine-Launched Ballistic Missiles (SLBM's) and Intercontinental Bombers

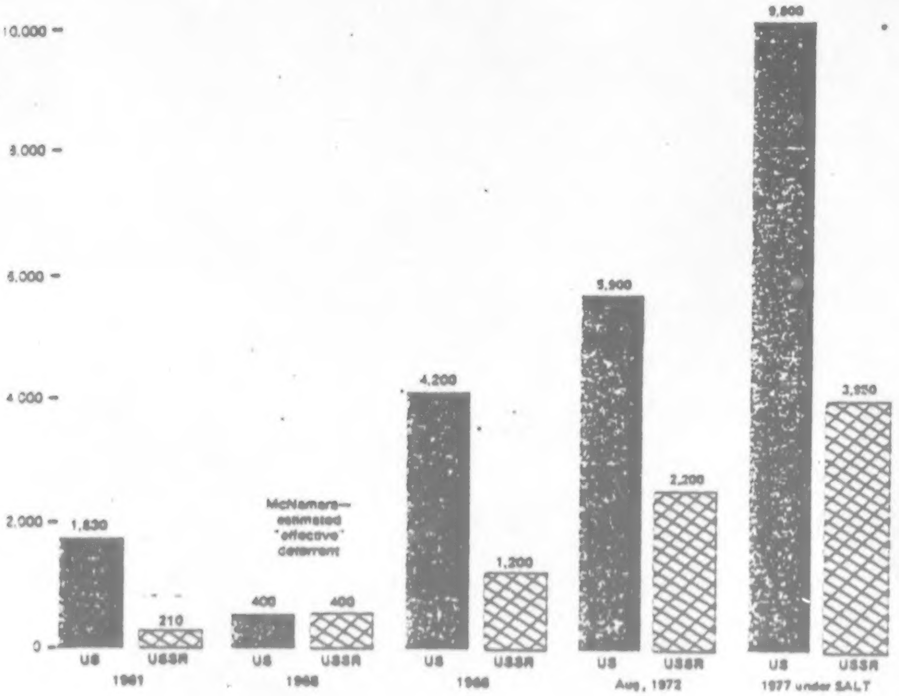


Fig. 4.20

Since World War II, the United States has spent $1.3 trillion on military programs. The Soviet Union has spent an estimated $1 trillion. A large part of these huge expenditures has been used to build nuclear weapons.

When both countries signed the SALT accords in 1972, they agreed to limit defensive missiles to very low levels and to stabilize the number of offensive missiles for five years. However, SALT does not halt the nuclear arms race.

Here are some general concepts related to drawing bar charts:

1. The bars may be horizontal (as in figure 4.18) or vertical (as in figure 4.20). It is simply a matter of convenience and space requirements.

2. The width of each bar should be equal. The space between each bar should be equal. There should always be a space between bars; if there is no space, we have a histogram which will be covered in the next section.

3. Remarks about choosing scales for graphs, as discussed in the last section, are also pertinent for bar charts. A scale should be chosen that will make interpolation easy with regard to reading the graph and interpreting it. As with the graphs and circle charts, sometimes just a rough sketch showing approximate relative relations will suffice to give insight with regard to the quantities plotted.

Bar charts are often presented in a pictograph form. Figure 4.21 below illustrates birth and death rates, for the world and various regions in 1970.[35]

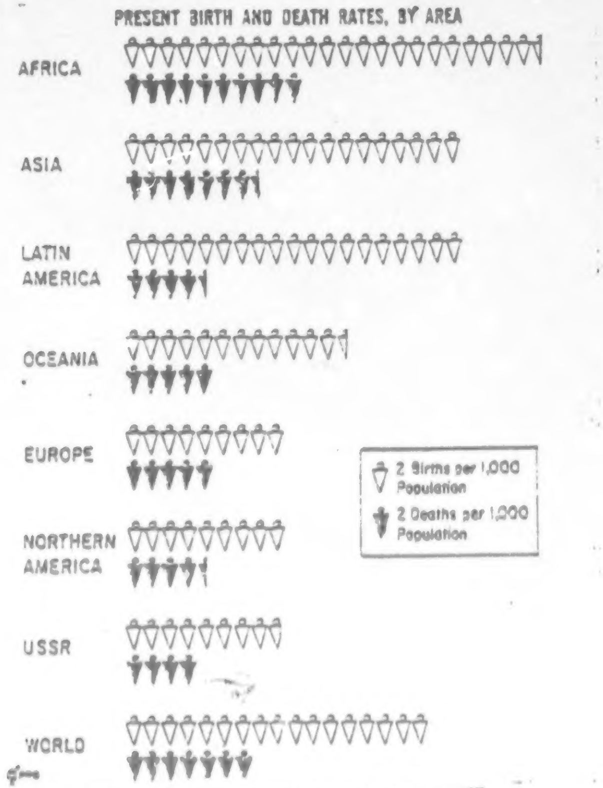


Fig. 4.21

Source: Population Reference Bureau, Inc. 1970 World Population Data Sheet. April 1970.

Worldwide, for each thousand persons there are 20 more births than deaths a year. The rate is higher in Latin America, Africa and Asia.

Let us analyze population growth in the world in 1970 using the pictographs. For the world, there are 17 figures representing births and 7 figures representing deaths. As indicated in figure 4.21 each figure represents 2 births per thousand or 2 deaths per thousand. Hence

$$\text{world birth rate} = 17 \cdot 2/1000 = 34 \text{ per thousand}$$

$$\text{world death rate} = 7 \cdot 2/1000 = 14 \text{ per thousand}$$

$$\text{net population increase} = 20 \text{ per thousand}$$

$$20 \text{ per thousand} = 2 \text{ per hundred or } 2\%$$

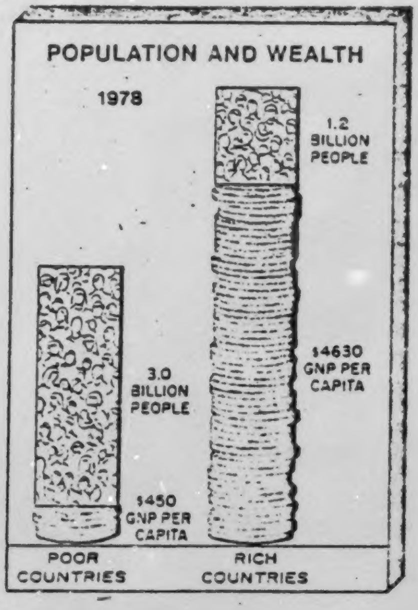
$$\left(\frac{20}{1000} = \frac{x}{100} \qquad x = \frac{2000}{1000} = 2\%\right)$$

The doubling time for the world can be obtained from

$$\text{doubling time} = 70/\%\ \text{increase} = 70/2 = 35 \text{ years.}$$

(The above equation was discussed in Chapter 3, and will be verified in Chapter 8.)

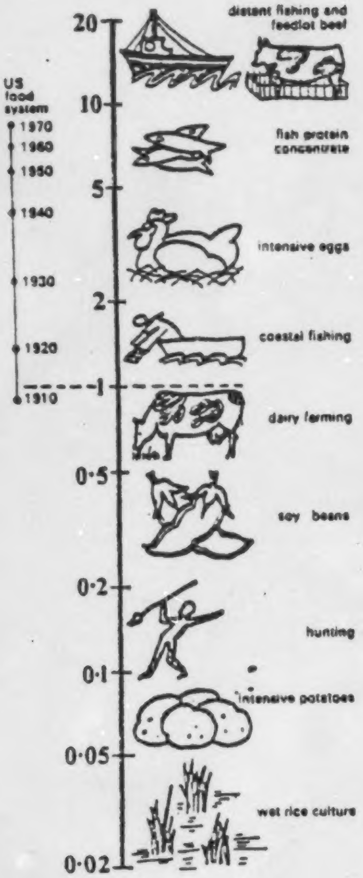
The pictographs in figure 4.22 below illustrate that most of the world's people (3.0 billion out of 4.2 billion) live in poor countries and that the average GNP per capita in rich countries is more than ten($4630/$450) times that in the poor countries.[36]



Source: Population Reference Bureau and United Nations

Fig. 4.22

CALORIES INPUT TO PRODUCE
ONE CALORIE OUTPUT



Left of the line is the calorie-in to calorie-out ratio of the entire U.S. food system. Right of the line is that ratio for food production only. Reprinted from the UN *Development Forum*, November 1974. Source: John S. and Carol E. Steinhart, "Energy Use in the U.S. Food System," *Science*, April 19, 1974.

Another type of pictograph is shown in figure 4.24.[38] The calorie-in to calorie-out scale shown on the left indicates that this ratio (for the entire U.S. food system) changed from less than 1 in 1910 to almost 10 in 1970. This change is due to the greatly increased mechanization and use of chemical inputs such as pesticides and fertilizers on American farms, and the increased consumption of meat. The pictograph on the right shows that while it takes only about 0.03 fuel calories to produce a food calorie from a wet rice culture, it takes about 12 fuel calories input for each food calorie output from cattle raised in a feedlot. When processing, refrigeration, packaging, and transportation are taken into account, the difference is even greater.

Fig. 4.24

Sometimes, if care is not taken, pictographs can mislead. Consider Figure 4.25.[39] The decrease in home heating stock looks larger than it really is because (1) the scale does not start with zero but with 50 (this makes the relative differences in height look greater) and (2) the Aprill 1977 house is drawn wider as well as taller (since we tend to compare the areas of the houses and not their heights, the differences seems greater).

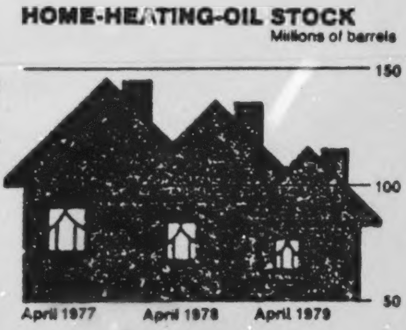


Fig. 4.25

A bar chart giving the same information with less chance of misleading is shown in figure 4.26.

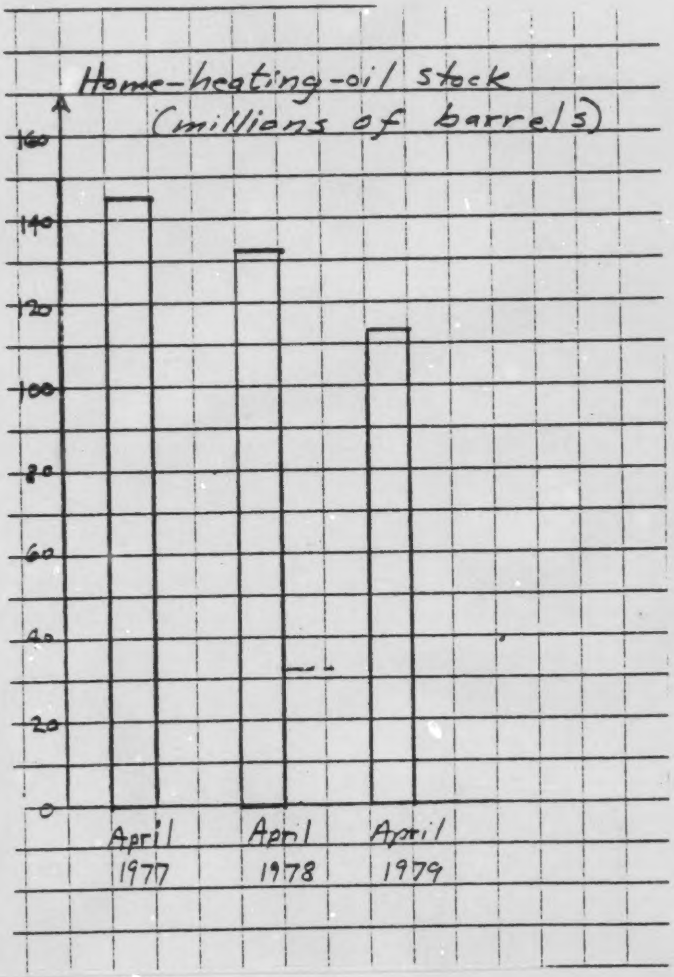


Fig. 4.26

Exercises

1. Using the 1979 World Population Data Sheet, plot bar charts for each of the following variables: a) Doubling time; b) Infant mortality; c) % under 15 years of age; d) Life expectancy at birth; e) Per capita GNP, US$ for the following countries: United States, Netherlands, USSR, Brazil, Taiwan, Algeria, Cuba, India, Sri Lanka, Mali. Select a range of values in each case so that

you get between 10 and 15 rectangles. Can you make any general statements by comparing the bar charts?

Plot bar charts for the following data (problems 2 through 6)

2. U. S. Imports of raw materials, 1976[40]

Material	% imported	Material	% imported
aluminum	87	manganese	98
chromium	89	nickel	71
cobalt	98	tin	85
copper	15	tungsten	59
iron	21	zinc	59
lead	15		

3. The automobile uses:[41]

60% of all rubber used in the U. S.; 10% of all aluminum;
20% of all steel; 7% of all copper; 13% of all nickel;
35% of all zinc; 50% of all lead

4. Physical Quality of Life Index

The PQLI (Physical Quality of Life Index) is based on an average of indexes related to life expectancy, infant mortality, and literacy rate.[42]

Country	PQLI	Country	PQLI
U. S.	95	Iran	52
Netherlands	96	Cuba	85
USSR	91	India	41
Brazil	66	Nicaragua	55
Taiwan	87	Mali	14

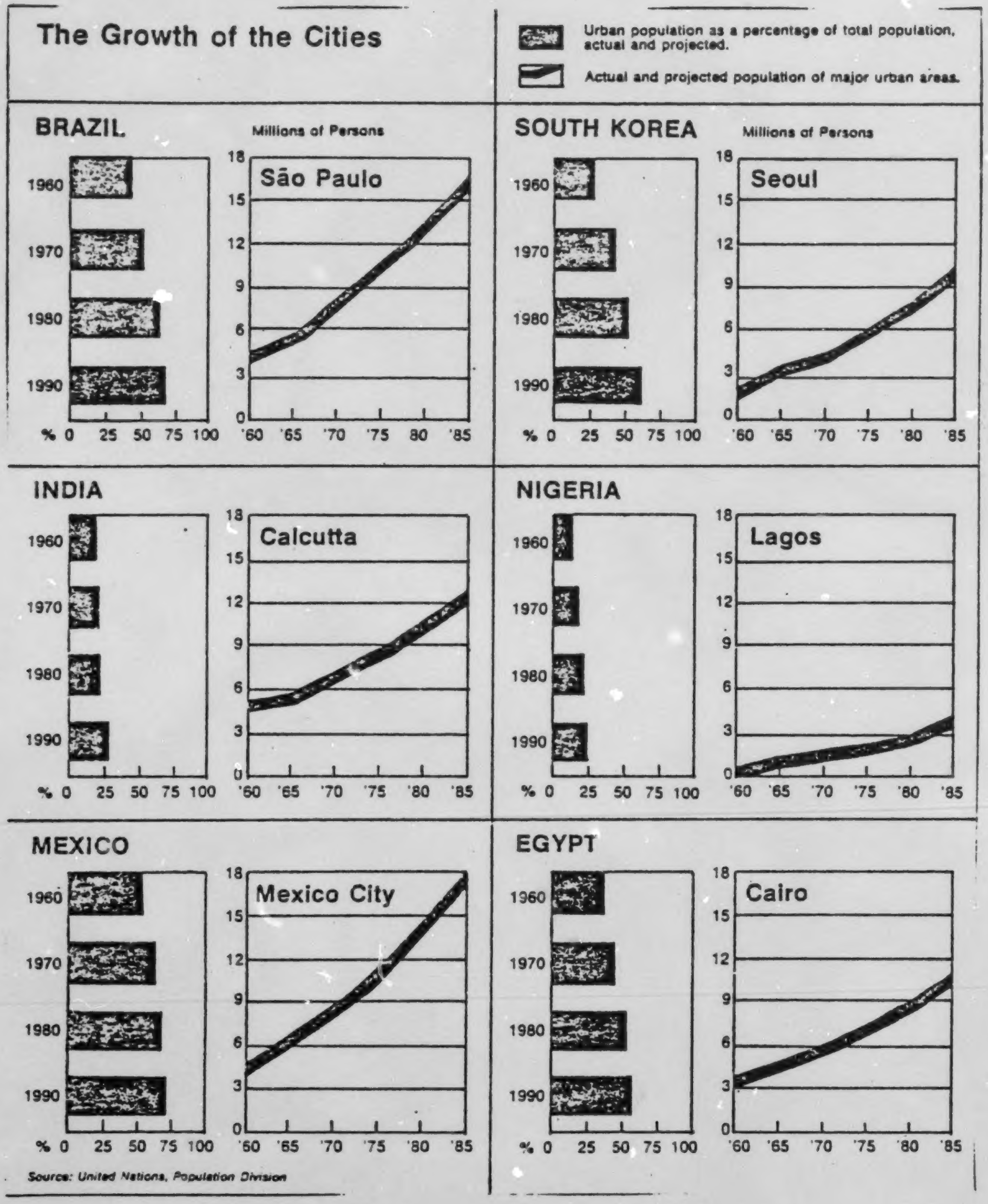
5. Income Distribution[43]

a) Mexico, 1969	% of national income	b) Ecuador, 1970	% of national income
1) richest 20%	64%	richest 20%	73.5%
2) 2nd 20%	16%	2nd 20%	14.5%
3) 3rd 20%	9.5%	3rd 20%	5.6%
4) 4th 20%	6.5%	4th 20%	3.9%
5) poorest 20%	4.0%	poorest 20%	2.5%

6. World crude petroleum production and consumption, 1976[44] (show production and consumption bars for each country next to each other).

Country	Production (million barrels/day)	Consumption (million barrels/day)
U. S.	9.7	17.3
Canada	1.6	1.8
Western Europe	1.0	14.0
Far East & Oceania	2.6	8.3
Latin America	4.6	3.8
Middle East	22.0	1.8
Africa	5.9	1.0

7. Figure 4.27 illustrates the tremendous growth of cities in Third World countries.[45] Compute approximate percent increases in population for: a) Brazil and Sao Paulo, b) India and Calcutta, c) South Korea and Seoul, d) Mexico and Mexico City

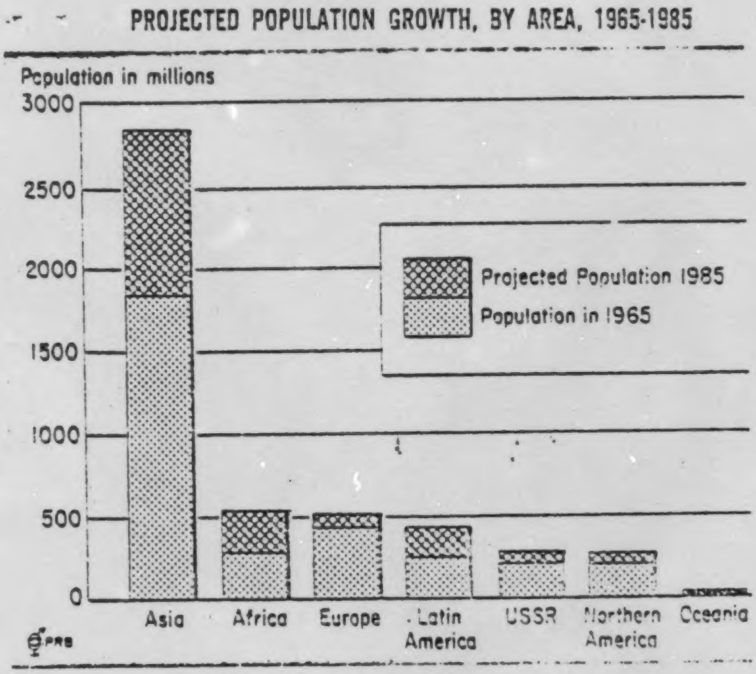


Source: United Nations, Population Division

Fig. 4-27

46

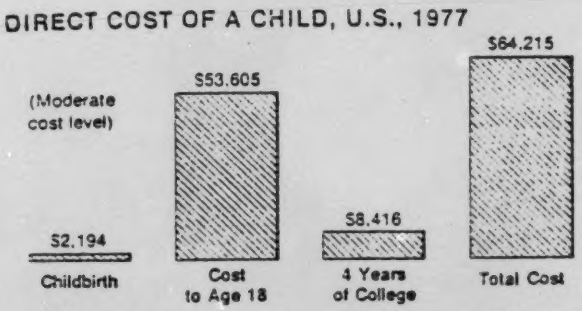
8. Use the bar charts shown below to estimate projected absolute and relative (percent) increases in population from 1965 to 1985 for the various regions of the world shown.


PROJECTED POPULATION GROWTH, BY AREA, 1965-1985

Source: Population Reference Bureau, Inc. *1970 World Population Data Sheet.* April 1970, Man's Population Predicament,

Fig. 4.28

9. According to the bar charts below, what percent of the total cost of raising a child in 1977 is needed for ; a) childbirth; b) raising the child to age 18; c) 4 years of college?

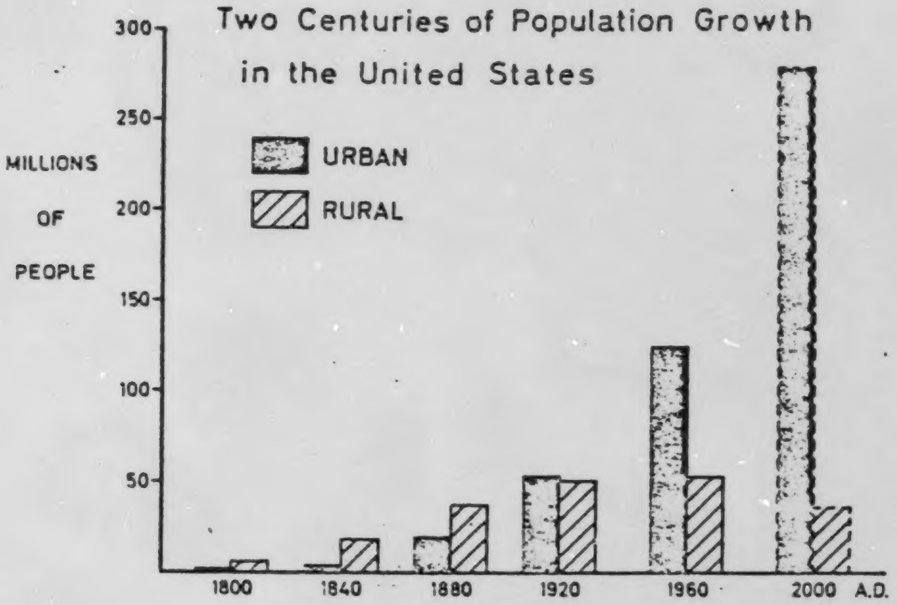

DIRECT COST OF A CHILD, U.S., 1977

Source: Thomas J. Espenshade, "The Value and Cost of Children", *Population Bulletin* (April 1977), Table 15 : 14.

Fig. 4.29

10. Using the bar charts below, estimate the ratio of urban to rural population in the United States in:

a) 1840; b) 1880; c) 1920; d) 1960; e) 2000.



Source of data - U.S. BUREAU OF THE CENSUS, *Reprinted in the New Republic, Jan. 10, 1970*

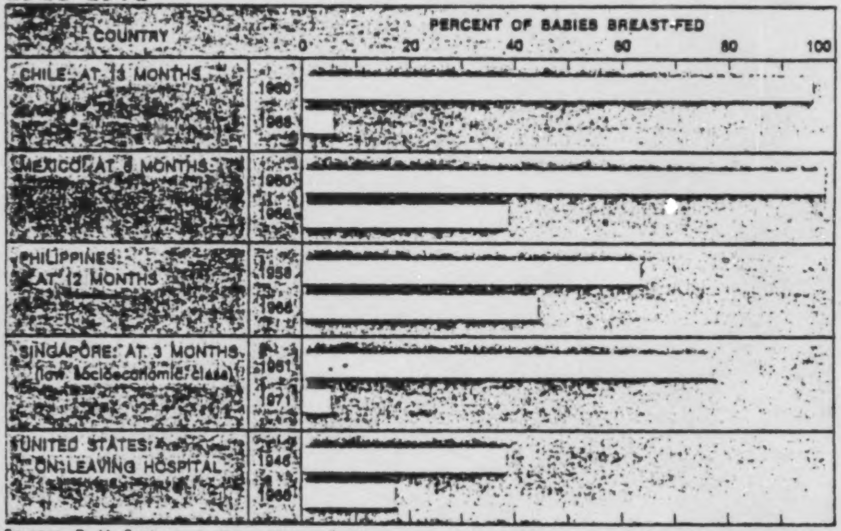
Figure 4-30

11. Figure 4.31 shows the tremendous decrease in breast feeding in selected countries and years.[48]

a) What was the percent decrease from 1960 to 1968 in babies breast fed in Chile?

b) What is the ratio of the babies breast fed in Mexico at 6 months in 1966 compared to those breast fed under these conditions in 1960?

Extent of Breast Feeding in Selected Countries and Years, 1946–1971



Sources: D. M. Sanjur and others, "Infant Feeding and Weaning Practices in a Rural Pre-industrial Setting," *Acta Paediatrica Scandinavica*, Supplement 200 (1970), p. 15; M. L. Gabucan-Dulay, "Current Feeding Patterns as Observed among 1,000 Filipino Infants," *Philippine Journal of Pediatrics*, vol. 19 (April 1970), p. 101; F. B. Mönkeberg, "The Effect of Malnutrition and Environment on Mental Development," *Proceedings of Western Hemisphere Nutrition Congress II, San Juan, Puerto Rico, August 1968* (Chicago: American Medical Association, 1969), pp. 216–20; Wong Hock Boon, *Breastfeeding in Singapore* (Singapore: J. L. Morison Son and Jones [F. E.] Pte. 1971); and Herman F. Meyer, "Breast Feeding in the United States," *Clinical Pediatrics*, vol. 7 (December 1968), p. 709.

Fig. 4.31

12. Refer to figure 4.32.

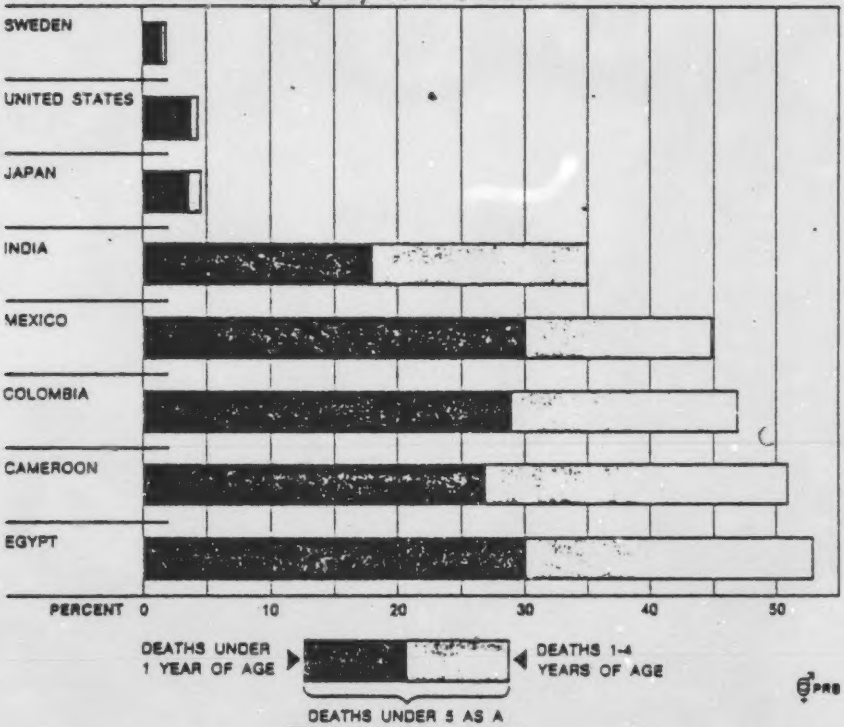
a) Of all deaths in Egypt (around 1971), what percent were children under 1 year of age? What percent were children aged 1 - 4? What percent were under 5 years of age?

b) Repeat part (a) for Mexico

c) What is the ratio of the percent of all deaths to children under 5 occurring in Egypt compared to the percent of such deaths occuring in the U.S.?

d) Repeat part (c) in comparing Mexico and Sweden.

. Deaths to Infants Under One Year of Age and to Children Under Five as a Percentage of Total Deaths



Source: United Nations, *Demographic Yearbook, 1971* (New York, 1972).

Fig. 4.33

13. Refer to the bar charts in figure 4.33.

a) What was the decrease in farm population from 1940 to 1977? What percent decrease does this represent?

b) What was the decrease in the number of farms from 1940 to 1977? What percent decrease does this represent?

c) What was the increase in the average farm size in acres from 1940 to 1977? What percent increase does this represent?

d) Compute ratios of 1940 values to 1977 values for parts (a), (b), (c), above.

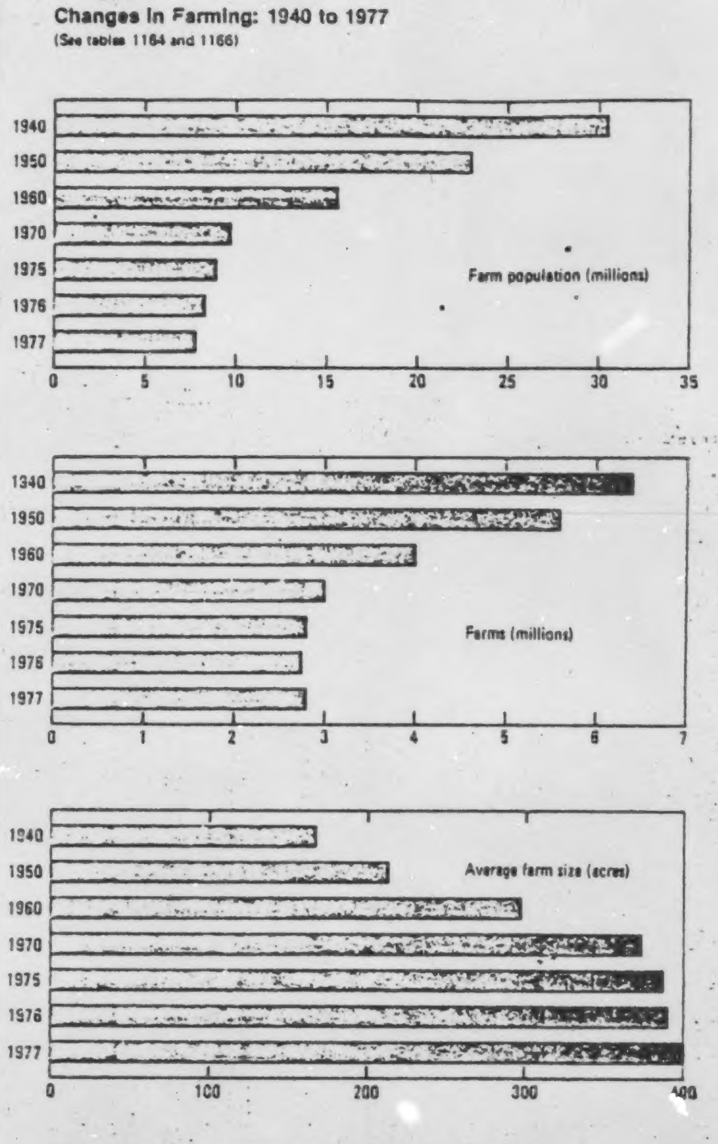


Fig. 4.33a

Source: Chart prepared by U.S. Bureau of the Census. Data from U.S. Department of Agriculture.

4.4 Histograms

Another valuable way of graphically representing data is by means of a histogram. It is very much like a bar chart but there is no space left between bars. Histograms are valuable for continuous (non discrete) variables and problems where areas under the curve are required.

The following example illustrates how to draw a histogram.

Example 4.4

An ecologist has developed an instrument to indicate average air pollution readings. Assume that the following are her readings in an urban area for 50 consecutive days.

56.9	46.2	47.8	46.7	38.6
41.6	55.7	51.9	38.9	54.2
67.2	47.3	29.6	33.2	67.8
69.4	31.6	39.2	53.6	60.5
30.8	58.3	52.0	48.3	45.6
29.4	65.3	45.8	59.7	50.7
81.2	41.7	59.2	44.2	74.1
83.3	80.3	39.6	73.2	50.1
67.5	69.7	70.8	75.6	46.2
46.4	40.8	78.2	64.2	45.9

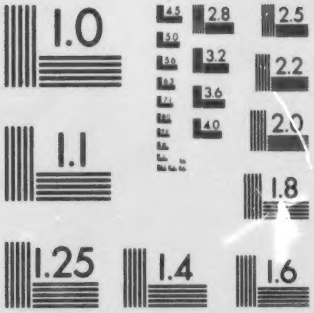
Table 4.4

We wish to organize the data and draw a histogram.

Solution

To make the measurements easier to interpret, we first rewrite them in numerical order, by rows:

25


1.0
4.5
2.8
2.5
5.0
5.6
3.2
2.2
6.3
7.1
3.6
8.0
2.0
4.0
1.1
1.8
1.25
1.4
1.6

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

29.4	29.6	30.8	31.6	33.2
38.6	38.9	39.2	39.6	40.8
41.6	41.7	44.2	45.6	45.8
45.9	46.2	46.2	46.4	46.7
47.3	47.8	48.3	50.1	50.7
51.9	52.0	53.6	54.2	55.7
56.9	58.3	59.2	59.7	60.5
64.2	65.3	67.2	67.5	67.8
69.4	69.7	70.8	73.2	74.1
75.6	78.2	80.3	81.2	83.3

Table 45

We next wish to group the data. To do this, we first compute the range.

Range = highest measurement - lowest measurement = 83.3 - 29.4 = 53.9 units.

For a histogram, we generally want 10 to 15 intervals. If the range is small or there is a small amount of data, a smaller number of intervals may be used.

Using 10 intervals for a preliminary calculation, we get

$$\text{width of interval} = \frac{53.9}{10} = 5.39$$

We will use a width of interval equal to 5 units; the interval should be an integral number of units, and 5 unit intervals will give us 11 intervals.

Because the lowest air pollution measurement is 29.4, we will start the histogram with a lower boundary of 29. Since any measurement between 28.5 and 29.5 is considered in the 29 category, our real lower boundary will be 28.5. Since the width of each interval is 5, the 1st interval will go from 29 to 33 and have real boundaries of 28.5 and 33.5. In the same way, the 2nd interval will go from 34 to 38, and have real boundaries of 33.5 and 38.5. Results are tabulated below for use in drawing the histogram. Note that there are 7 measurements in the interval from 39-43 (real boundaries 38.5 to 43.5). These 7 measurements are 38.6, 38.9, 39.2, 39.6, 40.8, 41.6, and 41.7.

Interval	Real Boundaries	Frequency
29-33	28.5-33.5	5
34-38	33.5-38.5	0
39-43	38.5-43.5	7
44-48	43.5-48.5	11
49-53	48.5-53.5	4
54-58	53.5-58.5	5
59-63	58.5-63.5	3
64-68	63.5-68.5	5
69-73	68.5-73.5	4
74-78	73.5-78.5	3
79-83	78.5-83.5	3

Table 4.6

Next we draw the histogram as shown below.

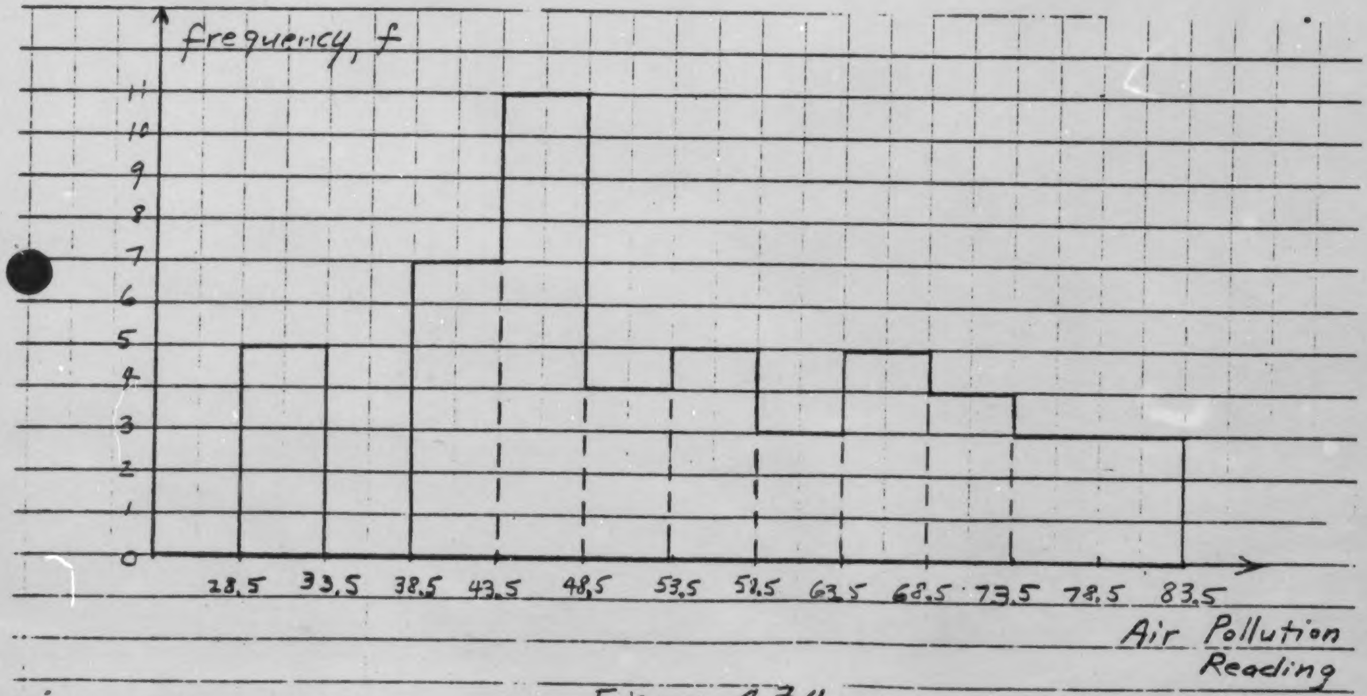


Figure 4.34

We have shown the real boundaries for each interval. Sometimes histograms are drawn using the integral values (28-33, 34-39, etc.).

Note that this is not the only correct histogram that could have been drawn. If we had used an interval of 4 units, we could have obtained another histogram with 14 intervals which is still within our specified range of 10 to 15 intervals.

Example 4.5

Using values from the 1979 World Population Data Sheet, draw a histogram for the Physical Quality of Life Index (PQLI). Use about 10 intervals.

Solution

The PQLI varies from 14 (for Niger, Mali, and Guinea-Bissau) to 96 (for the Netherlands). It is convenient to take our intervals as 10-19, 20-29, 30-39, 40-49, 50-59, 60-69, 70-79, 80-89, and 90-99. This gives 9 intervals, which is adequate. The real boundaries for the interval 10-19 are 9.5 to 19.5; for 20-29, they are 19.5 to 29.5, etc.

Results are tabulated below.

Interval	Real boundaries	Tally	Frequency
10-19	9.5-19.5	~~1111~~ ~~1111~~	10
20-29	19.5-29.5	~~1111~~ ~~1111~~ ~~1111~~ ~~1111~~ 1	21
30-39	29.5-39.5	~~1111~~ ~~1111~~ ~~1111~~ 1	16
40-49	39.5-49.5	~~1111~~ ~~1111~~ ~~1111~~ 11	17
50-59	49.5-59.5	~~1111~~ ~~1111~~ 1	11
60-69	59.5-69.5	~~1111~~ 1	6
70-79	69.5-79.5	~~1111~~ ~~1111~~ ~~1111~~ ~~1111~~	20
80-89	79.5-89.5	~~1111~~ ~~1111~~ ~~1111~~ ~~1111~~ 1111	24
90-99	89.5-99.5	~~1111~~ ~~1111~~ ~~1111~~ ~~1111~~ ~~1111~~ ~~1111~~	30

Table 4.7

The tally column is for counting. Since the 1st 4 countries on the population data sheet have values of PQLI between 40 and 49, we put 4 vertical lines in the 40-49 interval. Next comes Sudan with a PQLI of 34; we put a vertical line in the 30-39 interval. Since Tunesia is the 5th country to have a PQLI between 40 and 49, we put a horizontal line across the 4 vertical lines indicating that so far we have 5 countries with PQLI's between 40 and 49. By continuing this process, we obtain the frequencies shown in the table. Using these frequencies, we draw the histogram shown below (Figure 4.35).

The histogram shows again the great gaps in the quality of life in various parts of the world. Note that a total of 64 countries (10 + 21 + 16 + 17) have a PQLI below 49.5 while 74 countries (20 + 24 + 30) have a PQLI above 69.5.

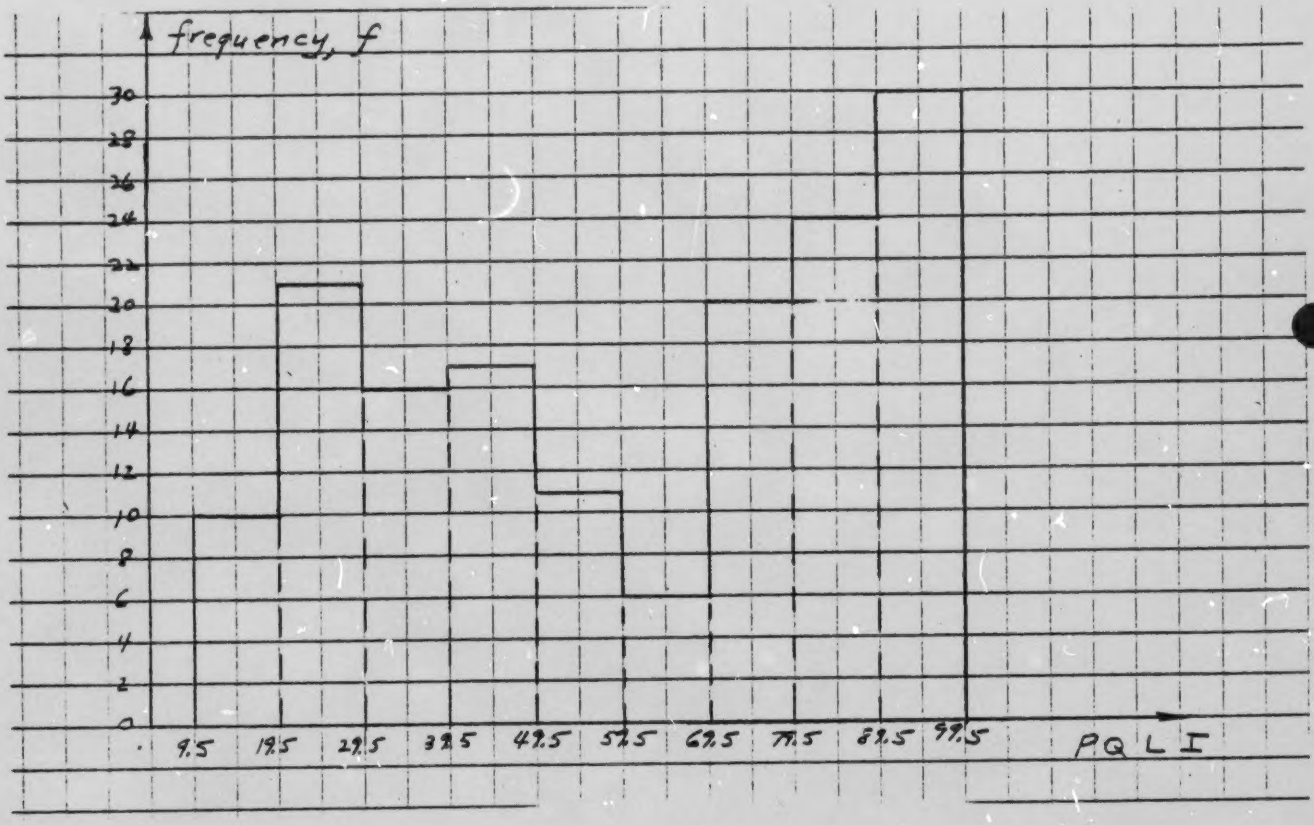
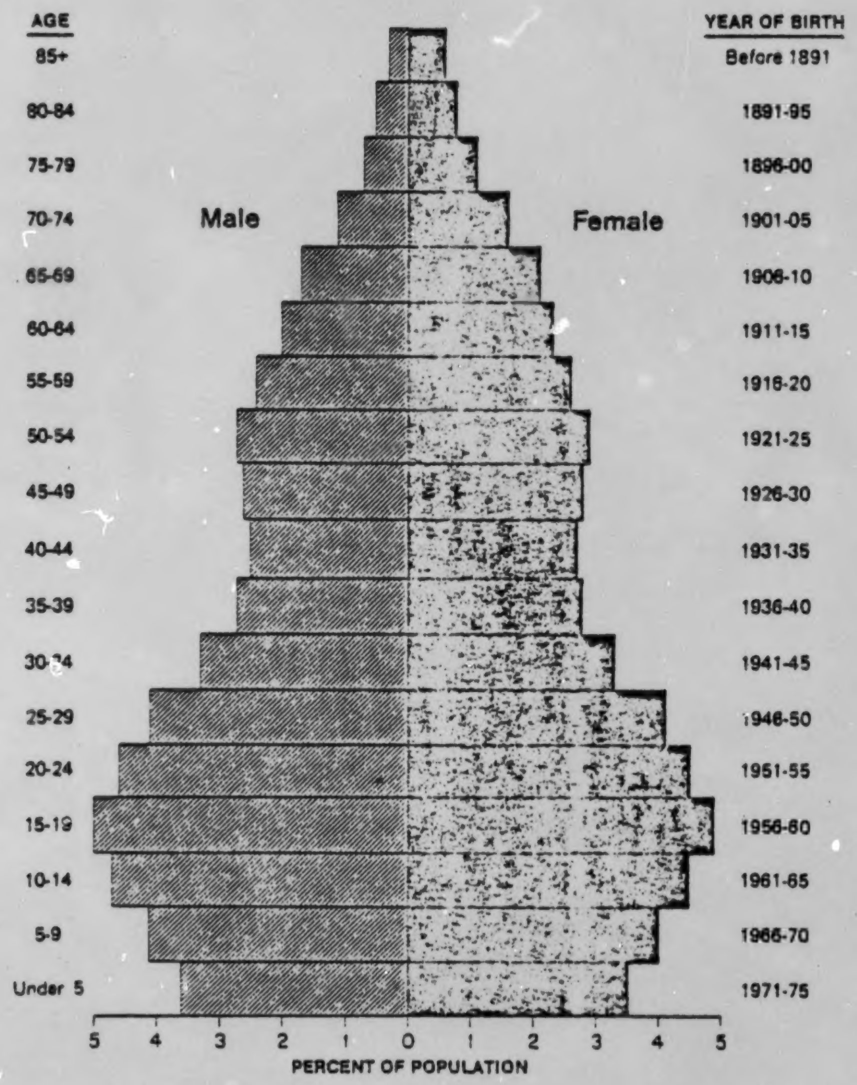


Fig. 4.35

An important example of a histogram is the population pyramid. It has significance in considering future population growth, as discussed in chapter 3. The population pyramid for the United States is shown below.[51] It is really two histograms; one for males on the left side, and one for females on the right side. Note that the interval "under 5" has a real upper boundary of 5 (not 4.5 as in most histograms), the interval 5-9 has real boundaries of 5 and 10, and so on. In other words a person is not considered 10, for example, after the age of 9.5, but only when his or her 10th birthday is reached.



Population Pyramid of the United States on January 1, 1976

Source: U.S. Bureau of the Census.

Fig. 4.36

Much valuable information can be readily observed from the population pyramid. The U. S. birth rate has been decreasing after the 1956-60 period. There was a "baby boom" after World War II (note intervals 1946-50, 1951-55, and 1956-60). The percent of the population that is female is noticeably greater than that which is male for all intervals equal to and above the 65-69 age interval.

Example 4.6

Using figure 4.36, calculate the percent of the U. S. population that was below 15 years of age in 1976.

Solution

We need the percentages for both males and females for the intervals under 5, 5-9, and 10-14. Recall that the true upper boundary for the 10-14 interval is 15; any one 15 years old or above would be placed in the 15-19 interval.

We read off the percentages from the population pyramid as accurately as we can, as follows

	Age	Percent
Male,	under 5	3.6
	5-9	4.1
	10-14	4.7
Female,	under 5	3.5
	5-9	4.0
	10-14	4.5
		24.4

Table 4.8

About 24.4% of the U. S. population was below 15 years of age in 1976. This agrees closely with the 24% indicated for the U. S. population below 15 years of age on the 1978 World Population Data Sheet.

A frequency polygon is another valuable type of graph. It is obtained by simply connecting the midpoints of the tops of each bar of the histogram.

The frequency polygon for the data and histogram of example 4.4 in this section is shown below. The frequency polygon connects points a-b-c-d-e-f-g-h-i-j-K.

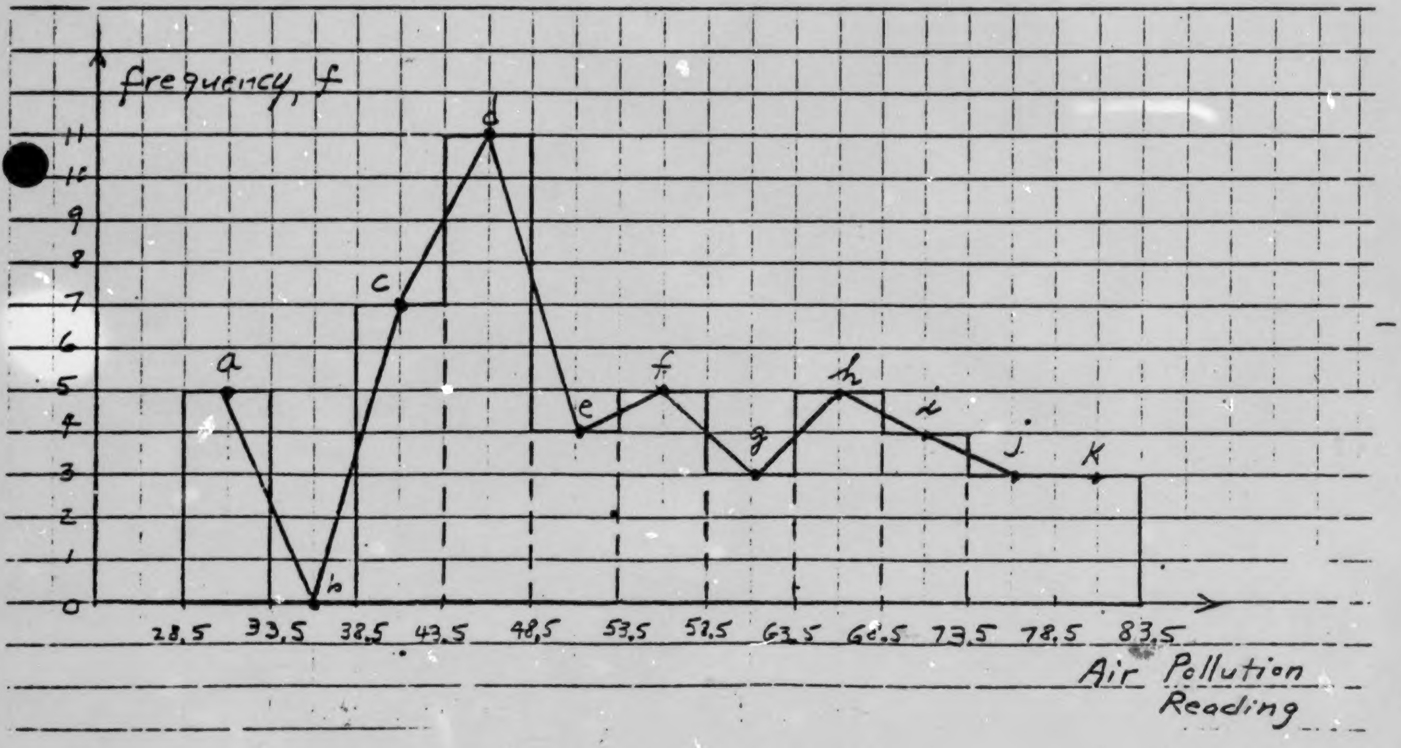


Fig. 4.37

The frequency polygon for the PQLI histogram is shown below (fig. 4.38), points a through i.

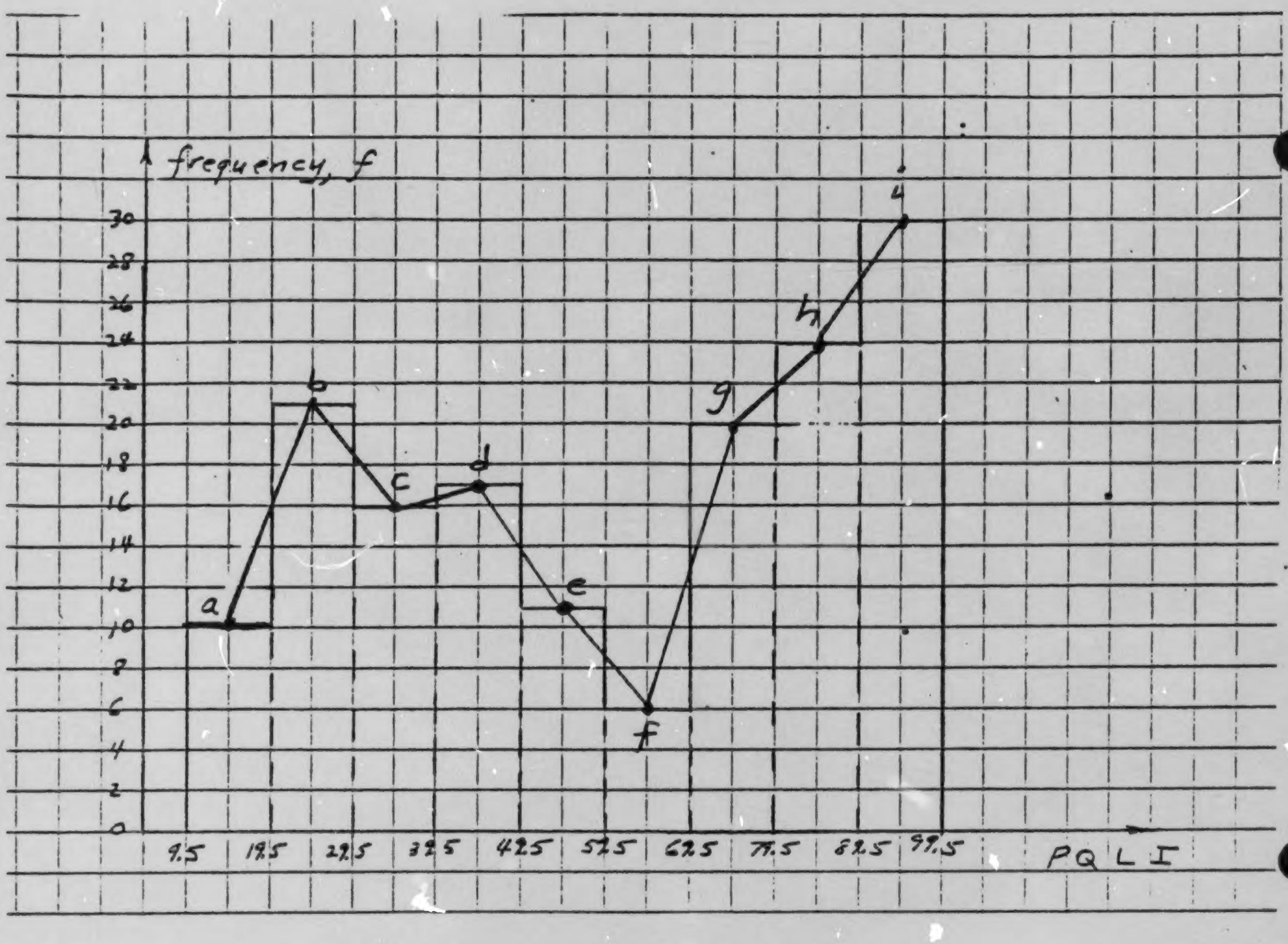


Fig. 4.38

Exercises

1. Using the Population Reference Bureau World Population Data Sheet, draw histograms with about ten to fifteen intervals, and frequency polygons for each of the following variables. (Consider countries only.):

a. mid-1979 population estimate

b. birth rate

c. death rate

d. rate of natural increase

e. number of years to double population

f. infant mortality rate

g. population under 15 years (percent)

h. population over 64 years (percent)

i. life expectancy at birth (years)

j. urban population (percent)

k. per capita gross national product (US$)

2. Assume that the following values represent the number of thousands of tons of garbage (rounded to the nearest thousand tons) collected on 50 consecutive working days. Draw a histogram and a frequency polygon. Use between 10 to 15 intervals.

20	26	26	22	29
27	23	25	19	25
23	27	19	31	20
27	21	20	25	23
20	22	28	29	23
23	30	17	22	19
29	24	21	27	28
31	28	20	28	25
18	26	22	23	25
20	25	23	20	30

3. The following twelve numbers represent the percentages of population under 15 years of age in 12 distinct regions of Asia, Africa and Latin America: 45, 44, 44, 42, 40, 43, 43, 44, 36, 46, 40, 43. Using 5 intervals, draw a histogram and a frequency polygon.

4. For the age-sex pyramids shown below,[52] compute for both Mexico (a rapid growth country) and Sweden (a no growth country)

a) percent of population under 15 years of age;

b) percent of population above 64 years of age;

c) the dependency ratio.

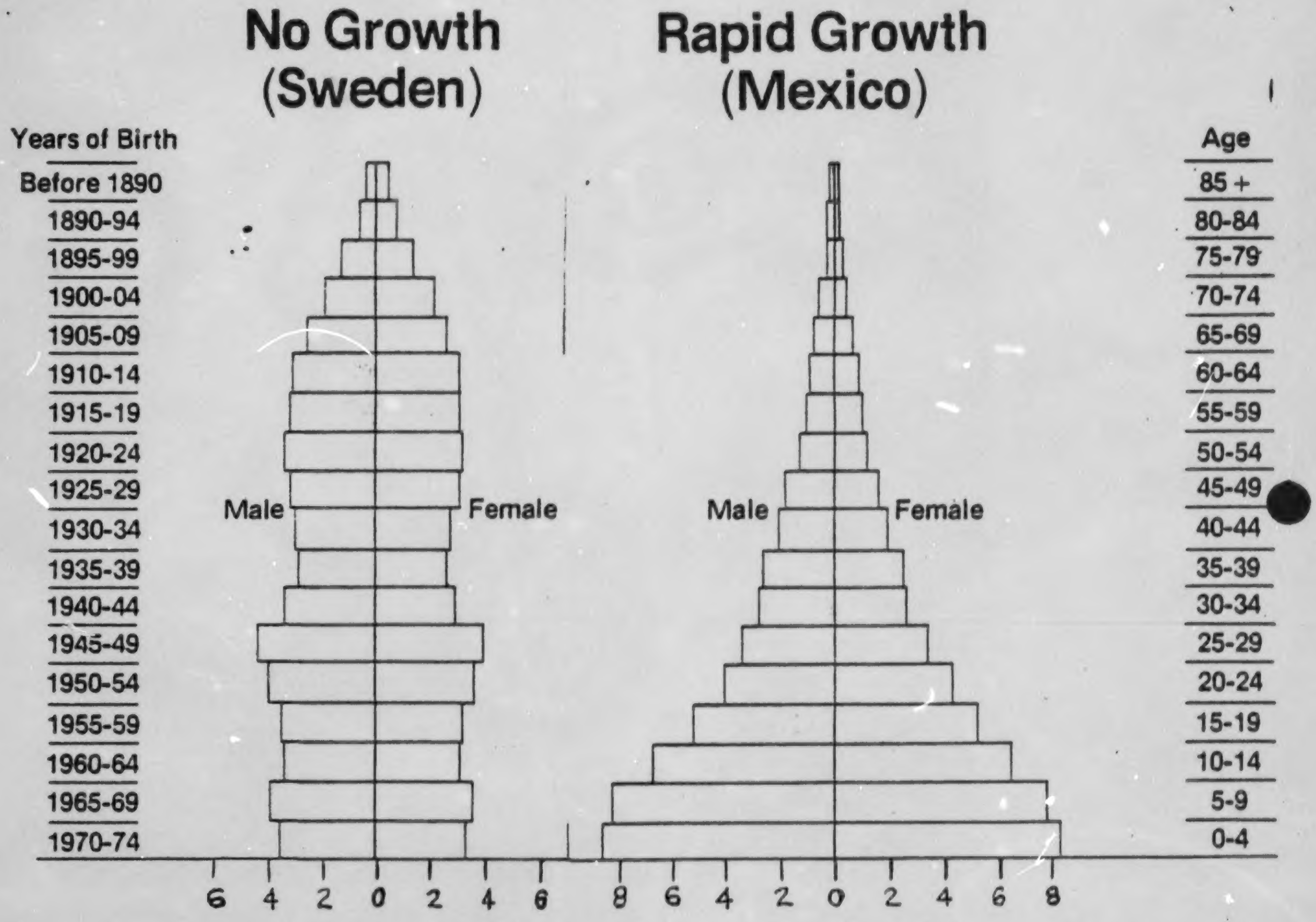


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Fig. 4.39

5. Draw a histogram illustrating the age distribution in the U. S. in 1970,[53] using the data shown below:

Age range	U. S. Population (in thousands)
under 5	17,154
5-9	19,956
10-14	20,789
15-19	19,070
20-24	16,371
25-29	13,477
30-34	11,430
35-39	11,106
40-44	11,981
45-49	12,116
50-54	11,104
55-59	9,973
60-64	8,617
over 65	20,066

Notes

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Chapter 5 Sequences

A British parson, Thomas Robert Malthus (1766-1834) stated that the food supply or means of subsistence tends to increse as an arithmetic sequence and the population tends to increase as a geometric sequence. This is one of the earliest statements relating food supply and population growth and has since been the focus of much controversy.

In this chapter we will define sequences, consider arithmetic and geometric sequences, and relate these to several issues related to human survival.

A sequence is a group of terms that follows a fixed pattern or law. We should be able to tell the next terms from the preceding ones. For example, given 1, 4, 7, 10 we see that we are starting from 1 and that the pattern seems to be that each term is 3 more that the preceding term. The next terms are easily obtained by just continuing the pattern, that is, continuing to add 3 to give 13, 16, 19, ... The three dots mean etc., or and so on, and indicate that the sequence continues without a limit.

Example 5.1

Find the general law and write the next three terms for the following sequences:

a. 1, 4, 9, 16

b. 1, 1, 2, 3, 5, 8, 13

a. We note that the differences between the terms keep increasing by 2 $(4 - 1 = 3,\ 9 - 4 = 5,\ 16 - 9 = 7)$, hence we add 9, 11, and 13 to get the next 3 terms; 25, 36, and 49.

Further note that each value of the sequence is a square number.

$$1 = 1^2 \quad 4 = 2^2 \quad 9 = 3^2 \quad 16 = 4^2, \text{ etc.}$$

Hence the general rule is that each term is a square; the tenth term, for example, would be 10^2, or 100. Note that the 3 terms that we found above check: $25 = 5^2$, $36 = 6^2$, and $49 = 7^2$

b. In the sequence 1, 1, 2, 3, 5, 8, 13 note that the 1st two terms add up to the third term, the 2nd and 3rd terms add up to the 4th term, etc. Hence the general rule is that any term (after the first two) is the sum of the two previous terms. Using this rule, the next three terms of the sequence would be:

8 + 13 = 21

13 + 21 = 34

21 + 34 = 55

This sequence is known as the Fibonacci sequence, after Leonardo Fibonacci who discovered it.

Arithmetic sequences obey the general rule that you add a constant to get the next term. For example, 3, 8, 13, 18 is an arithmetic sequence. We start with 3 and add the constant 5. The next terms would be 23, 28, 33, etc.

Is 10, 9, 8, 7, 6, ... an arithmetic sequence? Are we adding a constant to get the next term in each case? Recall the rule that subtracting is equivalent to adding a negative value.

At this point it is instructive for you to make up some arithmetic sequences. Try them out on other students and have them make up some that you can check out too.

Geometric sequences obey the general rule that you multiply by a constant to get the next term. For example, 2, 6, 18, 54 would be an example of a geometric sequence. Here we start with 2 and multiply by 3 to get the following terms. The next terms after 54 would be 3 x 54 = 162, 3 x 162 = 486, etc.

Would 1, 1/2, 1/4, 1/8, ... be a geometric sequence? If so, what are we multiplying by to get the next terms? Recall the concept that dividing by 2 and multiplying by 1/2 give the same result.

Is 1, 1, 1, 1, ... a sequence? Can we predict the next terms? Is it arithmetic?

If so, what do we add to get the next term? Is it geometric? If so, what do we multiply by to get the next term? What do you conclude?

Again, it is instructive for you to make up several examples of geometric sequences and try them out on other students.

A special case of the geometric sequence is the binary sequence. Here the term that we multiply by is 2. An example of the binary sequence is

1, 2, 4, 8, 16, 32, ...

This is a very important sequence because many important factors relating to global survival double in relatively uniform periods of time.

Let us now refer back to the statement of Malthus and relate it to what we have learned. Let us assume that the food supply starts with 20 units and increases by 50 units in a given time (an arithmetic sequence). We'll also assume that the population starts at ten units and doubles in the same time period (a geometric (binary) sequence). We get the results below.

Time period	0	1	2	3	4	5	6	7
Food supply	20	70	120	170	220	270	320	370
Population	10	20	40	80	160	320	640	1280

Note that although the food supply starts ahead of the population and initially increases its lead, it is eventually overtaken and passed by increasingly greater amounts. This is the nature of rapid geometric growth compared to arithmetic growth.

Was Malthus right? Only partly. In terms of population growth he was, if anything, conservative in that population, as we indicated in more detail elsewhere, has been growing faster than a geometric sequence; it has been doubling in shorter and shorter periods of time. However, he did not foresee the tremendous technological strides that would enable very rapid growth of agricultural productivity that has enabled food production to keep up with population. However, at least half the world's people are undernourished or malnourished today and the question is how long can agricultural production continue to increase to meet increasing demands without badly damaging the soil and water on which it depends.

A concept that is closely related to binary sequences is that of doubling time. Doubling time is the number of years for some quantity to double. This has been discussed in Chapter 3 in terms of doubling times for population. Many factors relating to human survival have been doubling in relatively fixed periods of time. For example,

Quantity	Doubling time
solid waste	14 years
noise pollution (cities)	10 years
electrical energy demand	10 years (until 1973)

The 1979 World Population Data Sheet gives the doubling time for the countries of the world. Recall that the doubling time can be approximated using the equation: doubling time = 70 ÷ percent increase.

The importance of doubling time is illustrated in the following riddle which is given in The Limits to Growth by D. Meadows and others.

> Suppose you own a pond on which a water lily is growing. The lily plant doubles in size each day. If the lily were allowed to grow unchecked, it would completely cover the pond in 30 days, choking off the other forms of life in the water. For a long time the lily plant seems small and so you decide not to worry about cutting it back until it covers half the pond. On what day will that be?

If the pond is completely covered with the water lilies after 30 days, it must be half covered one day before that, or after 29 days. You thus have only one day to save the pond. This riddle shows very dramatically the effects of geometric growth with regard to reaching a limit. And many things related to global survival are growing geometrically. Are we in "the 29th day" with regard to air pollution or the pollution of the oceans? If so, we have very little time to make major changes. Lester Brown has called his latest book *The Twenty-Ninth Day* to indicate that the last quarter of the 20th Century is, in effect, our last day to save our pond.

Note the application of this concept to population growth. It took all of the world's history ("29 days") for the world's population to reach 4 billion people in 1976. Projections are that that number will be doubled by about 2016. Again, as much of a change in the last time period as in *all* the time periods before it.

The difference between arithmetic and geometric growth is related to the difference between simple and compound interest.

For simple interest, a constant rate is applied to an *unchanged* principal (starting amount). For example, suppose we had a principal of $100 and a simple interest rate of 7% per year. Our money would build up as shown below:

1st year	$100.00	(our original principal)
	x0.07	(7% simple interest rate)
	$ 7.00	(interest for the 1st year)
	$100.00	(add the original principal)
	$107.00	(our total after 1 year)
2nd year	$100.00	(with simple interest, we use our original principal again)
	x0.07	(7% simple interest rate)
	$ 7.00	(interest for the second year)
	$107.00	(add the total after 1 year)
	$114.00	(our total after the 2nd year)

Note that the increase is constant, $7.00 per year. After 10 years, the total gain would be 10 x $7.00 = $70.00. This is an example of arithmetic growth. It is shown graphically in Figure 5.1. With 7% simple interest, it would take 100/7 = over 14 years for our money to double.

When interest is compounded, the constant rate is applied to the sum of the principal *and* the accumulated interest, at each of the designated time intervals.

Let us assume again that we have a principal of $100 and an interest rate of 7% per year, but this time we have compound interest. This time our money would increase as follows:

1st year	$100.00	(our original principal)
	x0.07	(7% (compound) interest rate)
	$ 7.00	(interest for the 1st year)
	+ $100.00	(add the original principal)
	$107.00	(our total after 1 year)

Note that after the 1st time period, simple and compound interest give the same result.

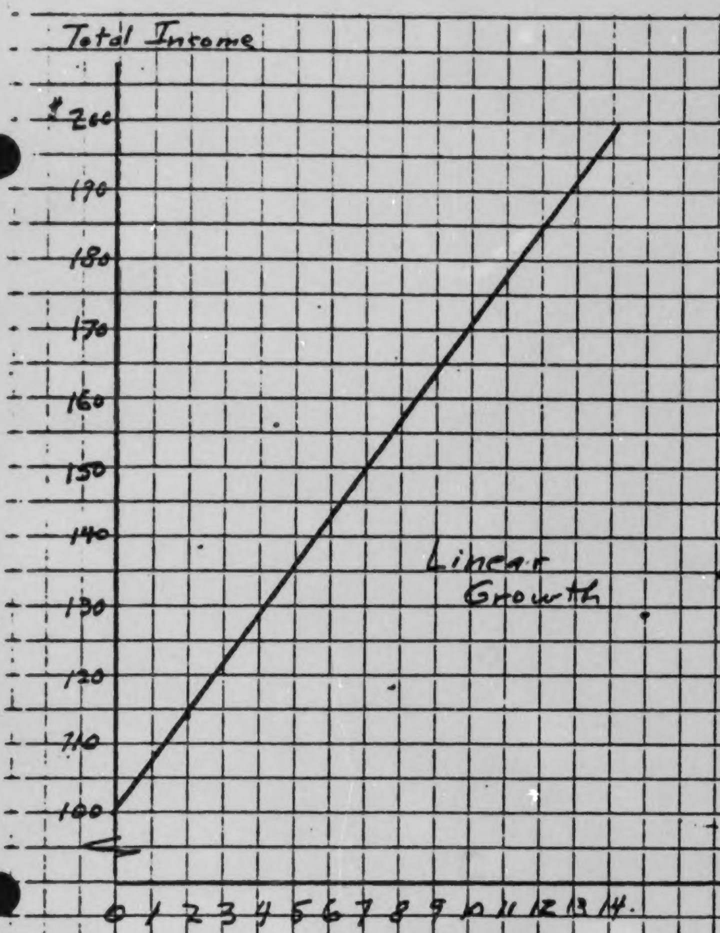
2nd year	$107.00	(for compound interest, we use the original principal + accumulated interest)
	x0.07	(7% interest rate)
	$ 7.49	(interest for the 2nd year; note that it is already $0.49 more than the 2nd year interest for simple interest)
	+ $107.00	(add the total after 1 year)
	$114.49	(the total after the 2nd year)

Using a hand calculator, we get the following further results:

	Total	Annual increment
end of 3rd year	$122.50 ($114.49 x 1.07)	$8.01
4th year	$131.08 ($122.50 x 1.07)	$8.58
5th year	$140.26	$9.18
6th year	$150.08	$9.82
7th year	$160.59	$10.51
8th year	$171.83	$11.24
9th year	$183.86	$12.03
10th year	$196.73	$12.87

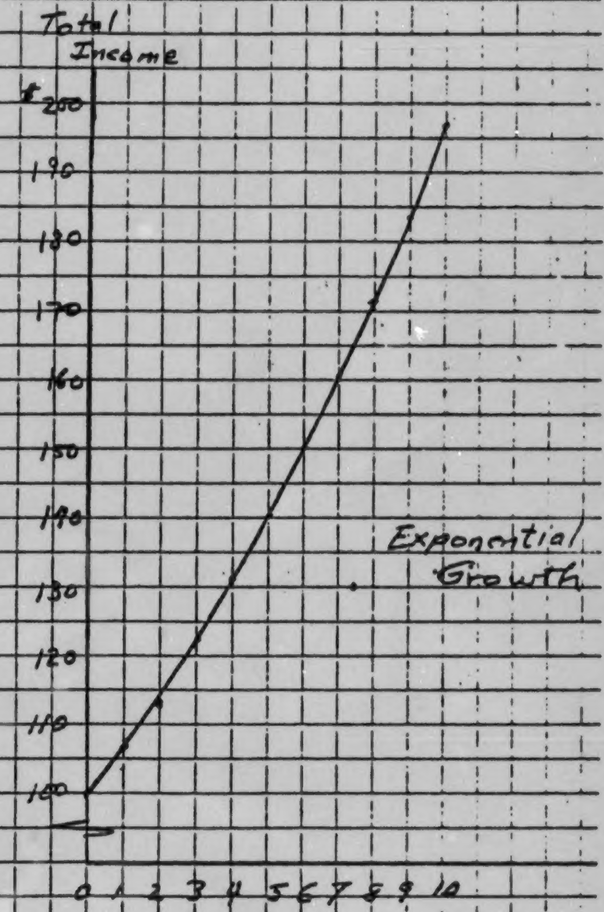
Observe that the annual increments keep increasing, unlike the case for simple interest where each annual increment is constant ($7.00, in our example). Note that our money has nearly doubled at the end of 10 years. This checks the previously discussed equation: 70/growth rate (%) = 70/7 = 10 years. Note that this is 4 years less than the doubling time when we have simple interest.

This growth is geometric because we multiply the previous result by a constant (1.07). This kind of growth is pictured in Fig. 5.2.



Growth of $100 at 7% simple interest

Fig. 5.1



Growth of $100 at 7% compound interest

Fig. 5.2

Arithmetic growth is often called linear (straight line) growth (see Fig. 5.1). Geometric growth is frequently denoted as exponential growth. Its graph is generally called a J curve (see Fig. 5.2). The graphs of world population growth and energy consumption versus time have this shape. Note the much more rapid growth of the exponential curve compared to the linear curve. This difference in growth becomes even greater as time goes on.

An important difference between two geometric sequences is shown in Fig. 5.3.

The Two-or Three-Child Family

Average family size makes a difference for the family and for society if continued over several generations.

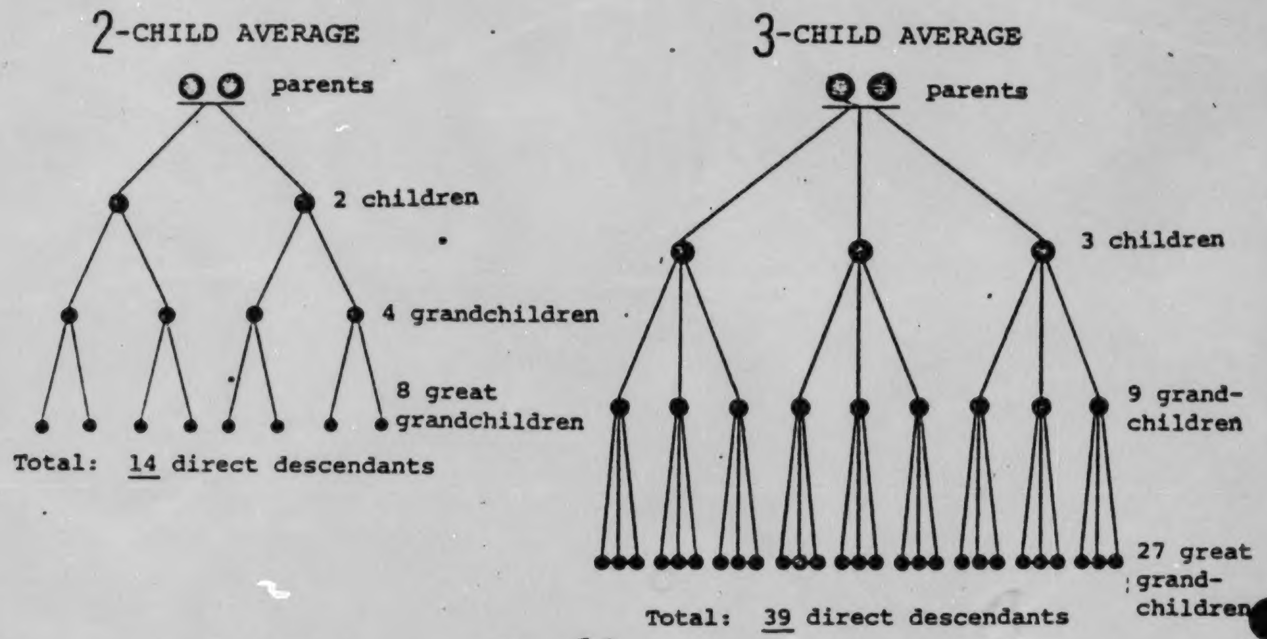

2-CHILD AVERAGE
parents
2 children
4 grandchildren
8 great grandchildren
Total: 14 direct descendants
3-CHILD AVERAGE
parents
3 children
9 grandchildren
27 great grandchildren
Total: 39 direct descendants

Fig. 5.3

Note how much faster the 3-child average family builds up compared to the 2-child average family (39 direct descendants compared to 14 direct descendants). In Chapter 4 there are graphs showing how U. S. population would grow under conditions of (a) 2 children per couple and (b) 3 children per couple.

Another important application of geometric sequences is that of the decay of radioactive materials. The decay is expressed in terms of the half-life, or the time for one half of the radioactive material to decay. For example, consider the table below which indicates the decay of a radioactive material with a half-life of 500 years (at the end of every 500 year period there is half of what was left at the start of the period).

	Material left	Years from now
1)	1 unit	0
2)	1/2	500
3)	1/4	1000
4)	1/8	1500
5)	1/16	2000
6)		

Complete row 6. About how many years would it take from the start until there would be less than 1% of the original radioactive material left?

This type of analysis is very significant with regard to the storage of waste from nuclear power plants. Some types of radiation have very long half-lives. At present there is no safe method of storing the nuclear wastes for long periods of time.

One way to visualize the concept of half life is to consider the tossing of a large number of coins, say 20,000 coins. Whenever a coin lands on heads it is removed and the rest of the coins are tossed until all have landed on heads. On each toss, approximately half of the coins will land on heads; thus each toss will have about half as many coins as the previous toss. We can show the approximate results for the first 5 tosses as follows:

toss	coins
1	20,000
2	10,000
3	5,000
4	2,500
5	1,250

Exercises

The use of a hand calculator will be very helpful in solving many of the following problems. Some will be solved using logarithms in chapter 8.

1. Indicate for each of the following whether or not they are sequences. For the sequences, indicate which are arithmetic, which are geometric, and which are neither. Also, indicate the next two terms for the sequences.

   a. 1, 1/4, 1/16, 1/64 ...
   b. 1, 5, 10, 25, 50 ...
   c. 3, 8, 13, 18 ...
   d. 2, 2, 4, 6, 10, 16 ...
   e. 25, 20, 15, 10 ...
   f. 1, 3, 9, 27 ...

2. In the example of the water lilies and the pond, how much of the pond would be covered after the 28th day, the 27th day, the 23rd day? Would it have appeared safe for the owner of the pond to take a one-week vacation after the 23rd day? Why? What would have been the result if he had?

3. Make up a. 3 arithmetic sequences
   b. 3 geometric sequences
   c. 3 sequences that are neither arithmetic or geometric.

4. Electrical power plants in the U. S. now occupy about 12 square miles.[2] The total area of the U. S. is about 3 million square miles.[2] If the electrical energy demand continued to double every 10 years, as it has done in recent years (it has started to level off in the last few years), how long would it take for the entire U. S. to be covered by electrical power plants? Assume no changes in the size of the power plants.

5. Mexico's population has been doubling every 20 years. How many Mexicans will there be a century from now for every Mexican there today if its present population growth rate continues? Neglect migration effects. Can Mexico's

population continue to grow much longer at current rates? Why or why not?

6. What is the ratio of the U. S. population to that of Mexico today (use the population data sheet)? What will this ratio be in 80 years, if Mexico's population doubles every 20 years and our population doubles every 80 years?

7. Noting that we each have or had 2 parents, 4 grandparents, 8 great grandpatents, etc., how many ancestors did we have 20 generations back?

8. Assume that a page is about 1 millimeter (mm.) thick (about 1/254 of an inch). If the paper is folded, it becomes 2 mm; if folded again, it becomes 4 mm, etc. Assume that the paper could physically be folded many times. Check that 8 folds would make it about 1 inch thick. About how many folds would (theoretically) make its thickness reach from New York City to

a) Los Angeles, California (3000 miles away)

b) the moon (240,000 miles away)

c) the sun (93 million miles away)?

Note that 1 mile = 5,280 feet.

9. Here is another problem which demonstrates how rapidly numbers in a binary sequence increase. A father agrees to give his son an allowance of a penny on the first day of the month, 2 pennies for the second day, 4 pennies for the third day, and so on, doubling the amount each day. How much money would the son get on the 30th day of the month? How does this compare with the amount he would receive on the 30th day if he received a penny on the first day and a penny increase each day?

10. The present rate of production of electrical energy in the United States is only one-millionth of the rate of solar energy flow to the earth. If the rate of electrical energy production in the U. S. continues to double every ten years, how long would it take until the rate equals that of solar energy flow to the earth?

11. The level of pollution in a lake is doubling every 10 years. If the lake is now one-eighth polluted, in how many years will it be fully polluted?

12. Country A has a population or 200 million people which is doubling every 60 years. Country B has a population of 10 million people which is doubling every 20 years. At these rates, what will these populations be in 120 years from now? Estimate the number of years for country B's population to equal that of country A.

13. If we have a principal of $1,000, how much would we have after 1 year, 2 years, 3 years, 4 years, and 5 years, if we invested the money at a) 8% simple interest, and b) 8% compound interest? What is the approximate doubling time in each case?

14. Carbon 14 has a half-life of 5700 years. If archeologists find bones which have only about 1/16th of the original amount of carbon 14, about how old are the bones?

15. The half-life of uranium 238 is about 5.4 billion years. How long would it take for uranium 238 to decay to 1/8th of its present quantity?

16. A certain radioactive material has a half-life of 10 days. What fraction of the sample will remain after 30 days?

17. Suppose that a radioactive sample has a half-life of 10 years and that it now has a strength of 640 curies. If it can be safely disposed of when its strength has diminished to 10 curies, how many years will we have to wait? (Curies are units of radioactivity.)

18. A certain resource is being depleted exponentially at the rate of 5% per year. (This means that each year there is 95% of what there was the previous year.)

a) How long does it take to use up one-half of the amount available at a certain time?

b) If there were 250 million tons available in 1970, how much would you expect to be available in 1998?

Note

1. Adapted from Options, A Study Guide to Population and the American Future, Population Reference Bureau, 1973.
2. Joseph Priest, Problems of Our Physical Environment, Reading, Massachusetts, Addision-Wesley Publishing Co., 1973, p. 26.

Chapter 6 Population Growth Mathematics

The future growth of population for the world and for various regions and countries will be a most important factor with regard to global survival. In this chapter we will consider a mathematical model related to population growth.

In the real world there are many factors which must be considered with regard to population growth. These include the age structure (what percent of the people are in various age periods), the male-female ratio, what percent of the people get married, at what age couples marry, at what age couples have children, how many children couples have, etc. Thus it is very hard to make predictions about population changes.

We will now set up a simplified model (set of assumptions) that will help us get insight into population growth. Even with its many simplifications, many concepts related to population growth can be better understood through relatively simple computations (only addition, subtraction, multiplication, and computation of percents). This model is based on that of L. Schaefer, Population, Environment, Society - A Teachers Resource Manual, pages 65 to 69. We will expand the applications and implications of the work in that book.

First, some necessary definitions:

1. Generation - the time span from the birth of a woman until the end of her childbearing years.
2. Doubling Time - the number of years for a population to double.
3. Birth Cycle - the number of years from a woman's birth to the beginning of her childbearing years.

Next, some basic assumptions for our model of population growth (these can be modified for the analysis of a particular country or region):

1. A generation is thirty years,
2. The birth cycle is fifteen years,
3. Each person will live exactly 60 years,

4. The sex ratio remains constant - half male and half female. Everyone gets married (happily, we hope) and has children.

Example 6.1

We will first consider population growth when the average number of children per couple is 4. (Computation of the average number of children per couple is considered in chapter 10.) We start with the conditions shown below:

Age → 0 15 30 PR 60

Birth Cycle	Number of Children C	Number of People of Reproductive Age R	Number of People past the Reproductive Age PR1	PR2	Total Population
0	20	0	0	0	20

Table 6.1

Note: C stands for the childhood period, assumed to be from ages 0 to 15. We are starting with 20 children and no people in any other age categories. The start with 20 is purely arbitrary; any even number could be used, and 20 is convenient.

R stands for the reproductive period, assumed to last from ages 15 to 30. PR stands for the post-reproductive period (the period when people are assumed to be too old to have children), ages 30 to 60. To keep the various periods all equal to 15 years, the post-reproductive period is divided into PR1 (post-reproductive period one), ages 30 to 45 years, where people are grandparents, and PR2 (post-reproductive period two), ages 45 to 60, where people are great grandparents.

All of this should become clearer as we show the simple calculations necessary to go through several birth cycles.

Let us continue our chart as shown below:

Age → 0 15 30 45 60

Birth Cycle	C	R (couples)	PR1	PR2	Total
0	20	0	0	0	20
1	40	20 (10)	0	0	60 = 40 + 20

10 couples

20 people in the reproductive period

Table 6.2

Going from birth cycle 0 to birth cycle 1, 15 years have elapsed. The 20 children are all 15 years older and have all moved into the reproductive period (R). Based on our assumptions, we have 10 females and 10 males, and thus 10 happy marriages. Since there is an average or 4 children per couple, we get a total of 10 • 4 or 40 children. As indicated, the 10 in parenthesis in the R column means 10 couples. We still have no people in the post-reproductive periods; hence the total population has gone up to 60 people.

You may now want to take a sheet of paper and continue the table for several more birth cycles up to birth cycle 6, and then compare your results with those shown below.

Age → 0 15 30 45 60

Birth Cycle	C	R (couples)	PR1	PR2	Total
0	20	0	0	0	20
1	40	20(10)	0	0	60
2	80	40(20)	20	0	140
3	160	80(40)	40	20	300
4	320	160(80)	80	40	600
5	640	320(160)	160	80	1200
6	1280	640(320)	320	160	2400

Table 6.3

Comments on the above calculations:

1. For every birth cycle, the children of the previous birth cycle move into the reproductive period, marry, and have an average of 4 children per couple. Thus, for example, the 80 children born in birth cycle 2 move into the reproductive period in birth cycle 3. There are 40 males and 40 females; 40 couples are formed. Couples have an average of 4 children. Hence, there is a total of 160 children in birth cycle 3.

2. Those people in post-reproductive period two for a given cycle all die at the end of that cycle, since they reach age 60. Note that the 20 people in post reproductive period two (PR2) of birth cycle 3 do not appear in birth cycle 4.

3. Note the numbers in the columns for the successive cycles. What kind of sequences are formed? Note the exception in the totals column until we reach birth cycle 3. What do you think is the reason for this exception?

4. We can only make conclusions on the doubling time for the population after all 4 age categories are filled; that is starting with birth cycle 3.

5. Note that the total population of 2400 for birth cycle 6 can be calculated from three different points of view:

a. The total population equals the sum of the numbers of people in the childhood, reproductive, and post-reproductive periods.

Total = people in (C + R + PR1 + PR2)

Total = 1280 + 640 + 320 + 160 = 2400

b. The total population for any birth cycle equals that of the previous birth cycle plus the increases in population (the difference between births and deaths during that birth cycle).

Total population = Old population + births - deaths

In birth cycle 6, 1280 children were born. Eighty people who were alive in birth cycle 5 died during birth cycle 6. The total population at the end of birth cycle 5 is 1200. Hence,

$$\text{New population} = 1200 + 1280 - 80 = 2400$$

c. The number of people in each age category doubled; hence there must be twice as many people at the end of birth cycle 6 as there were at the end of birth cycle 5, or 2 x 1200 = 2400 people.

6. Note that after birth cycle 3 when all age categories have people, the total population for a birth cycle is twice that of the previous birth cycle. Hence, the doubling time for population is one birth cycle, or 15 years. If no changes were made, what would be the population for our model community after 15 more years, 30 more years, 45 more years, 60 more years? What effects would this have on the environment, use of resources, crowding, etc., in the community?

7. For birth cycle 6, the percent of the population under 15 years of age is

$$\frac{\text{population under 15 years of age}}{\text{total population}} \times 100\% = \frac{1280 \text{ people}}{2400 \text{ people}} \times 100\% = 53.3\%$$

This is a very high percentage, representative of a very rapidly growing population. Mexico, a country whose population is increasing extremely rapidly, has 46 percent of its population under 15 years.

To summarize what we have found so far, with an average of 4 children per couple, the population growth is extremely rapid, with the population doubling every 15 years. There is tremendous momentum for continued growth due to the fact that over 50 percent of the population are under 15 years of age.

The community that started with only 20 children has very rapidly grown to

2400 people. Let us assume that this is starting to badly strain the environment, that crowding is occurring, that air and water are becoming polluted, and that the general quality of life is beginning to deteriorate. Let us further assume that local demographers (population experts) are predicting another doubling of population in just 15 years and claiming that this will have very extreme effects on many aspects of the quality of life.

What are some possibilities to help avoid this fate? (Note that we are only considering population changes here, but other factors such as changes in life styles, consumption, etc. should also be considered.)

Suppose that it is suggested that the population growth halt immediately - that is, that there be no increase in population at all in birth cycle 7. This is called zero population growth (ZPG). Let us see what is involved in this.

Since there is to be no population increase, the population at the end of birth cycle 7 (denoted as 7a below, since other possibilities will be considered) must be 2400 (the same as that at the end of birth cycle 6).

Age	0	15	30	45	60
Birth Cycle	C	R (couples)	PR1	PR2	Total
6	1280	640(320)	320	160	2400
7a	?	1280(640)	640	320	2400

Table 6.4

How many children could be born during birth cycle 7a? Since the population is stable, the number of children born must be equal to the number of people who die during this birth cycle, or 160 (see PR2 column of birth cycle 6).

What difficulties are related to this?

1. Since there are 640 couples in birth cycle 7a, the average number of children per couple during this cycle is 160/640 or just one in four. This means

on the average, for every four couples, three will have no children at all and one will have just one child. Imagine the problems involved in trying to obtain such a condition!

2. Note the tremendous decrease in the number of people in the childhood period, from 1280 to 160, or from 53 percent of the population to about 7 percent. Consider this in terms of the economy and the many people dependent on children for a livelihood, such as teachers, manufacturers of toys, producers of baby foods, etc. Such a rapid change would have very disruptive effects on many lives.

However, these difficulties must be considered alongside the very negative effects of not taking action such as heavy pollution, insufficient food, resource scarcities, etc. This illustrates the importance of anticipating problems and taking proper action before the situation becomes critical.

Since an immediate change to ZPG causes some major problems, let us consider a more gradual change. We will consider what happens if there is an average of 2 children per couple for the next birth cycles. We start again with the results for birth cycle 6. It would be instructive for you to make computations up to birth cycle 10 and compare them with the results below.

		0	15	30	45	60
	Birth Cycle	C	R (couples)	PR1	PR2	Total
	6	1280	640(320)	320	160	2400
2	7b	1280	1280(640)	640	320	3520
children	8b	1280	1280(640)	1280	640	4480
per	9b	1280	1280(640)	1280	1280	5120
couple	10b	1280	1280(640)	1280	1280	5120

Table 6.5

Note how the population continues to increase for 3 more cycles or 45 years from 2400 to 5120 and that it finally levels off at 5120 people. This is more than twice the population of 2400 at the time when the switch to two children per couple was started. Note however the tremendous improvement compared to a population of 19,200 (2400 → 4800 → 9600 → 19,200), which would have occurred at the end of birth cycle 9 if couples continued to have an average of 4 children (and there would still be 53% of the population under 15 years of age under those conditions, with the resultant population momentum effect).

Hence a change to 2 children per couple slows the population growth but does not stop it right away, due to the very high percent of the population under 15 years of age. This is the reason that the U. S. population has continued to grow recently even though the number of children per couple has been below the actual replacement level of 2.1 children per couple. It also reinforces the importance of the fact that today over 40 percent of the people in developing countries are under 15 years of age.

Observe the age distribution when zero population growth, ZPG, is finally reached. What percent of the population is in each age category?

Notice the large amount of useful population insights we have gained so far from our relatively simple population model. We have been able to gain insight into such concepts as doubling time, theoretical replacement level (2 children per couple), and zero population growth.

Example 6.2

Since population grows so rapidly when couples have an average of 4 children, let us see what happens when there is an average of only 2 children per couple. The other assumptions used in the previous example remain the same. Again, it will be instructive for you to make your own computations and check the results below.

Age	0	15	30	45	60
Birth Cycle	C	R (couples)	PR1	PR2	Total
0	20	0	0	0	20
1	20	20(10)	0	0	40
2	20	20(10)	20	0	60
3	20	20(10)	20	20	80
4	20	20(10)	20	20	80

Table 6.6

Here are some questions for you to consider with regard to the above table:

1. Is ZPG reached?

2. What is the maximum population reached?

3. Why isn't ZPG reached for the first few birth cycles?

4. What is the percent of the population for each age category after birth cycle 5? Compare this with the onset of ZPG for the previous case (birth cycle 9b).

5. How do you think the quality of life in this case would compare with that for the rapid growth rate of the previous case (4 children/couple)? Can you anticipate any difficulties?

6. What would happen if each couple had 4 children for the next several cycles (see exercise 4)?

At this point it would be very instructive to work out the case where the average couple has (a) 3 children, starting with just 64 children (see exercise 1), and (b) 1 child (negative population growth - see exercise 2).

Exercises

For each problem below, use the same assumptions as used in the sample examples, unless otherwise indicated.

1. Starting with just 64 people in the childhood period and assuming that couples have an average of 3 children, set up a table to show how the population increases and calculate the approximate doubling time. (We are starting with 64 children to avoid getting an odd number of people in the reproductive period in the early birth cycles.)

2. Starting with just 80 people in the childhood period, and assuming that couples have an average of 1 child, set up a table to show how the population changes. Calculate the time for the population to become half of its previous value.

3. Continue the calculations for zero population growth by completing the table for birth cycles 8a and 9a (immediately following birth cycles 6 and 7a in example 1). What would be the average number of children per couple for these birth cycles?

4. For the case where couples have an average of two children (example 2), find results for birth cycles 5, 6, 7, 8 under the assumption that each couple in these birth cycles has 4 children.

5. Starting with 512 children and assuming that couples have an average of 2.5 children, set up a table to show how the population increases and calculate the approximate doubling time. (If the number of people in a reproductive period is odd, assume that one person does not get married. If you get a decimal for the number of children, change your result to the nearest even whole number.)

6. Starting with 512 children and assuming that couples have an average of 1.5 children, set up a table to calculate population changes. (If the number of people in a reproductive period is odd, assume that one person does not get married. If you get a decimal for the number of children in any birth cycle, change your result to the nearest even whole number.)

7. Can you find any patterns from the various cases considered so far, where couples have had an average of 1, 1.5, 2, 2.5, 3, and 4 children per couple? Hint: Consider the factor by which the total population increases for each case, and relate this factor to the doubling time. Use the pattern to find the doubling time for the case where couples have an average of 5 children.

8. The analysis in this section does not take immigration or emigration into account. Here is an equation that considers migration. (An equation is just a statement that two terms or groups of terms are equal.)

$$P = P_o + B - D + I - E$$

where P = population at a given time

P_o = population at an earlier time

B = number of births

D = number of deaths

I = number of immigrants

E = number of emigrants

Hence our equation indicates that the population at a given time equals the population at an earlier time + the number of births - the number of deaths + the number of immigrants - the number of emigrants.

Assume that a country has 20 million people at the start of 1978 and during 1978 there are 600,000 births, 350,000 deaths, 100,000 immigrants and 50,000 emigrants.

a. What is the population at the end of 1978

b. What is the percent increase in population?

c. What percent of the population increase is due to net migration?

9. To consider the effects of people living longer, work out example 1 again assuming that each person lives 75 years. Add a post-reproductive period 3 (PR3) from 60 years to 75 years of age.

Chapter 7 "The Limits To Growth"

How long can rapid growth in population and industrialization continue before we reach some limiting factor? A book that treats this question in great detail is The Limits To Growth by D. Meadows and others (see Bibliography in the Appendix).

The Limits to Growth uses mathematics extensively in its analysis. Included are the following features:

1. It shows the significance of exponential (geometric) growth as it relates to population, resource depletion, pollution, industrialization, and demand for food.

2. It uses mathematical models to analyze the effects of growth.

3. It illustrates the importance of considering the relationships between variables (for example, how population growth affects pollution and agricultural production) and getting numerical values for these relationships.

4. It presents computer results that take many variables into account and also considers the effects of variations in these variables.

Many of the conclusions of this book are of great concern with regard to global survival; for example:

1. Levels of pollution are increasing exponentially and we don't know what the safe limits are.

2. At present rates of usage, we'll run out of many nonrenewable resources in the near future.

3. We're rapidly approaching the time when demand for agricultural land will exceed the amount of land available.

4. Most importantly, "If the present growth trends in world population, industrialization, pollution, food production, and resource depletion continue unchanged, the limits to growth on this planet will be reached sometime within the next one hundred years." (page 29)

The publication of The Limits To Growth stirred up much attention. Hundreds of thousands of copies were sold in the U. S. and millions were sold abroad. Many citizens became extremely concerned about the book's warning that the world is heading toward disaster unless there are major changes in values and methods of production. The computer printouts which indicated that exponential growth would soon reach a limit could not be ignored.

Many economists disagreed with the book's conclusions. They felt that such things as better technology, improved energy sources, more productive agricultural methods, recycling, and better pollution control, would enable growth to continue for a long time. There was much controversy. Many conferences were held to consider the book's analysis. Other books and articles were published defending and attacking the book's doomsday predictions.

Was The Limits To Growth correct? Only the future will tell. But considering some of the events which have occurred since the book was published - energy shortages, famines in several areas, increased inflation in much of the world, continuing decrease in environmental quality in many cases, and a failure of nations to agree on a law of the sea for sharing the ocean's resources or to better international economic relations, to name just a few - seem to indicate that we should certainly take the book's message seriously.

Since the publication of The Limits To Growth, in 1972, the Club of Rome, which backed the project, has been refining the predictions of The Limits To Growth and indicating what needs to be done to avoid the catastrophe that it predicts. They have published several books including Mankind At The Turning Point, Goals for Mankind, and RIO (Reshaping the International Order). These books are described briefly in the bibliography.

I am presenting in the next few pages a summary of the book in a reprint from the Zero Population Growth National Reporter, entitled, "The Limits To Growth - Implications for the United States," by Rodelia Hapke, May, 1972. This is followed by a series of questions to test your understanding of the material in the article. Your answering of these questions will sharpen your ability to gain useful information from articles which use mathematical information and techniques.

Following this is presented a 10-page study guide with many questions keyed directly to the book, The Limits To Growth. A study of this book along with this study guide will give much practice at obtaining data from charts, reading graphs, and analyzing the effects of exponential growth.

The Limits To Growth

Implications for the United States

by Rodelia Hapke

Zero Population Growth, Inc, 1972

The Limits to Growth (Universe Books, New York, 205 pages, $2.75 paperback. Available from ZPG National) is a popularized report of an 18 month study of the world system by an M.I.T. research team under the direction of Dr. Dennis Meadows.

This article will briefly skim the highlights of *Limits*, and then plunge directly into examining the implications of the book for policy formation of the United States.

Future world alternatives: stability or disaster?

The purpose of the MIT project was to build a model of trends in the world system until the year 2100, using the most complete information available. Five variables were used: population, food production, industrial production, resource depletion, and pollution. The model found (as one would intuitively suspect) that all growth models eventually end in collapse; and that the survival of man can be assured only by stabilizing the five main components of the world system.

What will result if current trends continue their present course? The computer yielded the projection shown in figure 1.

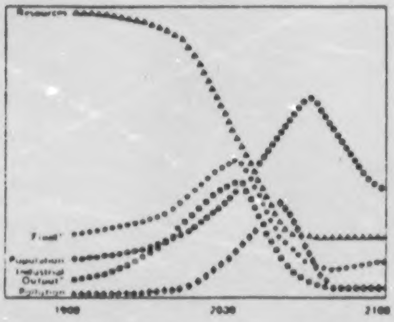


Figure 1 - MIT projection of current world trends. *Per Capita

"The 'standard' world model assumes no major changes in the physical, economic, or social relationships that have historically governed the development of the world system. All variables plotted here follow historical values from 1900 to 1970. Food, industrial output, and population grow exponentially until the rapidly diminishing resource base forces a slowdown in industrial growth. Because of the natural delays in the system, both population and pollution continue to increase for some time after the peak of industrialization. Population growth is finally halted by a rise in the death rate due to decreased food and medical services." (Limits, 124)

Thus, present trends lead to a substantial decline in population within the next 130 years. Using a computer, the MIT team examined future alternatives which would be more conducive to the happiness and survival of the human species. Various projections are considered in *Limits*, the alternative most leading to long term survival being shown in figure 2.

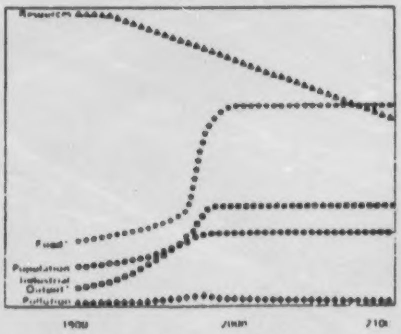


Figure 2 - MIT projection of stabilizing policies introduced in the year 1975 - The World in Equilibrium. *Per Capita

In this projection, worldwide population is *stabilized* in the year 1975, resource consumption and pollution generation per unit of industrial output are reduced to ¼ of their 1970 values, and the economic preferences of society are shifted from material goods to services.

Exponential Growth

To appreciate the magnitude of change that will be necessary to create a world in equilibrium, it is helpful to consider the difference between *linear* and *exponential* growth. Concerning linear growth, *Limits* writes:

> *"Most people are accustomed to thinking of growth as a linear process. A quantity is growing linearly when it increases by a constant amount in a constant time period. For example, a child who becomes one inch taller each year is growing linearly. If a miser hides $10 each year under his mattress, his horde of money is also increasing in a linear way. The amount of increase each year is obviously not affected by the size of the child nor the amount of money already under the mattress."* (Limits, 25-27)

In sharp contrast there is exponential growth, illustrated by the following two examples:

> *"Suppose you own a pond on which a water lily is growing. The lily doubles in size each day. If the lily were allowed to grow unchecked, it would completely cover the pond in 30 days, choking off the other forms of life in the water. For a long time the lily plant seems small, and so you decide not to worry about cutting it back until it covers half the pond. On what day will that be? On the twenty-ninth day, of course. You have only one day to save your pond."* (Limits, 29)

In 1800 there were 1 billion people. In 1930 there were 2 billion people. Today there are 3.7 billion. By

2000 there will be 7 billion people. (U.N. demographic statistic)

Limits emphasizes that the basic components of the world system — population, resource depletion, pollution, industrial output — all exhibit *exponential, not linear, growth.* Thus, like the water lily above, early stages of their growth may completely mislead us when making future projections. *Limits* emphasizes that it is vital that this exponential feature be conveyed to our leaders and the general public, since most of us generally think only in terms of linear growth.

Let us now turn to the basic components of the world system to determine what might be the role of the United States in stabilizing these various curves.

Growth in the world system: population.

Two of the five basic components of the world system serve to increase growth in the system. These two components are population and capital investment. The other three components (pollution, resource depletion, and possibilities for food production) act as limits to growth in the system.

What are the alternatives for population for the next 130 years? Figure 3 considers alternatives for the world as a whole.

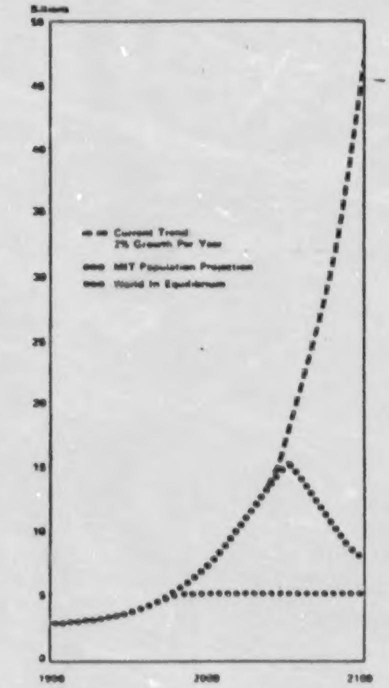


Figure 3 · World population alternatives.

Three possibilities are considered:

(1) Continuation of present trends (2% each year), with no interference from lack of food, resource depletion, or other factors.

(2) The population component of Figure 1 above: the MIT projection for population if we just stand by and let current trends for the world system take their natural course.

(3) The population component of Figure 2 above: a world which can survive in equilibrium for a long time into the future.

Turning from the world to the United States, three possibilities for population are represented in Figure 4.

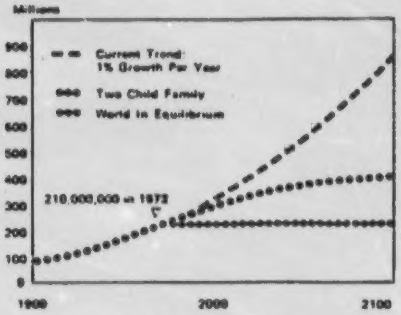


Figure 4 · United States population alternatives

These possibilities are:

(1) Continuation of present trends (1% growth each year), which leads to eight hundred million people by the year 2100.

(2) Adoption of the two child family as average family size (this seems to be the recommendation of the final report of the Population Commission).

(3) The population component of Figure 2 above: a world which can survive in equilibrium for a long time into the future.

What are the prospects for stabilizing population in the very near future by drastically lowering the birth rate? A comparison of the *age-structures* for the United States and the underdeveloped countries, will indicate the magnitude of this task (see figure 5).

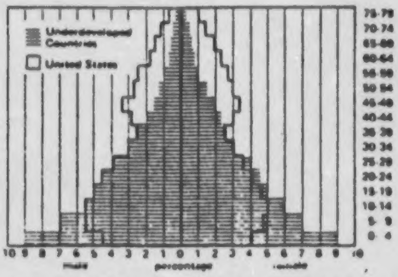


Figure 5 · Age structure of the United States and of the underdeveloped countries.

Due to the high proportion of young people in the U.S. population, an average family size of 1.2 children is required to *immediately* stop the U.S. population from growing. Yet, the world population is even younger: today more than half of the world's population is under 20 years of age. To *immediately* stabilize population world-wide by bringing births into balance with deaths would require an average family size of about 1 child per family.

Growth in the world system: economic growth.

Economic growth is the second impetus to growth in the world system. This factor is especially important because it has implications for resource depletion and pollution generation. Figure 6 compares the economic growth trend for the United States with a combined trend for the eight other most populous countries in the world (China, India, Pakistan, Indonesia, Japan, Brazil, Nigeria, Germany). These eight countries plus the United States contain 64% of the world's population. This graph also indicates the level of per capita income that would result from Figure 2 above: a world in equilibrium. In considering this level of per capita income, it is vital to note that much of it is taken up by food, and that economic preferences have been shifted from material goods to services (see figure 6).

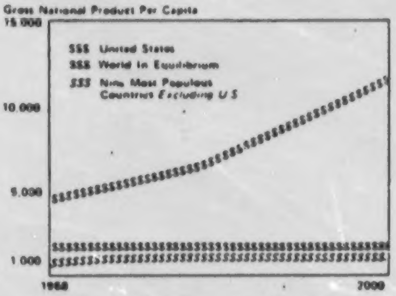


Figure 6 · Economic growth projections.

This graph indicates that stabilizing this component of the world system will have drastically different implications for the United States and other highly industrialized countries, as opposed to the underdeveloped, non-industrialized countries.

Since most countries presently have a very low GNP, stabilizing the economic component of the world system will place virtually no demands on them. To a large extent, these countries would continue current efforts to improve the quality of life for their citizens.

However, the demands that would be placed on the industrialized countries, particularly the United States and Japan, would appear to be enormous. These demands would fall into three main areas:

(1) The *stabilizing of industrial capital* in the year 1990.

(2) The *redirecting of present economic activity* away from the current emphasis on material goods with a relatively short life span, which consume vast quantities of resources. A redirecting of economic activity towards efforts to preserve the quality of soil, food production, production and distribution of contraceptives, and the production of material goods which are ecologically sound. A major part of such economic activity would be directed into *services* rather than the production of material goods.

(3) A *world-wide redistribution of income*, leading to a world-wide average income (food, services, and industrial output) of $1,800, equal to the present average income of Europe today, and about half of present U.S. average income.

The task of accomplishing this economic goal would appear to be enormous. The burden of stabilizing population would require herculean efforts from the underdeveloped countries. Achieving the economic goal would require equally herculean efforts from the United States and the other highly industrialized countries.

Limit to growth: resource depletion.

Today, with 6% of the world's population, the United States consumes 35% of the world's resources each year. All developed countries, with the exception of the Soviet Union, are net *importers* of most of the metals and ores on which their economies depend. Of the 36 basic resources needed to sustain a modern technological nation, the United States is a net importer of 27. Figure 7 indicates the expected time of depletion of many non-renewable resources, given current trends and taking into account presently known global reserves *(Limits, 56-59)*. This chart also indicates the percentage of these resources that the United States currently consumes.

Resource	*US Consumption As % Of World Total*	*Depletion Date Given Current Trends*	*Depletion Date Assuming Five Times Known Global Reserves*
Gold	26%	1981	2001
Copper	33%	1982	2002
Lead	25%	1992	2036
Mercury	24%	1984	2013
Natural Gas	63%	1994	2021
Petroleum	33%	1992	2022
Silver	26%	1985	2014
Tin	24%	1987	2033
Zinc	26%	1990	2022

Figure 7 · **Resource depletion.**

All the resources indicated on this chart will run out (or, more probably, become enormously expensive) before the year 2000. *Limits* considers the extremely optimistic assumption that we have somehow miscalculated present reserves, and estimates the depletion year if we have 5 *times* the currently known reserves. This wildly optimistic assumption postpones the day of reckoning by an average of less than 32 years. Like the lily in the pond considered above, our resource usage is increasing *exponentially*.

Which countries would suffer most from resource depletion? The countries that use them — the highly industrialized countries such as the United States. It is crucial to note that the United States uses resources not only for *luxury* items (TV sets, air-conditioning, etc.), but also for the very basic items of *survival* — the production and distribution of food, modern medical care, etc. It is in the self-interest of the industrialized countries to start now to reduce resource consumption. (Note that in Figure 2 above, a world in equilibrium, resource consumption is stabilized to ¼ of its 1970 value per unit of industrial production.)

Limit to growth: pollution.

With 6% of the world's population the United States contributes about 35% of the world's pollution each year. *Limits* documents that many pollutants in our environment are increasing at an exponential rate — CO_2 in the atmosphere; waste heat in the Los Angeles basin; the release of sulfates, chlorides, sodium and potassium into Lake Ontario; nuclear wastes; oxygen depletion in the Baltic Sea; and lead in the Greenland ice-cap.

What of future trends? In the *Environmental Handbook* Garrett DeBell sums up the situation by writing, "all power pollutes". Figure 8 indicates that American power consumption is expected to increase 500% in the next 30 years.

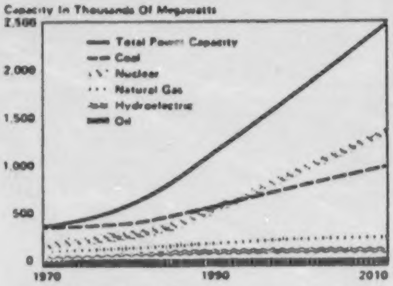


Figure 8 - **United States power projection.**

What will be the effect of such increasing pollution on human health and lifespan? Many people have mistakenly felt that somehow man is immune to the effects of pollution — that although other species (e.g. pelicans, fish of various sorts, robins) are susceptible to pollution, that somehow man will escape its effects by *avoiding* pollution. However, close examination of the facts shows that this is not the case. To avoid the mercury contamination of many fish, the pesticide contamination of fruits and vegetables, and the DDT contamination of animal fats, many Americans even today are drastically altering their eating habits and switching to "organic" foods. Yet, at least one newspaper report has claimed that there are pesticide residues in even the best of organic foods. Rising air pollution is creating an unhealthy situation for many citizens in our major cities; yet for economic reasons they find it "too expensive" to move.

If pollutants continue to rise exponentially each year, it will be harder and harder, and more expensive to avoid them as time passes. It cannot be overemphasized that the world and the entire ecosystem are interconnected; and that man is subject to the same laws as are the other animals of the animal kingdom.

Limit to growth: food.

Along with resource depletion and pollution, food production is the third constraint to worldwide growth. Today food production worldwide is barely keeping up with population growth. Figure 9 considers the limits to food production.

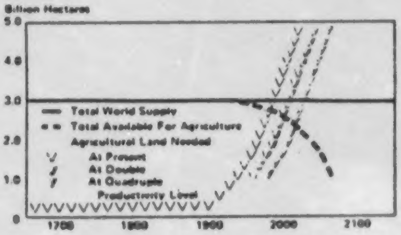


Figure 9 - **Arable land.**

Commenting on this graph, *Limits* writes: *"Total world supply of arable land is about 3.2 billion hectares (7.8 billion acres). About 0.4 hectares per person of arable land are needed at present productivity. 'The curve of land needed thus reflects the population growth curve. The light line after 1970 shows the projected need for land, assuming that world population continues to grow at its present rate. Arable land available decreases because arable land is removed for urban-industrial use as population grows. The dotted curves show land needed if present productivity is doubled or quadrupled."* (Limits, 50)

Because the need for arable land to keep up with growing population is growing *exponentially*, the doubling of food productivity postpones the food crisis for 30 years.

Both the lily in the pond in the example considered earlier, and the amount of arable land available for food production illustrate a striking feature about exponential growth: *the apparent suddenness with which such quantities approach a fixed limit.* The lily takes 29 days to cover half of the pond, and then only *one* additional day to cover the rest of the pond. The farmer has only one day in which to save his pond . . . When we turn to food production, the implications are equally startling and grim. Since world population is currently doubling every 35 years, the need for food is doubling every 35 years. Thus, 35 years before the maximum food growing capacity of the planet is reached, mankind will be using only *half* of this capacity. Then, in the next short 35 years, world population doubles, the need for food doubles . . . and suddenly there is a food crisis.

World in equilibrium: putting on the brakes.

Limits considers three models to achieve stability in the world system. The first is illustrated by Figure 2 above. This graph results from the adoption of the following policies:

1. Population is stabilized worldwide in 1975.
2. Capital investment increases at its current rate until 1990, at which time it is stabilized by setting the investment rate equal to the depreciation rate.
3. The following policies are introduced in 1975:
   - —resource consumption and pollution generation are reduced to ¼ of their 1970 values per unit of production.
   - —the economic preferences of society are shifted from material goods to services.
   - —capital is diverted into food production; vigorous programs of soil enrichment and preservation are effected.
   - —vigorous programs of resource recycling are instituted.
   - —industrial capital (plants, machines, etc.) is built to have a much longer lifespan.

Since the first and second of these policies would be difficult to achieve, *Limits* considers the following alternative program. In this case the policies just listed are modified in the following way:

1. The first policy above is modified as follows: Instead of *stabilizing population* in the year 1975, family size averages 2 children. This permits world population to double in size within 60 years.
2. The second policy above is modified as follows: Instead of stabilizing industrial capital in the year 1990, an attempt is made to maintain the same per capita output as in 1975. Since those little "capitas" are multiplying, this means that total industrial output doubles in 60 years.
3. All other policies mentioned above remain the same.

This set of policies leads to the projection illustrated in figure 10.

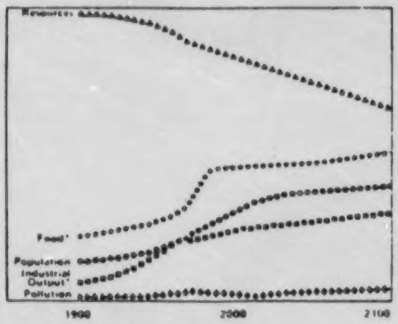


Figure 10 - **MIT projection of alternate stabilizing policies to those introduced in Figure 2. *Per Capita**

The third projection considered by the MIT team is the following. In this projection, the policies in the example just considered are put into effect in the year 2000 instead of in the year 1975. The consequences of this delay are given in Figure 11.

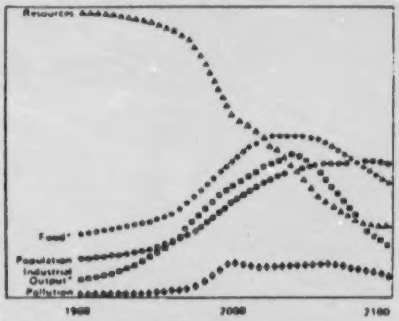


Figure 11 - **MIT projection of stabilizing policies introduced in the year 2000. *Per Capita**

Putting on the brakes: the role of the United States.

We have examined the five main components of the world system — population, resource depletion, industrial output, food production, and pollution generation. It is evident that the burden of stabilizing the world system will fall largely on the highly industrialized countries of the world, especially the United States. For when one looks at the five, components of the world system, the underdeveloped countries have virtually no effect on any of these components, except for the component of population. The United States will largely determine whether the components of resource depletion, and industrial output are brought under control . . . the United States will largely determine *how (not whether)* the world wide system will achieve equilibrium.

Such a task will be enormous. But if we are to accept the models and predictions of *Limits*, do we have any choice? As *Limits* emphasizes, "Accepting the nature-imposed limits to growth requires no more effort than letting things take their course and waiting to see what will happen. The most probable result of that decision, as we have tried to show, will be an uncontrollable decrease in population and capital." *(Limits, 169)*

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Study Guide For "The Limits To Growth - Implications for the United States"

1. What was the purpose of the MIT project?

2. What 5 variables were considered?

3. What did the model find?

4. What must be done to assure the survival of humanity?

5. What assumptions are made in the standard world model?

6. Consider figure 1 and the accompanying discussion.

    a. What years are considered?

    b. What was plotted from 1900 to 1970?

    c. Why does industrial growth slow down?

    d. Why do population and pollution continue to increase after industrialization slows down?

    e. What is the result of "natural delays in the system"?

    f. In what year, approximately, does population reach a peak?

    g. How does pollution vary for the following approximate time periods:

        (1) from 1900 to 1980 (2) from 1980 to 2030

        (3) from 2030 to 2070 (4) after 2070

7. Consider figure 2 and the accompanying discussion:

    a. How can you tell that stability is reached?

    b. Will that stability last for 300 years if no changes occur? Why or why not?

    c. What factors led to stability of the world system?

8. Briefly comment on the difference between linear and exponential growth.
   a. Draw a sketch showing each kind of growth (recall chapter 5).
   b. Give two examples of each type of growth.
   c. Relate these types of growth to arithmetic and geometric sequences.

9. What kind of growth do the basic components of the world system, such as population and pollution, exhibit?

10. Why is it essential that people, especially our leaders, be very aware of the nature of exponential growth?

11. What two basic components of the world system increase growth in the system?

12. What three components act as potential limits to growth in the world system?

13. Are the five components interrelated? Indicate a few examples of which components affect other components.

14. Which of the three population alternatives shown in figure 3 do you think is most likely in the future? Why?

15. Consider figure 4.
    a. Is the U. S. population still increasing?
    b. Why do many people think that our population has leveled off or is decreasing?
    c. Which of the three curves do you feel will most closely represent U. S. population growth in the future? Why?

16. Why are prospects for an immediate halt in U. S. population growth very slim?

17. Why are worldwide prospects for a stoppage of population growth even slimmer?

18. Consider figure 5.

    a. What percent of the people in the undeveloped countries are in the age category:

        (1) 0 to 4 years (really 0 to just under 5 years)

        (2) 5 to 9 years? (really 5 to just under 10 years)

        (3) 10 to 14 years? (really 10 to just under 15 years)

        (4) below 15 years of age?

19. Why must we consider economic growth as well as population growth?

20. Why would the stabilization of economic growth place greater demand on the U. S. and other industrialized countries than it would on the non-industrialized countries?

21. What are the three primary demands on industrialized countries with regard to stabilizing economic growth?

22. Comment on the feasibility of meeting these objectives.

23. Comment on the U. S. as a consumer of natural resources.

24. Are we self-sufficient with regard to the resources needed to maintain our economy?

25. Consider figure 7 and the accompanying discussion.

    a. What percent of the world's natural gas does the U. S. use?

    b. When will the world run out of natural gas, given current usage trends?

c. Repeat part b if we actually had 5 times the currently known reserves of natural gas.

d. Why are there so few years between the answers to the above two questions?

e. Why is it in the interest of the industrial nations to reduce resource consumption?

26. Name several pollutants that are increasing exponentially.

27. Consider figure 8.

a. In about what year is U.S. total power capacity projected to be (1) 500,000 megawatts ____________ (2) 1,000,000 megawatts ____________ (3) 2,000,000 megawatts ________________

b. What is the approximate projected doubling time?

28. Why is it essential to stop the exponential increase in pollution?

29. Consider figure 9 and the accompanying discussion.

a. Would doubling or quadrupling present productivity of agricultural land put off a food crisis for many years? Why?

b. Why has land available for agriculture been decreasing in recent years?

c. Relate figure 9 to the riddle of the water lilies in the pond.

30. Briefly summarize the policies that led to the stabilization in the world model in figure 2.

31. Consider figure 10 and the accompanying discussion.

a. What policy changes from those of figure 2 were made?

b. Do we still have stability, at least to the year 2,000?

32. Why the difference between figure 10 and figure 11?

    a. Does figure 11 show stability?

    b. What conclusion should we draw?

33. What should be the role of the U. S. with regard to future world stability?

34. Summarize the article in one paragraph.

35. What is your opinion about the article? Do you think that there should be no more growth? If there is further growth, what types of growth should it be? What other factors should be considered with regard to possible limits to growth?

STUDY OUTLINE FOR "THE LIMITS TO GROWTH"

1. As you read the book THE LIMITS TO GROWTH, answer the questions.
2. Read the "Forward."

   1) What are the objectives of the Club of Rome?

   2) What types of problems are facing men of all nations today?

   3) What five basic factors can ultimately limit growth on this planet?

3. Page 21. According to U. Thant (in 1969),

   1) How much time did we have left?______________________________

   2) What must we do?

   Have we made progress in these areas? Give some examples.

4. Page 24. Consider figure 1 (and pages 23, 24 & 25)

   1) Most people's concerns are where?

   2) The Club of Rome's concerns are where?

5. Pages 26-28

   1) What is a model?

   2) Why use a model?

   3) Why is a model needed to study the problems considered in this book?

   4) What 5 major trends was the model (in this book) designed to investigate?

   5) Is it a perfect model? Explain.

   6) What advantages does their model have over a mental model?

6. Page 29 read the conclusion carefully. Based on your general knowledge and things you've learned in this course about population, world food demand, and pollution, do you agree or disagree at this time with these conclusions? Briefly explain.

7. Page 31

   1) What kind of sequence is this?__________________________

   2) Write the 1st 4 terms of the sequence.

   3) Do you agree with Han Fei Tzu's conclusion?

8. Page 33, 34.

   1) Define exponential growth.

   2) Give 2 examples.

   3) Fill in this table of world fertilizer consumption. Doubling time is about ___________ years.

year	Thousand metric tons
	10,000
	20,000
	40,000

9. Page 35. Complete this table of world urban population

year	Millions of People	Doubling Time
	250	
	500	
	1000	
	2000	

10. Page 37.

    1) A population increasing at a 2% growth rate will double in ____________ years.

    2) The U.S. population is increasing at about 1%. At this rate our population would double in _________ years.

11. Page 38 & 39.

    1) Briefly define System Dynamics.

    2) Briefly define and illustrate a positive feedback loop.

12. Pages 40-45. World population growth.

    1) Compare the following table

Year	Billions of People	Doubling Time
1) 1650	1/2	
2) 1830	1	
3)		
4)		
5)		

Result for years beyond 1975 are projections or ex________________.

6) What is meant by "super"-exponential growth?

7) What causes positive feedback?

8) What causes negative feedback?

9) A change in the ______feedback loop is most important in rapid population rise. Death rates have decreased due to_______________.

10) At present rates of population growth, in 60 or 70 years we could have __________people in the world for every person now living.

13. Pages 45-51.

1) ______________has been growing even faster than population.

2) For industrial output, the positive feedback loop is caused by _________________________and the negative feedback loop is caused by _________________________.

3) Refer to figure 7. The 3 countries growing most rapidly are _____________, _____________________ and _____________. The country growing hardly at all is_______________. The country whose growth is arithmetic rather than________________ is _________________. By the year 2000, Argentina's GNP per capita (by extrapolation) should approach ______________. What is happening to the gap between rich and poor nations?

4) Referring to tables 2 and 3, compare the economic (GNP) and population growth rates for rich countries (U.S. and USSR, for example) and poor countries (Indonesia, for example).

14. Page 53. What does the quotation mean to you?

15. Page 55 and top of page 57.

1) The two main categories of necessities are

a)_______________________________ and

b)_______________________________

2) What is assumed about social conditions?

3) Is this a good assumption? Why or why not? If not a good assumption, why is it made?

16. Pages 56 to 63 Food.

1) Figure 8. Which areas get enough calories and proteins? ____________________. Which areas do not? Is this what you would expect?

2) What per cent of children die before their first birthday in

(a) Zambia____________________

(b) India & Pakistan____________

(c) Columbia____________________

These children are most likely the victims of____________

3) Can we easily grow much more food by putting more land under cultivation? _______Why or why not?

4) Figure 9. Total food production in the nonindustrialized areas of the world has_________________, but food per capita has________________ due to increases in______________________.

5) Figure 10. Arable Land.

a. When will we run out of agricultural land . . .

(1) at the present productiveity level? _________

(2) at twice the present productivity? _________

(3) at four times the present productivity? _________

b. Why are precise figures not important when exponential growth is concerned?

c. Why does each doubling of yield from the land cost more than the last one?

17. Pages 63-78 Nonrenewable Resources.

1) Table 4 and pages 69-71.

a. How long will our supply of chromium last?

i. if the current rate of use stays constant?

ii. if the rate of usage keeps growing exponentially?

iii. same as(ii) above but reserves are 5 times present reserves.

b. Repeat the above question for our supply of coal.

2) Figure 11.

a. Relate this figure to the data in Table 4.

b. Compare these cures to those on page 60.

3) Figure 12.

a. When does the chromium usage rate reach a maximum?____________

b. Why does the usage rate start to decrease?

4) Repeat for figure 13. How many years are gained by doubling reserves?

5) Unless usage rates slow down, most non-renewable resources will be very expensive in ________ years.

6) What feedback loops are pushing us toward the depletion of the earth's resources?

7) Figure 14. Generally, as GNP per capita_________, energy consumption________. Which country uses most energy per person?

18. Pages 78-94 Pollution.

1) Figures 15, 16, 17, 20.

The pollutants plotted are all growing.__________ .

2) Fig. 17. Fill in the chart.

Stored nuclear wastes	year	doubling time
100	1981	—
200		
400		
800		

19. Pages 97-99.

    1) What are the 5 basic factors considered in this book?

    2) Do they relate to each other?_____

       Which factors effect what other factors?

    3) Briefly outline the 4 main steps in construction of the model.

20. Pages 99-103.

    1) Why does the world model not need to be extremely detailed?

       What are its limitations?

21. Pages 103-109.

    1) Figure 23

    The feedback loops for population are (a)____________________ and (b)________________________. For industrial capital, they are____________________________and____________________________.

    2) Figure 24.

       a) The amount of food grown is effected by

          a ________________

          b. ________________

          c. ________________

       b) Pollution is effected by

          a. ________________

          b. ________________

          and effects

          a. ________________

          b. ________________

    3) Figure 25.

    Industrial output effects__________and____________ and is effected by_______________,__________________ and_____________________.

22. Pages 110, 111 and 112 (top) THE WORLD MODEL.

   a. What factors effect life expectancy?

   b. What factors effect births per year?

   c. Is the world model simple or complex?

   d. Could the analysis be done without a computer?

23. Page 109 (bottom), pages 112-118. Quantitative Assumptions.

   (1) What is meant by quantifying a relationship?

   (2) Figure 27.

   What is the approximate average life expectancy for nutritional levels of 4000 calories?__________ 6000 calories?________________ 8000 calories?_________________________.

Is the relationship between nutrition and life expectancy quantified?

   (3) What relations are plotted (quantified) in

      a. Figure 29?

      b. Figure 30?

24. Pages 119-124.

   1) What factors effect fertility?

   2) The desired birth rate is effected by__________

   3) Figure 31.

      a. Generally as GNP rises, birth rate________.

      b. For the world, the average GNP per capita is__________, and the average births per thousand people per year is_____.

   4) Briefly consider the "value" of a child and the "cost" of a child.

   5) Compare the "value" and "cost" of a child in developed and undeveloped countries. Relate to population growth.

25. Pages 125, 126, 128 (Fig. 34).

    1) In figure 34, why can curves A & B be rejected?

    2) Is the exact curve known?

    3) Which curve is considered best? ____ Why?

26. First skip to pages 130 (bottom) and 131. Complete the table below.

SYMBOL	VARIABLE
———	POPULATION
- - - -	
▬▬▬	
- - — - -	
-•-•-•-•-	
B	

27. Pages 129, 131, 132 WORLD MODEL STANDARD RUN.

    1) The years covered vary from 1900 to ________________________.

    2) From 1900 to 1970, food, population and industrial output grow____________________.

    3) The cause of failure is______________________.

    4) Population growth reaches a maximum and then decreases due to a rise in__________________.

    5) Briefly discuss the birth and death rate curves.

28. Figure 36, pages 133, 134.

    1) Compare figures 35 and 36.

    2. What basic assumption is made for figures 35 & 36.

CHAPTER IV TECHNOLOGY AND THE LIMITS TO GROWTH

1. Figure 37 (page 140)

   1) What causes failure (stops growth) in this case?

2. Fig. 38. 1) Complete the table (based on the curve)

% Biological Oxygen Demand Reduction	$ per pound
30	
50	
70	
80	
90	
95	

2. Comment on this table.

3. Comment on Table 6, page 143 with relation to the above question. Is it feasible to remove all the pollutants from our environment?

4. Figure 38 (p. 144)
    1) What assumptions are made?
    2) The cause of limit to growth is ____________________ .
    3) Briefly trace the curves for resources, food, industrial output, population and pollution.

5. Repeat the question of 4 above for
    1) fig. 40 page 145
    2) Fig. 41 page 146
    3) Fig. 42 page 147

6. What is common for all of these figures with regard to the ultimate result (see page 149).

7. Page 149 - 151
    1) What are some time delays that are introduced in the model?
    2) Why are these time delays serious? Relate them to the overshoot mode.

8. Technology in the Real World
    1) What is the impact of technology on the essential problem of exponential growth in a finite and complex system?

9) Technological Side-effects
    1) What have been some side effects of the Green Revolution?
    2) From your knowledge, what are some other negative side effects of technological advancements?

Chapter V The State of Global Equilibrium

1) Pages 163 - 164. Summarize what has been stated so far re feedback loops, time delays, exponential growth, technology effects, etc.

2. Deliberate Constraints on Growth, pages 165 - 176

   1) What restraints are now applied?

   2) If only population is stabilized, is this enough?

   3) What does figure 45 show?

   4) What policies produced the behavior shown in figure 46?

   5) What are the differences in policy between those producing figure 46 and figure 47?

   6) What are the effects of delaying changes until the year 2000? Relate to figure 48.

   7) Summarize the key ideas in this section.

3. The Equilibrium State

   1) What is meant by the equilibrium state?

   2) What are the 3 chief requirements for its occurrence?

   3) What are some problems and some advantages of such a state? (see the next 4 or 5 pages in Limits To Growth).

SUMMARY (What have we learned?)

Summarize the book by considering the following questions:

1. What is the nature of the problem considered in the book?
2. What factors can limit growth on the planet?
3. Are there any grounds for optimism?
4. What should be done? When?

Chpater 8 Logarithms

With the current widespread use of hand calculators, logarithms have lost much usefulness as a computational tool. However, logarithms are very helpful in solving problems related to the growth of population and resource depletion. They are also essential to an understanding of the decibel unit which is used to measure sound intensity.

In this chapter we will present the basic principles of logarithms, and apply them to some problems of current global interest.

8.1 Exponents

Logarithms are exponents, so we start with a discussion of exponents. Consider 2^5. The number 5 is called the exponent or power and is said to occupy the superscript position. The number 2 is called the base.

2^5 is simply a shorthand way of writing $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$; it means 2 multiplied by itself five times. 2^5 is read as 2 raised to the 5th power.

Other examples using exponents are

$$5^2 = 5 \cdot 5 = 25$$

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

In general, x^n means the base x raised to the n^{th} power and indicates that the base x is to be multiplied by itself n times. Hence

$$x^4 = x \cdot x \cdot x \cdot x \qquad (4\ x\text{'s})$$

$$x^7 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \qquad (7\ x\text{'s})$$

A number having a base raised to a negative exponent (such as 5^{-2}) is equal to one divided by the same base raised to the positive (absolute) value of the exponent. Hence $5^{-2} = 1/5^2$. Further examples are

$$3^{-4} = \frac{1}{3^4} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{81}$$

$$4^{-3} = \frac{1}{4^3} \quad \frac{1}{4 \cdot 4 \cdot 4} = \frac{1}{64}$$

Multiplication of numbers with exponents

$2^4 \cdot 2^3$ means $(2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$

$2^4 \cdot 2^3 = 2^{4+3} = 2^7$

When multiplying two numbers which have the same base, add the exponents of the numbers being multiplied, to get the exponent of the product. The base remains the same. Further examples are

$$5^2 \cdot 5^4 = 5^{2+4} = 5^6$$

$$3^5 \cdot 3^{-2} = 3^{5+(-2)} = 3^3 = 3 \cdot 3 \cdot 3 = 27$$

In general

$$\boxed{x^a \cdot x^b = x^{a+b}}$$

Eq. 8.1

Division of numbers with exponents

$\frac{2^7}{2^3}$ means $\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

$\frac{2^7}{2^3} = 2^{7-3} = 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

When dividing two numbers which have the same base, subtract the exponent of the denominator from the exponent of the numerator to get the exponent that the base is raised to. Further examples are,

$$\frac{3^5}{3^2} = 3^{5-2} = 3^3 = 27$$

$$\frac{4^2}{4^{-3}} = 4^{2-(-3)} = 4^{2+3} = 4^5$$

In general,

$$\boxed{\frac{x^a}{x^b} = x^{a-b}}$$

Eq. 8-2

Raising a number with an exponent to a power

$$(2^3)^4 \text{ means } 2^3\cdot 2^3\cdot 2^3\cdot 2^3 = (2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2) = 2^{12}$$

$$(2^3)^4 = 2^{3\cdot 4} = 2^{12}$$

When raising a number with a base, x, and an exponent, m, to a power, n, the result is the base, x, raised to the power m times n. In equation form,

$$\boxed{(x^m)^n = x^{m\cdot n}} \qquad \text{Eq. 8.3}$$

Further examples are

$$(2^5)^3 = 2^{5\cdot 3} = 2^{15}$$

$$(5^2)^4 = 5^{2\cdot 4} = 5^8$$

$$(x^4)^3 = x^{4\cdot 3} = x^{12}$$

Equations 8.1, 8.2, and 8.3 will be helpful in computations using logarithms.

8.2 Powers of ten

A special case of exponents is powers of 10. Consider 10^4. This means $10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$. The exponent, 4, tells us how many times the 10 should be multiplied by itself. Alternately, we can look at the exponent as telling us how many places to shift a decimal point. For example $1.0 \times 10^4 = 10{,}000.0$. The exponent is +4 and the decimal point shifts 4 places to the right.

Consider the following:

$$10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$$

$$10^2 = 10 \cdot 10 = 100$$

$$10^1 = 10 = 10$$

$$10^0 = 1 = 1$$

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10 \cdot 10} = \frac{1}{10^2} = 0.01$$

$$10^{-3} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{10^3} = 0.001$$

Note:

1. There is a pattern. As the exponents decrease by one on the left side, each number on the right side is one-tenth of the one above it.

2. Positive exponents tell us how many times we multiply by 10. Negative exponents tell us how many times we divide by 10.

3. $10^0 = 1$. To verify this, consider $10^4/10^4 = 10^{4-4} = 10^0$. But we know that $10^4/10^4 = 1$ since any number divided by itself equals one. Hence $10^0 = 1$.

The basic rules for exponents, which were considered in the last section, can be applied to numbers written as powers of 10. For example

$$10^3 \cdot 10^5 = 10^{3+5} = 10^8 \qquad \boxed{10^a \cdot 10^b = 10^{a+b}} \qquad \text{Eq. 8.4}$$

$$10^7 \div 10^3 = 10^{7-3} = 10^4 \qquad \boxed{10^a \div 10^b = 10^{a-b}} \qquad \text{Eq. 8.5}$$

$$(10^4)^2 = 10^4 \cdot 10^4 = 10^{4 \cdot 2} = 10^8 \qquad \boxed{(10^a)^b = 10^{a \cdot b}} \qquad \text{Eq. 8.6}$$

8.3 Scientific Notation

Any number can easily be expressed in terms of a power of ten. Hence,

$$2300 = 2.3 \times 1000 = 2.3 \cdot 10^3$$

$$16{,}753 = 16.763 \times 1{,}000 = 16{,}753 \cdot 10^3$$

$$0.029 = 2.9 \div 100 = 2.9 \times \frac{1}{100} = 2.9 \cdot 10^{-2}$$

A number may be expressed as a power of ten in several different ways.

For example,

$$2300 = 2.3 \cdot 10^3$$

$$2300 = 23 \cdot 10^2$$

$$2300 = 230 \cdot 10^1$$

$$2300 = 23{,}000 \cdot 10^{-1}\text{, etc.}$$

It has been agreed, for standardization, to write base ten numbers in a form called scientific notation. This form involves a number, equal to or greater than one and less than 10, times 10 raised to a power which is an integer (an integer is a positive or negative whole number, or zero).

Hence 2300 would be written as 2.3×10^3 in scientific notation.

Here are examples of how to convert numbers to scientific notation.

Example 8.1

Write 30,000,000 in scientific notation.

Solution

$$\underbrace{30,000,000}_{\text{decimal point shift}} = 3.0 \cdot 10^n$$

We must shift the decimal point 7 places to the right to get from 3.0 to 30,000,000. Hence $n = 7$, and $30,000,000 = 3.0 \cdot 10^7$

Example 8.2

Write 0.000 000 012 in scientific notation.

Solution

$$\underbrace{0.000\ 000\ 012}_{\text{decimal point shift}} = 1.2 \cdot 10^n$$

Here we must shift the decimal point 8 places to the left to get from 1.2 to 0.000 000 012. Hence $n = -8$, and

$$0.000\ 000\ 012 = 1.2 \cdot 10^{-8}$$

8.4 Logarithms

As previously pointed out, logarithms are exponents. We will only be dealing in this book with logarithms with base 10 (called common logarithms). The connection between logarithms and powers of 10 can be seen in the following relationships.

a) $1000 = 10^3$

b) $\log_{10} 1000 = 3$

The latter equation is read as the logarithm(log, for short) of 1000, to the base 10, is 3. Note that the logarithm, 3, is just the exponent or power to which 10 is raised. Since we will only be dealing with base 10, we will simplify equation b above, as follows:

$$\log 1000 = 3 \quad \text{(base 10 understood)}$$

To reinforce this concept that logarithms equal exponents, consider the following examples:

$$\begin{array}{rl}
10{,}000 = 10^4 & \log 10{,}000 = 4 \\
1{,}000 = 10^3 & \log 1{,}000 = 3 \\
100 = 10^2 & \log 100 = 2 \\
10 = 10^1 & \log 10 = 1 \\
1 = 10^0 & \log 1 = 0 \\
0.1 = 10^{-1} & \log 0.1 = -1 \\
0.01 = 10^{-2} & \log 0.01 = -2 \\
0.001 = 10^{-3} & \log 0.001 = -3 \\
0.0001 = 10^{-4} & \log 0.0001 = -4
\end{array}$$

(base 10 understood whenever no base is given)

In each case the logarithm on the right is equal to the corresponding exponent, or power of 10, on the left. In general

$$\boxed{A = 10^n \longrightarrow \log A = \log 10^n = n}$$

Eq. 8.7

Since logarithms are exponents, operations on logarithms follow the same laws as operations on exponents.

Let $A = 10^a$ and $B = 10^b$

Multiplication

$A \cdot B = 10^a \cdot 10^b = 10^{a+b}$ (from Eq. 8.4)

$\log (A \cdot B) = \log (10^a \cdot 10^b) = \log (10^{a+b}) = a + b$ (from Eq. 8.7)

But, since $A = 10^a$, $\log A = a$ (from Eq. 8.7)

and, since $B = 10^b$, $\log B = b$ (from Eq. 8.7)

Hence, $\boxed{\log (A \cdot B) = \log A + \log B}$ Eq. 8.8

The logarithm of the product of two numbers is the sum of the logarithms of the two numbers. This rule can be extended to the product of more than two numbers. That is, $\log (A \cdot B \cdot C \cdot D) = \log A + \log B + \log C + \log D$

Note that the use of logarithms converts a problem in multiplication to a simpler problem in addition.

Example 8.3

Compute 360·42, using logarithms.

Solution

Generally, we wouldn't use logarithms for a simple problem like this. We can easily find the answer, 15,120, by hand or with a calculator. We are using this example to illustrate how logarithms work in a multiplication problem and to give you practice in using them.

Let $x = 360 \cdot 42$

Then, from Eq. 8. , ($\log (A \cdot B) = \log A + \log B$), we get

$\log x = \log 360 + \log 42$

Using a calculator (if you have any problem working with logarithms on your calculator, contact your instructor),

$\log 360 = 2.556302$

$\log 42 = \underline{1.623249}$

$\log 360 + \log 42 = 4.179551 = \log x$

x is the number whose log is 4.179551.

This is expressed as antilog x = 4.179551.

Again, using a hand calculator, we get x = 15119.972, or x = 15,120 (rounded off). We know that we have the correct answer, since it checks the answer obtained from direct multiplication.

Note that we have used 6 digits after the decimal point for the logarithms. Generally logarithm tables give only 4 digits after the decimal point. Use of a hand calculator enables us to get more accurate results. When we don't need so much accuracy, we can use less digits.

Division

$$\frac{A}{B} = \frac{10^a}{10^b} = 10^{a-b} \qquad \text{(from Eq. 8.5)}$$

$$\log\left(\frac{A}{B}\right) = \log\left(\frac{10^a}{10^b}\right) = \log(10^{a-b}) = a - b \qquad \text{(from Eq. 8.7)}$$

Again, $\log A = a$, $\log B = b$

Hence

$$\boxed{\log\left(\frac{A}{B}\right) = \log A - \log B} \qquad \text{Eq. 8.9}$$

The logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator.

Logarithms convert a problem in division to a simpler problem in subtraction.

Example 8.4

Calculate $\frac{4,217,324}{228.327}$, using logarithms.

This can be done by long division, but it would take time, and care would have to be taken to avoid any error. Direct division by a hand calculator gives a result of 18470.54. Again, we will calculate the result using logarithms to give further practice in using logarithms in solving problems.

Let $x = \frac{4,217,324}{228.327}$

Then, from Eq. 8.9 ($\log A/B = \log A - \log B$), we get

$$\log x = \log 4,217,324 - \log 228.327$$

$$\log x = 6.625037 - 2.3585573$$

$$\log x = 4.2664797$$

$$x = \text{anti log } (4.2664797) = \underline{18,470.55}$$

Our result checks that obtained from direct division, except for the discrepancy of 1 in the 7th place, due to round off. Our answer can be rounded off to 18,471 if a result correct to the nearest whole number is sufficient.

Raising to a power

$$(A)^n = (10^a)^n = 10^{a \cdot n} \qquad \text{(from Eq. 8.6)}$$

$$\log (A^n) = \log (10^a)^n = \log 10^{a \cdot n} = a \cdot n \qquad \text{(from Eq. 8.7)}$$

Since $\log A = a$,

$$\boxed{\log A^n = n \log A} \qquad \text{Eq. 8.10}$$

The logarithm of a number raised to a power n is equal to n times the logarithm of the number.

Thus, for example, $\log 3.9^{2.61} = 2.61 \log 3.9$

Logarithms convert a problem in raising a number to a power to a simpler problem in multiplication.

Example 8.5

Compute 3^4 using logarithms

Solution

Here too we can easily get the answer without using logarithms. It is $3 \cdot 3 \cdot 3 \cdot 3 = 81$. But we want to solve it also by logarithms to reinforce the principles of this chapter.

$$\text{Let } x = 3^4$$

$$\log x = \log 3^4 = 4 \log 3 \qquad \text{(from Eq. 8.10)}$$

$$\log x = 4(0.477121) = 1.908485$$

$$x = \text{antilog}\ (1.908485) = \underline{81} \qquad \text{(it checks)}$$

Example 8.6

Compute $(3.2513)^{2.847}$ using logarithms

This problem requires the use of logarithms or a calculator. If your calculator can raise numbers to a power, you can get the result directly, 28.696333. If not, you'll need logarithms as shown below.

$$\text{Let } x = (3.2513)^{2.847}$$

$$\log x = \log\ (3.2513)^{2.847}$$

$$\log x = 2.847 \log 3.2513 \qquad \text{(Eq. 8.10)}$$

$$= 2.847(0.512057) = 1.4578264$$

$$x = \text{antilog}\ (1.4578264) = \underline{28.696333}$$

Our result checks that found above from raising a number to a power. With practice, many of the steps shown here can be eliminated and you can get results faster.

Note the similarity of the equations for exponents and the equations for logarithms. The base for the logarithms is left out since we are only considering base 10.

$x^a \cdot x^b = x^{a+b}$ (Eq. 8.1) $\log AB = \log A + \log B$ (Eq. 8.8)

In equation 8.1, we add exponents; in equation 8.8, we add logarithms.

$\dfrac{x^a}{x^b} = x^{a-b}$ (Eq. 8.2) $\log \dfrac{A}{B} = \log A - \log B$ (Eq. 8.9)

In equation 8.2, we subtract exponents; in equation 8.9, we subtract logarithms.

$(x^m)^n = x^{m \cdot n}$ (Eq. 8.3) $\log A^n = n \log A$ (Eq. 8.10)

In equation 8.3, we multiply the exponent m by n; in equation 8.10, we multiply the logarithm by n.

Exercises

1. Express the following numbers in scientific notation:

   a. 3,640,000 b. 0.000 32 c. 325×10^4 d. 214×10^{-5}

2. Write the following numbers out in full length.

   a. 3.0×10^6 b. 3.2×10^{-4} c. 1.19×10^5

3. Perform the operations indicated and simplify.

   a. $10^5 \times 10^3$ b. $10^9 \times 10^{-3}$ c. $(2 \times 10^4)(5.3 \times 10^3)$

   d. $(10^{15}/10^7)$ e. $(4 \times 10^8)(3 \times 10^5) \big/ (2 \times 10^6)$

4. Solve for x in each case using logarithms.

   a. $x = 326 \cdot 24$

   b. $x = 473/36$

   c. $x = (3.12)^{4.71}$

   d. $x = \frac{(28)(35)}{(12)(7)}$

   e. $x = \sqrt{7} = 7^{\frac{1}{2}}$

8.5 Applications of Logarithms to Population Growth, Resource Depletion, and Sequences

We will now use the principles of logarithms to solve problems related to population growth.

A population that is growing geometrically may be expressed as

$$\boxed{P = P_o\ (1 + r)^t} \qquad \text{Eq. 8.70a}$$

where P = population at the end of a given period

P_o = population at the start of the period

r = relative increase (or decrease) per year, expressed as a decimal

t = the number of years in the period

Note that the above equation represents a geometric sequence because it represents an increase in population by a factor $(1 + r)$ every year.

Equation 8.7a is similar to the equation that would be used to compute the money in a bank account that increases according to a compound interest law, as discussed in chapter 5.

Example 8.6

Let us first find a general equation for the time to double a population.

Solution

In this case P will be twice P_o. Hence,

$$P = 2\,P_o = P_o(1 + r)^t$$

$$2 = (1 + r)^t \qquad \text{(both sides divided by } P_o\text{)}$$

$$\log 2 = t \log (1 + r) \qquad \text{(from Eq. 8.10)}$$

$$\boxed{t = \frac{\log 2}{\log (1 + r)} = \frac{0.30103}{\log (1 + r)}} \qquad \text{Eq. 8.11}$$

In chapter 3, we used the equation,

$$\text{doubling time} = \frac{70}{\text{growth rate (\%)}}$$

The table below compares doubling times from that equation with doubling times from equation 8.11.

rate of increase (%)	r	1 + r	log (1 + r)	$\frac{.30103}{\log (1 + r)}$	$\frac{70}{\text{inc. (\%)}}$
1%	0.01	1.01	0.0043214	69.7 yrs.	70.0
2%	0.02	1.02	0.0086002	35.0	35.0
3%	0.03	1.03	0.0128372	23.4	23.3
4%	0.04	1.04	0.0170333	17.7	17.5
5%	0.05	1.05	0.0211893	14.2	14.0
6%	0.06	1.06	0.0253059	11.9	11.7
7%	0.07	1.07	0.0293838	10.2	10.0
8%	0.08	1.08	0.0334238	9.0	8.8
9%	0.09	1.09	0.0374265	8.0	7.8
10%	0.10	1.10	0.0413927	7.3	7.0

Table 8.1

It can be seen from the close agreement of columns 6 and 7 above that the equation 70/growth rate (%) is valid for computing doubling times. However, as the rate of population growth increases, it becomes less and less accurate and shouldn't be used for rates of growth of 15% or more.

Example 8.7

A population changes from 200 million people to 362 million in 20 years. (a) Find the rate of increase. (b) How long did it take the population to increase from 200 million to 300 million people?

$$P = P_o(1 + r)^t$$

Here P = 362 million people

P_o = 200 million people

$r = ?$

t = 20 years

(a) 362 million = 200 million $(1 + r)^{20}$

$$\frac{362}{200} = 1.81 = (1 + r)^{20}$$

$\log 1.81 = 20 \cdot \log (1 + r)$ (from Eq. 8.10)

$\log 1.81 = 0.257679$ (using hand calculator)

$$\frac{0.257679}{20} = \log (1 + r) = 0.0128840$$

The antilog of 0.0128840 = 1.0301 (using hand calculator)

$1 + r = 1.0301$

$r = 0.03$ or 3%

(b) $P = P_o(1 + r)^t$

300 million = 200 million $(1.03)^t$

$$\frac{300}{200} = 1.5 = (1.03)^t$$

$\log 1.5 = t \log 1.03$

$$t = \frac{\log 1.5}{\log 1.03} \quad \frac{0.1761}{0.01284} \text{ years} = \underline{13.7 \text{ years}}$$

Example 8.8

A certain natural resource is being depleted at the rate of 5% per year.

a) How long does it take to use one-half of the amount that is available today?

b) If there were 150 million tons of this resource available in 1975, how much would be available in the year 2000?

We can modify our equation for population growth so that it applies to resources as follows:

$$\boxed{R = R_o(1 + r)^t} \qquad \text{Eq. 8.12}$$

where R = amount of resources at the end of a given period

R_o = amount of resources at the start of the period

r = rate of change of the resource; r is negative if the resource is being depleted

t = the number of years in the period.

Note that this equation does not take into account the discovery of new resources which would have to be added in to get the total amount of resources at any time.

a) Here, $R = \frac{1}{2}R_o$ and $r = -0.05$. Note the negative sign for r, since resources are being consumed each year.

$$\frac{1}{2}R_o = R_o(1 + (-0.05))^t$$

$$\frac{1}{2} = (0.95)^t \qquad \text{(dividing both sides by } R_o\text{)}$$

$$\log\left(\frac{1}{2}\right) = t \log 0.95 \qquad \text{(Eq. 8.10)}$$

$$t = \frac{\log 0.5}{\log 0.95} = \frac{-0.3010}{-0.0223}$$

$$t = \underline{13.5 \text{ years}}$$

b) $R = R_o(1 + r)^t$

Here R_o = 150 million tons

R = ? million tons

$r = -0.05$

$t = 2000 - 1975 = 25$ years

$$R = 150 \text{ million tons} \cdot (1 + (-0.05))^{25}$$

$$= 150 \text{ million tons} \cdot (0.95)^{25}$$

$$\log R = \log 1.5 \times 10^8 + \log (0.95)^{25} \qquad \text{(Eq. 8.8)}$$

$$\log R = \log 1.5 \times 10^8 + 25 \log (0.95) \qquad \text{(Eq. 8.10)}$$

$$\log R = 8.1761 + 25(-0.02228)$$

$$= 8.1761 - 0.5570 = 7.6191$$

$$R = 41{,}552,\ 772 \text{ tons}$$

$$R = \underline{41.6 \text{ million tons}}$$ (rounded off)

This result seems reasonable, since 25 years represents almost two time periods in which the amount of resources is halved, and ½·½·150 million tons is 37.5 million tons.

Another equation that is valid for a population that is growing geometrically such that the rate of population increase is proportional to the population at any time is

$$\boxed{P = P_o \cdot 2^{t/K}} \qquad \text{Eq. 8.13}$$

where P_o = the population at an initial time

P = the population at a later time

t = the elapsed time from the initial time

K = the doubling time

To check the correctness of this equation, consider the following:

when $t = 0$ $\quad P = P_o \,.\, 2^{\,0/K} = P_o \cdot 2^0 = P_o \cdot 1 = P_o$

$t = K$ $\quad P = P_o \cdot 2^{K/K} = P_o \cdot 2^1 = 2P_o$

$t = 2K$ $\quad P = P_o \cdot 2^{2K/K} = P_o \cdot 2^2 = 4P_o$

$t = 3K$ $\quad P = P_o \cdot 2^{3K/K} = P_o \cdot 2^3 = 8P_o$

We see that the population doubles every K units of time. This checks with the definition of a doubling time.

Example 8.9

Mexico's mid-1979 population was 67.7 million people and its doubling time is 20 years. If current rates of population growth continue, what will Mexico's population be in 50 years?

Solution

$$P = P_o \cdot 2^{t/K}$$

In this case, P_o = Mexico's present population = 67.7 million people

P = Mexico's population in 50 years (from 1979)

t = the elapsed time (from 1979)

K = the doubling time = 20 years

Substituting, we get

$$P = 67.7 \cdot 2^{50/20} = 67.7 \cdot 2^{2.5}$$

$$\log P = \log 67.7 + 2.5 \log 2$$

$$= 1.8306 + 2.5(0.30103)$$

$$= 1.8306 + 0.7526$$

$$\log P = 2.5832$$

$$P = \underline{383 \text{ million people}}$$

Note the tremendous increase in Mexico's population in 50 years, if current population growth rates continue. It is

$$\frac{383 - 67.7}{67.7} \times 100 = 465.7\%$$

An equation for a radioactive element which decays at a rate which is proportional to the amount present is

$$M = \frac{M_o}{2^{t/h}} \qquad \text{Eq. 8.14}$$

where M_o = the amount at an initial time

M = the amount at a later time

t = the elapsed time from the initial time h = the half-life (the elapsed time for the amount of a substance or radiation to be half of what it was at the start of the period)

The following calculations show the correctness of this equation.

when $t = 0$ $$M = \frac{M_o}{2^{0/h}} = \frac{M_o}{2^0} = \frac{M_o}{1} = M_o$$

when $t = h$ $$M = \frac{M_o}{2^{h/h}} = \frac{M_o}{2^1} = \tfrac{1}{2} M_o$$

when $t = 2h$ $$M = \frac{M_o}{2^{2h/h}} = \frac{M_o}{2^2} = \tfrac{1}{4} M_o$$

Observe that for every change of time equal to h units, we have half of what we had at the beginning of the period. This checks with the definition of half life.

Example 8.10

Twenty percent of a radioactive material decays in 50 years. What is its half life?

We use $$M = \frac{M_o}{2^{t/h}}$$

Since 20% of the radioactive material decays in 50 years, 80% is still left. Hence $M = 80\% \, M_o = 0.80 \, M_o$

$$0.80 \, M_o = \frac{M_o}{2^{50/h}} = M_o \cdot 2^{-50/h}$$

$0.80 = 2^{-50/h}$ (dividing both sided by M_o)

$\log 0.80 = \frac{-50}{h} \log 2$ (From Eq. 8.10)

$h = -50 \cdot \frac{\log 2}{\log 0.8}$ (Solving for h)

$$h = -50 \cdot \frac{(0.30103)}{(-0.09691)}$$

$h =$ 155.3 years

Many sequence problems can be done quickly using logarithms.

Example 8.11

How many ancestors did we have 20 generations ago?

One generation ago we have two ancestors, our parents.

Two generations ago, we have four ancestors, our grandparents.

We continue in tabular form.

generations back	ancestors	
1	$2 = 2^1$	
2	$4 = 2^2$	
3	$8 = 2^3$	(note the pattern)
4	$16 = 2^4$	
n	2^n	
20	$? = 2^{20}$	

The required value $n = 2^{20}$. Hence,

$$\log n = 20 \log 2 \qquad \text{(Eq. 8.10)}$$

$$\log n = 20(0.30103)$$

$$\log n = 6.0206$$

$$n = \text{antilog } 6.0206 = \underline{1{,}048{,}576} \text{ ancestors}$$

This checks the result obtained by multiplying 2 by itself 20 times.

Exercises

1. A population is growing geometrically at the rate of 1.5% a year.
   a. Compute the doubling time for the population.
   b. How many years would it take for the population to grow from 200 million people to 500 million people?
2. A geometrically growing population increases from 240 million people to 380 million people in 20 years.

   a. What is the percent increase in population per year?
   b. What is the approximate doubling time?

3. A certain natural resource is being depleted at the rate of 4% per year.
   a. How long does it take to use one-half of the amount that is available today?
   b. If there were 300 million tons of this resource available in 1975, how much would be available in the year 2000?
4. A certain radioactive material has a half-life of 50 years. What is its rate of decay (% decrease per year)?
5. Using the mid-1979 population and doubling time of Brazil from the 1979 World Population Data Sheet (chapter 3), compute the projected population of Brazil in (a) 50 years and (b) 70 years.
6. A radioactive substance has a present value of radioactivity of 500 curies and a half life of 20 years. How much radioactivity will there be after (a) 20 years (b) 40 years (c) 50 years?

The following 4 problems involve geometric (binary) sequences, chapter 5.

7. Solve exercise problem 4 chapter 5, using logarithms.
8. Solve exercise problem 8 chapter 5, using logarithms.
9. Solve exercise problem 9 chapter 5, using logarithms.
10. Solve exercise problem 10 chapter 5, using logarithms.

8.6 Applications of Logarithms to Noise Pollution

There has been increasing awareness of the growing problem of noise pollution and the threats it poses to our health and our sanity. Teenagers have been found to be suffering from hearing problems due to long exposure to amplified rock music. Loud noise has been found to raise blood pressure, cause psychic stresses, increase the pulse rate, cause muscle tension, and affect the endocrine and other glands. Excessive noise lowers the efficiency of people trying to work or study.

The recent controversy over landing rights for the SST (supersonic transport plane) demonstrated that people are beginning to protest to prevent excessive noise in their neighborhoods.

Sound intensity is measured in decibels (dB). This unit is named after Alexander Graham Bell (1847-1922) who invented the telephone and did much research on the nature of sound. Since there is a very large range of sound intensity values, the decibel scale is a logarithmic scale which depends on the ratio of the intensity of the sound being considered to that of a reference sound.

The equation that is used is

$$\text{Sound intensity (dB)} = 10 \log (I/I_o) \qquad \text{Eq. 8.15}$$

where I = intensity of the sound in watts/square meter (w/m^2)

I_o = reference intensity = 10^{-12} w/m^2

I_o is the threshold of hearing; it is the lowest intensity that can be heard by some people.

Example 8.12 A certain sound has an intensity of 0.1 watts/square meter. How many decibels does this represent?

$$\text{Here,} \quad \frac{I}{I_o} = \frac{0.1\ w/m^2}{10^{-12}\ w/m^2} = \frac{10^{-1}}{10^{-12}} = 10^{11}$$

Hence, sound intensity (dB) = $10 \log (I/I_o) = 10 \log 10^{11} = 10 \cdot 11 = \underline{110\ \text{dB}}$

Here is a tabulation of how various sound intensities in decibels affect the average person.[1]

Decibel level	Effect
30	very quiet
50	sleep interference
70	annoyance threshold
90	very annoying
100	discomfort
130-140	acute pain

For reference purposes, here is a list of approximate decibel ratings for various sounds.[2]

Sound	Decibel Rating	Sound	Decibel Rating
threshold of hearing	0	vacuum cleaner	70
rustling leaf	10	garbage disposal	80
broadcast studio	20	motorcycle, 20 feet away	90
soft whisper	30	newspaper press	100
bedroom	40	riveter	110
light traffic noise, 100 feet away	53	thunder clap	120
air conditioning unit, 20 feet away	60	jet at takeoff	150

Every increase of 10 dB means that a sound is ten times louder. Hence, a newspaper press (100 dB) is $10 \cdot 10 \cdot 10 = 1{,}000$ times louder than a vacuum cleaner (70 dB).

Example 8.13

We're 10 feet away from two identical speakers which each have intensities of 90 dB. Find the total intensity from the two speakers.

We cannot just add the two intensities in decibels to get the total intensity in decibels. We must first obtain the total intensity in watts/square meter (w/m^2) for the two speakers.

For each speaker, $I(dB) = 10 \log (I/I_0)$

$$90 = 10 \log \frac{I}{10^{-12}}$$

$$\log \frac{I}{10^{-12}} = 9$$

$$\frac{I}{10^{-12}} = 10^9$$

$$I = 10^9 \cdot 10^{-12} = 10^{-3} \text{ w/m}^2, \text{ for each speaker}$$

Now, we can compute the total intensity of the two speakers in decibels.

$$I(\text{dB}) = 10 \log \frac{I}{I_o}$$

$$= 10 \log \frac{2 \times 10^{-3}}{10^{-12}}$$ (Intensity for 2 speakers is 2×10^{-3} w/m^2)

$$= 10 \log 2 \times 10^9$$

$$= 10(\log 2 + \log 10^9)$$

$$= 10(0.301 + 9)$$

$$= \underline{93 \text{ dB}}$$

Note that the intensity in terms of watts per square meter has doubled, but the intensity in terms of decibels has only increased by 3 decibels.

Exercises

1. A person makes a sound that has 100 thousand times the power of the faintest audible sound. What is the sound power in decibels?
2. The sound of a certain vacuum cleaner in a room has 100 million times the intensity of the faintest audible sound. What is the intensity of the sound in decibels?
3. A certain sound source has a power rating of 80 dB. What is the intensity of this sound in w/m^2?
4. Suppose that a single jet plane produces a sound level of 100 dB on take-off, at a given distance from the runway. What would be the sound level in decibels at this same position if 8 of these jets took-off at the same time?

5. A four engine jet plane is on a runway. The sound of each engine is 100 dB. What is the sound level when one engine is running? two engines? three engines? four engines?

Notes

1. Department of Air Resources, New York City EPA, pamphlet "Noise makes you sick."
2. Amos Turk, et al, Environmental Science, Philadelphia, W. B. Saunders Company, 1978, p. 532.

Chapter 9 Introduction to Statistics; Sampling

9.1 Introduction to Statistics

Applications of statistics and statistical reasoning have increasingly entered many fields of endeavor in recent years, including many related to global issues. H. G. Wells was prophetic when he stated many years ago: "Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write."

Statistics aims to provide understanding of issues through the consideration of numbers. The study of numbers (data) is divided into three categories:

1. collection of data
2. description and presentation of data
3. drawing conclusions from data

In this book, we consider the collection of data in this chapter, the description and presentation of data in the next three chapters, plus chapter 4 (graphical methods), and the drawing of conclusions from data in chapters 14 and 15. The aim of the presentations given here is to introduce you to statistical methods, and show applications to global issues. For a more extensive treatment of statistics, consult a basic textbook in the subject.

Statistics is a powerful tool of analysis. It can give insight that can be very helpful in making decisions. It is essential that all citizens know the rudiments of statistics since so much of what they daily read in the newspapers and see on television is based on statistics.

While realizing the great power of statistics in analyzing a situation, we should be aware that statistics can be used to mislead, especially by people who wish to push a point of view. We should be skeptical of any statistical argument, even those in this book, until we have investigated the source of the information, how the information was obtained, and if there are any factors that have not been considered. Keeping the following frequently quoted statements in mind, may be helpful:

1. "Figures don't lie, but liers figure."
2. "There are three kinds of lies; lies, damn lies, and statistics." Disraeli
3. "Statistics is the art of drawing a crooked line from an unwarranted assumption to a foregone conclusion."
4. "Many politicians use statistics much like a drunk uses a lamp post - more for support than for illumination."

In chapter 4, we indicated several ways in which graphs and bar charts can give a misleading impression. In the next chapter, we will show that there is a need to be careful in the use of various measures of central tendency, to avoid uncertainty.

9.2 Definitions of Some Statistical Terms

In this article we will define and illustrate some statistical terms that will be of value in this and the next few chapters on statistics.

A population is the set of all quantities of a certain kind. There are many different kinds of populations, depending on the purpose of an investigation. If we wish to know the opinions of adults on Staten Island, we could define the population as all residents on Staten Island, 18 years of age, and above. Other populations could be all college students, all people who voted Democratic in the last presidential election, all college graduates in the U. S., all steel bars used in nuclear power plants, all petro-chemical factories in New Jersey, etc. The census held every year in the United States considers all people in the United States as the population for its study.

Since it is generally difficult or very costly to study an entire population, a sample of the population is considered. Elements of a good sample are discussed in the next article. A sample consists of elements drawn from a population in order to predict characteristics of the population. To get information on the viewing habits of Americans, the Nielsen sample is used; this consists of about 1200 families that are monitored in terms of the TV shows that they watch. To obtain information about the views of his or her constituents, a politician can obtain the views of a portion (sample) of them.

A parameter is a property of a population. It could be an average income, a maximum GNP per capita, a number of crimes committed, etc. The important point is that it must be a property which takes the entire population into account.

A statistic is a property of a sample. If a sample is a good sample (see the next article), a statistic will be valuable in predicting a parameter (a property of a population). Much of the controversy in the science of statistics is related to the reliability of inferring a property of a population from a property of a sample.

A random sample is one in which each element of the population has an equal chance of being in the sample. If we wanted a random sample of all the countries on the 1979 World Population Data Sheet, we could put each country on a separate card, shuffle the cards well, and then select 20 cards, say, as our sample. A random sample may not be a representative sample. For example, it is possible that all or most of the 20 countries selected in the example above will be from Africa, or developed countries, etc.

A stratified random sample is one in which the number of elements of each type in the sample is related to the number of elements of each type in the population. For example, we might choose one or two countries from each region on the World Population Data Sheet for our sample to assure some geographical and economical balance. A sample of college students might be stratified so that freshmen, sophomores, juniors, and seniors were proportionately represented. A sampling of adults could be stratified according to age, income, religion, political party affiliation, etc. It is important that the sample within each stratification be random.

A distribution is the variation in values for a population or sample. If we are considering PQLI values for the countries of Northern Africa, the distribution would be 41, 44, 43, 40, 34, and 46 (see the 1979 World Population Data Sheet). If we are studying the per capita GNP of Middle America, the distribution would be $1,240, $570, $790, $450, $1,100, $830, and $1,220 (see Data Sheet).

9.3 Sampling

Suppose we wanted to know the quality of drinking water in our town. We wouldn't have to test all of it. A sampling of the water at various locations would give us much information. Suppose we wanted to know if people would be willing to spend 10% more for a quieter lawn mower. We couldn't ask every potential customer. Here too, a sampling of opinion would be very valuable. In many other cases related to human survival, information can be obtained relatively quickly and inexpensively using a sample.

However, we must be very careful in projecting results from a sample to a population. There are three essential elements that must be met if a sample is to be meaningful.

1. The sample must be large enough. Would asking just 3 or 4 people their opinion on an environmental issue give a good indication of a general population's views? Clearly not. Many more people would have to be surveyed. The size of sample required depends on circumstances. For example, to test the effectiveness of the Salk Polio vaccine, a very large sample was necessary, since very few people normally contracted polio. The smaller the probability of something happening (see Chapter 12), the larger the sample should be.

2. The survey must be based on a random sample of people or situations. This is where many samples lose their validity. For example, surveying a group

of people at a convention of environmentalists as to their views on air pollution would not produce meaningful results for the general population. In the same way asking subscribers to the Wall Street Journal their opinion on some economic matter would not give a viewpoint representative of everyone. Taking samples of air only in the middle of Central Park or just at a toll booth outside a tunnel would not give representative results for New York City.

In order for a sample to be random, every person or location must have an equal opportunity to be part of the survey. The lack of randomness is where many surveys become meaningless. It is far better to survey a small group of people chosen randomly than a much larger group which is not a random sample. Sometimes it is valuable to use a stratified random sample, so that all groups in a population are adequately represented.

3. The possibility of bias must be considered. People don't always tell the truth. They like to give answers pleasing to the questioner. They often give answers consistent with an image of themselves rather than the true reality. People might say that they think we should change our life styles in order to give more to poor, hungry people, but will they make such changes? People may claim they'll pay more for less polluting equipment, but will they follow through when the time to purchase comes?

The question in the survey must be carefully selected so as to be free from bias. For example, consider the following question: "Since we already have enough nuclear weapons to destroy the whole world, do you believe we should reduce the defense budget?" Clearly this question is not objective; it is attempting to elicit a negative answer by implying that we already have more weapons than we need.

Consider the following question used recently by a national polling organization: "Most people eat quite a lot of meat. But some people are vegetarians and don't believe in eating any meat. Would you describe yourself as a strict vegetarian, or pretty much a vegetarian, or one who eats meat but is careful about how much, or as one who eats meat quite often and regularly?"

There are some problems with this. It is not only a question, but it also makes some statements. It implies at least two things:

1. It is "normal" to eat plenty of meat.

2. Vegetarians are part of a small minority and they may be viewed as radical if they don't eat any meat.

Another problem with the "question" is that certain phrases such as "strict vegetarian" and "pretty much a vegetarian" may mean different things to different people.

It is very difficult but essential that questions be worded in such a way that they are completely clear and objective.

One way in which experimenters eliminate unconscious bias is through the use of a placebo. In testing whether vitamin C is valuable in reducing the number of colds children get, a sample of children was given a solution with vitamin C and a second "control" sample was given a placebo, a solution identical to that given to the other children, but without the vitamin C. None of the children knew whether they were given vitamin C or the placebo, so that their views on symptoms would not be influenced by that knowledge.

Sometimes a "double-blind" experiment is performed. Here both the investigator and subject don't know if the actual substance being tested or a placebo has been used in a particular case. This method was used in testing the Salk polio vaccine. It was felt that if doctors knew which children had received the polio vaccine, their judgment in a difficult case might be influenced.

To summarize, a good survey or sample must be (1) large enough, (2) random, and (3) free from bias.

Successful surveys can be used to get positive actions regarding human survival. For example, a survey showing that people favor a mandatory deposit on beverage containers, nutritious foods in vending machines, or more developmental aid to poor countries, can be used to help get action on such issues.

The best way to learn about samples is to carry one out. Below is an example of some questions that can be asked in a survey. Try these. Make up more. Find an issue or issues that you are very interested in. Get a sample of opinion on these issues. Use it to help generate action. But see that the sample is a valid one by making sure that it meets the 3 conditions discussed above.

These questions can be used as the basis of many mathematics problems related to basic calculations, graphs, averages, correlation, and hypothesis testing.

MATH AND ENVIRONMENT SURVEY

1. How many children are in your family (yourself and your brothers and sisters)?__________

2. How many children do you expect to have?________

3. How much total annual income do you feel is needed to support a family of four in New York City?_____

4. How serious do you feel the world's population crisis is?____

   1) not at all serious
   2) of slight concern
   3) of moderate concern
   4) serious
   5) very serious

5. Do you smoke cigarettes?

6. About what per cent of College of Staten Island students do you think smoke? ___________________ (cigarettes)

7. Do you think that smoking should be permitted in classes at our school?_________

1) Definitely not ___ 2) No___ 3) don't care___ 4) yes___

5) definitely yes___

8. What per cent of C.O.S.I. students do you think come to school by car?_____

9. Do you come to school by car?______ If yes, how many passengers including the driver are in the car, on the average?________

10. Do you think that vending machines at our school should have at least 50% nutritious foods such as fruit, juice, nuts, etc.

    1) definitely not____ 2) No___ 3) don't care____ 4) yes___

    5) definitely yes____

11. Do you think that beverages (beer or soda) should come in containers that have a mandatory deposit of at least 5 cents?

    1) definitely not___ 2) no___ 3) don't care___ 4) yes___

    5) definitely yes___.

12. The nations of the world should cut the amount of money spent on military affairs and provide more for health, nutrition, education and development. Do you agree?

    1) definitely not___ 2) no___ 3) don't care 4) yes___
    5) definitely yes___

Exercises

1. Plan and carry out a survey to find the average number of passengers in cars coming to your campus.

2. Set up and conduct a poll to find out if people would be willing to pay 10% more for a quieter lawn mower.

3. Outline a plan to carry out a survey to investigate how much litter there is at your school.

4. Poll students at your school on some current controversial issue. Analyze the data and indicate your conclusions. The next 6 chapters will provide tools for the analysis.

5. In what way(s) could the results of the following surveys be misleading?

   a. You ask the students in your environmental class if they feel that there should be a mandatory deposit on beverage containers.

   b. The National Rifle Association sends a questionnaire to its members to get their opinions on gun control legislation.

   c. A drug company asks 4 doctors to give an opinion on one of its products.

   d. You toss a coin ten times to see if it is a fair coin.

   e. You send a letter to all College of Staten Island graduates of the class of 1970, in order to obtain information about their incomes.

   f. A large number of people are called on the telephone to find out if they support a certain political candidate.

   g. To find out how people feel about police service in a community, a police officer is sent to a random sample of 300 addresses in the community to administer a questionnaire.

   h. A city council person uses letters from constituents to assess their views on an issue soon to be voted on.

i. Ann Landers, the advice columnist, based on a letter from a young couple, asked her readers "If you had it to do over again, would you have children?" Her results: 70 percent of the parents who responded said they would not have children if they made the choice again.

j. To assess public reaction to a speech made by a congressman, a reporter interviewed the first 50 people willing to give their views.

9.4 Sampling to Estimate Wildlife Populations

One result of recent rapid increases in human population and pollution has been threats to many species of wildlife. To ensure continuation of a wildlife species, it is necessary to have information about its population. Since wildlife generally do not stay in one location, it is difficult to get accuract information. One method of estimating the population of a given species is the "tag and recover" approach.

This method can be described by analogy. Suppose we have a container with a very large number of marbles. We wish to estimate the number of marbles, but it would take too long to count them. Suppose we take out 100 marbles from the container, mark them, and return them to the container. We then make sure that the marbles are mixed well, so that the marked marbles are randomly distributed among all the marbles. Assume that we then take out 200 marbles from the container, and that 18 of the marbles are found to be marked. We can then estimate the total number of marbles in the container from the following proportion:

$$\frac{\text{number of marked marbles}}{\text{total number of marbles}} = \frac{\text{marked marbles taken out}}{\text{marbles taken out}}$$

In this case,

$$\frac{\text{100 marked marbles}}{\text{total number of marbles}} = \frac{\text{18 marked marbles out}}{\text{200 marbles out}}$$

Cross multiplying, we get

$$18\cdot(\text{total number of marbles}) = 100\cdot 200$$

$$\text{total number of marbles} = \frac{100\cdot 200}{18} = 1111.1$$

The estimated total number of marbles is 1,110.

This approach can be applied to wildlife when the population considered is confined to an area with natural boundaries, such as an island or a small pond. We would catch some members of the wildlife population, tag them, and then release them. Later, we would recapture members of the species, and count the number of wildlife that were tagged. We could then estimate the number of that species in the region from the following proportion (for a given animal or fish):

$$\boxed{\frac{\text{number of tagged animals released}}{\text{total animals in the region}} = \frac{\text{number of tagged animals captured}}{\text{total animals captured}}} \quad \text{Eq. 9.1}$$

Steps should be taken to insure that the wildlife tagged and those recaptured are random samples by using random locations and times.

Example 9.1

Three hundred bass were captured, tagged, and released. Later, 15 out of 200 bass captured in the pond were found to be tagged. Estimate the number of bass in the pond.

Solution

We can apply the following proportion:

$$\frac{\text{tagged bass released}}{\text{total bass in pond}} = \frac{\text{tagged bass captured}}{\text{total bass captured}}$$

$$\frac{\text{300 tagged bass released}}{\text{total bass in pond}} = \frac{\text{15 tagged bass captured}}{\text{200 bass captured}}$$

Cross multiplying, we get

$$15\cdot(\text{total bass in pond}) = 300\cdot 200 = 60{,}000$$

$$\text{total bass in pond} = \frac{60{,}000}{15} = 4{,}000$$

Exercises

1. A fishery biologist takes 300 bass from a lake, marks them with tags, and releases them. Two weeks later, he nets another sample of 200, of which 30 are marked. What is the bass population in the lake?

2. Three hundred members of a certain animal species were captured, tagged, and released. Several weeks later, a sample of 100 members of that species was found, with ten of them having tags. Estimate the number of members of that species in the closed region.

3. Estimate the population of bass in a lake for each of the following situations:

    a) 400 tagged bass released; later, 25 out of 200 bass captured had tags.

    b) 200 tagged bass released; later 15 of 50 bass recaptured had tags.

Chapter 10 Measures of Central Tendency

Frequently when we have a large amount of data, we are interested in summarizing it to make it more understandable. One method of describing numerical data is by using a measure of central tendency or an average value. We shall apply measures of central tendency to several problems related to global survival in this chapter.

The life expectancy for a baby born in the world in 1979 was 60 years.[1] This does not mean that every baby born in 1979 will live 60 years. Some will live longer (barring major global calamities) and some will live less than 60 years. This value is a measure of central tendency, an average value.

The gross national product per capita (per person) in 1979 for the world was $1800.[2] This does not mean that for every person in the world, $1800 of goods and services were produced. Again, we have a measure of central tendency or average value.

There are 3 frequently used measures of central tendency: the arithmetic mean, the median, and the mode.

The arithmetic mean or mean, for short, is the value obtained by adding up all measurements or quantities in a distribution and dividing by the number of measurements or quantities. Hence, arithmetic mean = $\frac{\text{total of all quantities}}{\text{number of quantities}}$. This is frequently abbreviated

$$\mu = \frac{\Sigma x}{n}$$

where μ (Greek letter mu) is the arithmetic mean or mean

x is an individual measurement or quantity

Σx means add up all the individual measurements or quantities (Σ, the Greek letter sigma, is generally used as a short hand for summation)

n is the number of measurements or quantities

This equation is valid for a population. When we are dealing with a sample, the same equation is valid, but $\overline{X}$ is frequently used to denote the mean, instead of μ; hence $\overline{X} = \frac{\Sigma x}{n}$ is the form of the equation for a sample. Since both equations give the same arithmetic mean, we shall always use equation 10.1 ($\mu = \frac{\Sigma x}{n}$) in this book.

When we refer to an average value from now on we will mean the arithmetic mean, unless otherwise noted.

Example 10.1

Ten families have the following number of children: 3, 2, 5, 1, 1, 3, 0, 1, 2, 0. Compute the arithmetic mean.

Solution

To compute the arithmetic mean, we use

$$\mu = \frac{\Sigma x}{n}$$

where Σx = the total number of children

$\Sigma x = 3 + 2 + 5 + 1 + 1 + 3 + 0 + 1 + 2 + 0 = 18$

$n = 10$ (the number of families)

Thus $\mu = \frac{18}{10} = 1.8$

This value looks reasonable (close to what we would expect) with regard to the given data since there are 3 values greater than 2 and 5 values less than 2. Note that we need not get a whole number for the average value, even though each couple has a whole number of children.

The median is the middle value when all measurements or quantities are listed in numerical order. Half of the values are above the median and half are below. When there are an odd number of values, the median is the middle one. When there are an even number, the median is the average of the two middle values.

The mode is the measurement or quantity that occurs most often. If there is no numerical value that occurs more than once, there is no mode. If the two highest numerical values occur ~~more than once~~ the same number of times, it is considered to be a bimodal case. If three or more ~~quantities~~ equal peaks occur, ~~the same amount of times;~~ there is again considered to be no mode.

Example 10.2

Compute the median and mode for the data of example 1.

To compute the median, we list the data again, this time in numerical order

0, 0, 1, 1, 1, 2, 2, 3, 3, 5
↑
median (average of two middle values) = 1.5

Since there are an even number of values the median is the average of the two middle values, or $\frac{1 + 2}{2}$ = 1.5 children.

The mode is the value that occurs most often, or 1 child.

Example 10.3

A survey was taken to find out how many children various couples wanted, with the following results:

Children desired per family	Number of families
0	7
1	16
2	36
3	24
4	12
5	3
6	2

Compute the arithmetic mean, median, and mode for the children desired per family.

The key information is summarized in the table below. The total number of families surveyed (n) is 100. To get the total number of children desired by the 100 couples we multiply the values in columns (1) and (2) for each row. For example, there are 24 couples who desire 3 children. Hence a total of 24 x 3 = 72 children are desired by these 24 couples. Σx is the total of all values in column 3, the total of all children wanted by the 100 couples, or 235.

(1) Children desired per family	(2) Number of families	(3) = (1) x (2) Total children desired
0	7	0
1	16	16
2	36	72
3	24	72
4	12	48
5	3	15
6	2	12
	100	235

The mean is the total number of children (235) divided by the total number of families (100).

$$\mu = \Sigma x/n = 235/100 = \underline{2.35}$$

The median will be the average of the 50th and 51st values, since there are 100 families. Since there are 23 families (7 + 16) having less than 2 children and 41 families having more than 2 children, the median number of children must be 2.

The mode is the value for children desired per family that occurs most often. If we listed children desired per family in a row we would have 7 zeros, 16 ones, 36 twos, etc. Since most families (36) want 2 children, the mode is 2 children per family.

Previously we have shown (chapter 6) that to have eventual population stability in a country, the theoretical value for children per couple is 2. In the real world this figure is 2.11 to take into account replacements needed due to accidents and other unexpected deaths. The arithmetic mean (2.35) in this survey is greater than the replacement level. Therefore, if the couples surveyed actually had the amount of children that they desired, they would contribute to an increase in population.

Each of the measures of central tendency has positive as well as negative features.

The mean uses all the data in a distribution of quantities. It is commonly known and can always be calculated. However, its value can be greatly influenced by a small percent of values that differ greatly from the other data.

The median is relatively easy to obtain. It requires no computation. As a middle value it has significance. Its weakness is that it only uses the middle value or values and does not use most of the quantities in a distribution. Note that 2, 3, 5, 100, 1,000,000 and 2, 3, 5, 6, 7 both have a median of 5.

The mode, like the median, requires no computation. It is valuable in cases where one or two values occur with great frequency. A disadvantage is that it does not represent all values in a given distribution of values. Also, the mode may not exist for a given population or it may not be unique (there may be 2 modes).

Example 10.4

The following data were obtained from the 1979 World Population Data Sheet for the countries of Middle Africa.

Country	Population (millions)	Per Capita GNP (US$)[3]
Angola	6.9	330
Cameroon, United Rep. of	8.3	340
Central African Empire	2.4	250
Chad	4.4	130
Congo, People's Rep. of	1.5	500
Equitorial Guinea	0.3	340
Gabon	0.5	3,730
Sao Tome and Principe	0.1	420
Zaire	28.0	130

~~a.~~ Compute the mean, median, and mode for the per capita GNP of the countries of Middle Africa

Solution

To compute the mean for the per capita GNP of the 9 countries of Middle Africa, we add up the per capita GNPs for the countries and divide by the number of countries, 9.

The sum = 330 + 340 + 250 + 130 + 500 + 340 + 3,730 + 420 + 130 = $6,170

The arithmetic mean is found from

$$\mu = \frac{\Sigma x}{n} = \frac{\$6,170}{9} = \$685.6$$

The mean per capita GNP for the countries of Middle Africa is $685.6

The median is the middle value. We rewrite the values in numerical order.

130

130

250

330

340 ←—— Middle value = the median = $340

340

420

500

3,730

The median per capita GNP for the countries is $340.

There are two modes, $130 and $340, since both of these values occur twice. These are not significant values because if the average per capita GNP in say, Zaire and Equitorial Guinea shifted just slightly, there would be no mode at all. The mode is not considered to be an important measure of central tendency except in cases where there are only a few types of values such as shoe sizes, hat sizes, etc.

Note the very large difference between the arithmetic mean per capita GNP of $685.6 and the median per capita GNP of $340. This is due to the one large GNP per capita of $3,730 of Gabon. A disadvantage of the arithmetic mean is that it is greatly affected by one or more extreme values. The median is very little affected by extreme values. However, the median only takes the middle value or values of a distribution into account.

b. Next, compute the arithmetic mean for the GNP for the people (not countries) of Middle Africa.

To do this we must take the population of each country into account. The values given in the table are for GNP per person; thus the per capita GNP value for Zaire with 26.3 million people is far more significant than that for any other country. The total GNP for each country is found by multiplying the population of each country by its average GNP per capita (column 2 times column 3), as shown in the table below:

(1) Country	(2) Population (millions)	(3) Per Capita GNP (US$)	(4) = (2)·(3) Total GNP (millions of $)
Angloa	6.9	330	2,277
Cameroon, United Rep. of	8.3	340	2,822
Central African Empire	2.4	250	600
Chad	4.4	130	572
Congo, People's Rep. of	1.5	500	750
Equitorial Guinea	0.3	340	102
Gabon	0.5	3,730	1,865
Sao Tome and Principe	0.1	420	42
Zaire	28.0	130	3,640
Sum	52.4		$12,670

Again, the arithmetic mean can be found from

$$\mu = \Sigma x/n$$

where Σx = the sum of all the GNP values, or \$12,670 million

n = 52.4 million people

$$\mu = \frac{\$12,670 \text{ million}}{52.4 \text{ million people}} = \$241.8 \text{ per person}$$

This is close to the $250 given as the average GNP per capita for Middle Africa on the 1979 World Population Data Sheet. The discrepancy is due to round off; for example, if the actual population of Gabon is 0.52 say, rather than the 0.5 given, this would increase the arithmetic mean.

We cannot compute the median or mean for the per capita GNP for the people of Middle Africa, as the information on the World Population Data Sheet is for the countries of this region, not for individual people.

Note the large difference between the mean per capita GNP for the countries of Middle Africa ($685.6) and the mean per capita GNP for people in Middle Africa ($241.8). To understand this difference, consider Gabon, with a per capita GNP of $3,640. Since Gabon is one of only 9 countries in Middle Africa, this relatively very high value has a very great influence on the mean per capita GNP for the countries. But since Gabon has only 0.5 million of this region's over 50 million people (less than 1%), its high per capita GNP has little influence on the mean per capita GNP for the people of the area.

This problem illustrates that we have to be careful when dealing with measures of central tendency. We should always ask which measure of central tendency is being used and what data have been considered.

Here are some examples where an average value can be misleading.

1. The mean income level of a country may be relatively high, but this could be due to a small percent of the population who have very high incomes while the vast majority of people are in great poverty. For example, Brazil in recent years has made great strides in increasing its average income, but the majority of its people are now worse off economically due to a poor distribution of income gains.

2. The average population density of a country (number of people per square mile) may be relatively small, but if much of the country's area is mountainous, desert, or uninhabitable for other reasons, the density of the areas where people live can be very high.

Exercises

1. Survey your class or some other group as to how many children they have in their family. Count each half brother or sister as 1/2 person. Compute the mean, median, and mode.

2. Repeat for the number of children that each person in the group would like to have.

3. For those in the group that come to school by car, complete the table below and compute the mean, median, and mode.

Passengers per car	Number of cars	Total passengers

4. Compute the mean, median, and mode for the number of children per family under the following conditions:

Children per family	Number of families
0	8
1	20
2	34
3	20
4	11
5	4
6.	3

5. Assume the following (in thousands of tons) represents the amount of garbage collected in New York City on ten consecutive working days.

24, 21, 25, 26, 22, 20, 23, 24, 22, 27

Compute the mean, median, and mode.

6. Eighty couples have children as indicated in the table below:

Number of children	Number of couples
0	4
1	24
2	20
3	12
4	9
5	6
6	5

a. What is the mode for the number of children?

b. What is the arithmetic mean?

c. Compare the arithmetic mean to the replacement level (2.11 children/couple).

d. If there were 20 additional couples, how many children could they have (total for the 20 couples) so that the arithmetic mean for the children for all the 100 couples would equal the replacement level?

7. Complete the table below so that the arithmetic mean equals the replacement level. There are many correct answers, and it would be interesting to compare various answers.

Children desired per family	Number of families
0	
1	
2	36
3	
4	
5	2
	100 (total)

8. Count the number of passengers in 100 cars chosen at random that enter the school parking lot. Compute the arithmetic mean and mode for the number of passenger cars. Indicate how you could get a random sample of cars.

9. What is the mean life expectancy for the people living in Tropical South America? Be sure to take the population of each country in that region into account in making your computations. Compare your answer with that given on the data sheet. Also compute the mean life expectancy for the countries of Tropical South America.

10. Find the mean, median, and mode PQLI (Physical Quality of Life Index) for the countries of (a) Western Africa, (b) Western Europe.

11. Compute the mean, median, and mode for the following 10 incomes:

$10,000
$ 8,000
$ 7,000
$ 5,000
$11,000
$1,000,000
$ 9,000
$ 11,000
$ 8,000
$ 11,000

Comment on your results. Which measure of central tendency is most meaningful?

12. Life expectancy at birth is based on the mean age of people who died during the previous year.

Find the life expectancy if 10 people died in the previous year at the ages indicated in each of the following cases:

a) ages at death are

1, 1, 1, 28, 43, 80, 80, 80, 80, 80

b) ages at death are

65, 65, 65, 65, 65, 75, 75, 75, 75, 75

13. Find the mean, median, and mode for the PQLI of all the countries and other geopolitical entities on the 1979 World Population Data Sheet.

14. Repeat exercise 13 for infant mortality.

15. Repeat exercise 13 for per capita GNP.

16. Repeat exercise 13 for doubling time for population.

17. Repeat exercise 13 for the percent of population under 15 years of age.

Notes

1. 1979 World Population Data Sheet of the Population Reference Bureau, Inc.

2. Ibid

3. According to the 1979 World Population Data Sheet, these values are for 1976 or 1977.

Chapter 11 Variability

11.1 Introduction

A measure of central tendency (mean, median, or mode) by itself can be misleading. For example, a country's mean per capita income may have increased greatly, but the majority of people may be worse off if only a small percent of the people received very large increases. Consider the following two cases of income changes:

Country A		Country B	
% of people	income change	% of people	income change
10	+ $500	20	+ $100
20	+ $200	30	+ $ 80
40	0	30	+ $70
30	- $ 50	20	+ $50

In both countries the mean income increase is $75 (check this) but in country A only 30% of the people are better off and 30% are worse off, while in country B everyone's income has improved.

Just reporting that the income per capita in a country has increased can be very misleading. It can disguise increased poverty, hunger, and unrest.

According to the 1979 World Population Data Sheet, the average per capita GNP for the people of the world was $1800. However, this single figure disguises the fact that many countries had extremely low values ($110 for Ethiopia, for example) and many had very high values ($8,640 for the U. S., for example).

We need some measure that takes the variability (also called dispersion or spread or scatter) into account. Two common measures of variability that we will consider are the range and the standard deviation.

11.2 The Range

The range of a distribution of numbers is the difference between the highest term in the distribution and the lowest term in the distribution.

$$\boxed{\text{Range} = \text{highest term} - \text{lowest term}} \qquad \text{Eq. 11.1}$$

Example 11.1

In one ten day work period, the New York City Sanitation Department collected the following amount of garbage, in tons. Find the range.

22, 27, 24, 26, 29, 23, 21, 24, 30, 25

Solution

The highest value is 30 tons

The lowest value is 21 tons

Range = 30 - 21 = 9 tons

Example 11.2

Using the 1979 World Population Data Sheet, find the range in (a) GNP per capita; (b) infant mortality rate

Solution

a) According to the 1979 World Population Data Sheet:

The highest GNP per capita is $14,200 for the United Arab Emirates.

The lowest GNP per capita is $80 for Bhutan and Maldives

The range for GNP per capita is $14,200 - $80 = $14,120

This reflects the tremendous income gaps in the world today.

b) Afghanistan has the highest infant mortality rate, 226 per 1000 births.

Sweden has the lowest infant mortality rate, 8 per 1000 births.

The range for infant mortality rate is 226 - 8 = 218 deaths per 1000 births

Where a person is born in the world certainly makes a difference.

11.3 The Standard Deviation

The usefulness of the range is limited because it considers only the two extreme terms of a distribution. We will now illustrate how to find the standard deviation which utilizes every term of a distribution, and therefore is far more valuable in statistical analyses.

Example 11.3

For the following distribution of values, find the standard deviation:

2, 3, 5, 7, 8, 10, 14

Solution

Before giving the equation for the standard deviation, we will show the step-by-step process for obtaining it.

We will need deviations (differences) from the mean, so we first compute the mean.

$\Sigma x = 2 + 3 + 5 + 7 + 8 + 10 + 14 = 49$ (total of all terms)

$n = 7$ (number of terms)

μ = arithmetic mean = $\Sigma x/n = 49/7 = 7$

Next we find the deviation of each term from the mean. Results are summarized in the table below.

Terms x	Deviations from the mean $(x - \mu)$	
2	$2 - 7 = -5$	(2 is 5 units below the mean)
3	$3 - 7 = -4$	
5	$5 - 7 = -2$	
7	$7 - 7 = 0$	
8	$8 - 7 = 1$	
10	$10 - 7 = 3$	
14	$14 - 7 = 7$	(14 is 7 units above the mean)
$\Sigma = 49$	$\Sigma(x - \mu) = 0$	

Note that the sum of all the deviations from the mean, $\Sigma(x - \mu)$, equals zero. This will always occur and provides a very valuable check. It reflects the fact that the nature of the mean is such that the deviations of values greater than the mean must balance those less than the mean.

Of course, the fact that the sum of all deviations from the mean adds up to zero does not mean that there is no variability. A deviation of -3 from the mean is just as significant as one of +3; yet when these two values are combined, we get a net deviation of zero.

One way to make each deviation positive is to square it. This is done for each deviation in the following table:

Terms x	Deviations from the mean $(x - \mu)$	Squared deviations from the mean $(x - \mu)^2$
2	-5	$(-5)^2 = 25$
3	-4	$(-4)^2 = 16$
5	-2	$(-2)^2 = 4$
7	0	$0^2 = 0$
8	1	$1^2 = 1$
10	3	$3^2 = 9$
14	7	$7^2 = 49$
Sums	0	104

We next calculate the mean of the squared deviations, which is $\frac{\Sigma(x - \mu)^2}{n}$, and is called the variance.

Here $\Sigma(x - \mu)^2 = 25 + 16 + 4 + 0 + 1 + 9 + 49 = 104$

$n = 7$ (the number of terms)

Hence, variance $= \frac{\Sigma(x - \mu)^2}{n} = \frac{104}{7} = 14.86$

The final step in computing the standard deviation is to take the square root of the variance. The standard deviation is denoted by the Greek letter σ (sigma).

$$\boxed{\text{standard deviation} = \sigma = \sqrt{\text{variance}} = \sqrt{\frac{\Sigma(x - \mu)^2}{n}}} \qquad \text{Eq. 11.2}$$

For the given problem

$$\sigma = \sqrt{14.86} = \underline{3.85}$$

Note: Values of square roots should be found using your hand calculator. The mathematical review in the appendix shows how to get square roots using calculators which don't have square root buttons.

The significance of the standard deviation is that for a normal distribution (explained in chapter 14), 68% of all values fall within one standard deviation of the mean, and 95% of all values fall within 2 standard deviations of the mean. This is very important in statistical analyses of data and is considered in more detail in chapter 14. Exercise 2 for this chapter is intended to give you some feel for the significance of the value of the standard deviation for various distributions of values.

Here is a summary of the steps used to compute the standard deviation. It is helpful to tabulate values when carrying out these steps.

Step 1. Compute the mean

$$\mu = \Sigma x/n$$

Step 2. Calculate the deviation of each term from the mean $(x - \mu)$. Check that the sum of all deviations from the mean, $\Sigma(x - \mu)$, is equal to zero.

Step 3. Square all deviations: $(x - \mu)^2$

Step 4. Compute the mean of the squared deviations, which is the variance

$$\text{variance} = \frac{\Sigma(x - \mu)^2}{n}$$

Step 5. Compute the standard deviation, the square root of the variance,

$$\sigma = \sqrt{\frac{\Sigma(x - \mu)^2}{n}}$$

This equation gives the standard deviation for a population. For a sample, S is used instead of σ, to denote the standard deviation, and we divide by $n - 1$ instead of n. The sample standard deviation, S, is an approximation of the population standard deviation, σ. For large distributions, $n - 1$ and n will be nearly equal and the two equations will give about the same results. In this book, for simplicity, we shall illustrate the use of equation 11.2 (for σ) only.

We shall now compute the standard deviation in problems related to global issues.

Example 11.4

The following readings for the pollution index in a certain city are obtained: 24, 36, 28, 43, 50, 38, 20, 26, 35, 40. Compute the standard deviation for these values.

Solution

We first list all values in tabular form to facilitate computation. We have listed the values in numerical order (this is helpful, but not necessary).

	(1) x	(2) $(x - \mu)$	(3) $(x - \mu)^2$
	20	$20 - 34 = -14$	$(-14)^2 = 196$
	24	$24 - 34 = -10$	$(-10)^2 = 100$
	26	$26 - 34 = -8$	$(-8)^2 = 64$
	28	$28 - 34 = -6$	$(-6)^2 = 36$
	35	$35 - 34 = 1$	$1^2 = 1$
	36	$36 - 34 = 2$	$2^2 = 4$
	38	$38 - 34 = 4$	$4^2 = 16$
	40	$40 - 34 = 6$	$6^2 = 36$
	43	$43 - 34 = 9$	$9^2 = 81$
	50	$50 - 34 = 16$	$16^2 = 256$
Sums	340	0 (it checks)	790

We will follow the 5 steps in our outline.

Step 1. We compute the mean.

$\Sigma x = 340$ (the sum of all pollution readings)

$n = 10$ (there are 10 terms)

$$\mu = \frac{\Sigma x}{n} = \frac{340}{10} = 34$$

Step 2. We compute the deviation of each term from the mean (column 2 of the table). We sum all the deviations $\Sigma(x - \mu)$ and get zero, which provides a check on the computation of the mean.

Step 3. We square all deviations (column 3 of the table).

Step 4. We calculate the variance.

a. We add the squares of the deviations; $\Sigma(x - \mu)^2 = 790$

b. We divide by the number of terms, n, or 10.

$$\text{variance} = \frac{790}{10} = 79$$

Step 5. We obtain the standard deviation by computing the square root of the variance

$$\sigma = \sqrt{\text{variance}} = \sqrt{\frac{\Sigma(x - \mu)^2}{n}} = \sqrt{79} = \underline{8.89}$$

There is a shortcut equation for the standard deviation which is convenient for calculations. It is

$$\sigma = \sqrt{\frac{n\Sigma(x^2) - (\Sigma x)^2}{n^2}} \qquad \text{Eq. 11.3}$$

where n = the number of terms in the distribution

$\Sigma(x^2)$ = the sum of all the x^2 terms

$(\Sigma x)^2$ = (the sum of all the x's) squared

Equation 11.3 has the advantage that we do not first have to compute the mean, μ. It involves only one subtraction, which often gives a more precise result.

Applying this shortcut approach to Example 11.4, we first set up the table below.

x	x^2
20	400
24	576
26	676
28	784
35	1,225
36	1,296
38	1,444
40	1,600
43	1,849
50	2,500
$\Sigma x = 340$	$\Sigma(x^2) = 12{,}350$

$$\sigma = \sqrt{\frac{10(12{,}350) - (340)^2}{10^2}}$$

$$\sigma = \sqrt{\frac{123{,}500 - 115{,}600}{100}}$$

$$\sigma = \sqrt{\frac{7900}{100}}$$

$$\sigma = \sqrt{79} = \underline{8.89}, \text{ as before}$$

Example 11.5

A survey found the following results with regard to the number of children in 100 families. Find the mean and the standard deviation for the number of children per family.

Number of families (frequency, f)	Number of children x
14	0
16	1
30	2
20	3
10	4
6	5
4	6

Solution

Since we are given a tabulation of values, we need a table with values for $(x - \mu)$ and $(x - \mu)^2$, and also for the frequency, f, times x and times $(x - \mu)^2$.

	(1) f Number of families	(2) x Number of children per family	(3) $f \cdot x$ Total number of children	(4) $(x - \mu)$ Deviations from mean	(5) $(x - \mu)^2$ Square of deviations from mean	$f \cdot (x - \mu)^2$ Total of squares of deviations
	14	0	0	-2.3	5.29	74.06
	16	1	16	-1.3	1.69	27.04
	30	2	60	-0.3	0.09	2.70
	20	3	60	+0.7	0.49	9.80
	10	4	40	+1.7	2.89	28.90
	6	5	30	+2.7	7.29	43.74
	4	6	24	+3.7	13.69	54.76
Sums	100		230			241.00

Note how much space is saved compared to the listing of 100 values in a column. However, the computation of the standard deviation, σ, for tabulated values is a little tricky. We shall use our 5 step procedure again, modified to take the tabulation into account.

Step 1. Compute the mean.

The mean is the total number of children divided by the total number of families. Although there are only 7 rows, n is not 7, but 100, since there are 100 families. To get the total number of children, we multiply the number of families (column 1) by the number of children per family (column 2), and add the totals in column 3; the total number of children in the 100 families is 230.

$$\text{mean} = 230/100 = 2.3$$

Step 2. We compute the deviation of each term from the mean, $(x - \mu)$. See column (4). Note that the summation of all deviations from the mean $\Sigma(x - \mu)$ for the 7 rows is not zero. Again, this is due to the fact that we do not have 7 families; we have 100 families. If we multiply the deviation in each row by the number of families, f, having the corresponding deviation, we should get zero. That is, $\Sigma f\cdot(x - \mu)$ should be zero. Or,

$$14(-2.3) + 16(-1.3) + 30(-0.3) + 20(0.7) + 10(1.7) + 6(2.7) + 4(3.7) =$$

$$-32.2 - 20.8 - 9.0 + 14.0 + 17.0 + 16.2 + 14.8 = -62.0 + 62.0 = 0 \quad \text{(it checks)}$$

Step 3. Square all deviations.

This is done in column (5). Since we have tabulated data, we also need a column for frequency times the squares of deviations (column 6). This takes into account the fact that we have not one, but 14 families with an $(x - \mu)^2$ value of 5.29, not one, but 16 families with an $(x - \mu)^2$ value of 1.69, and so on.

Step 4. Compute the mean of the squared deviations, the variance.

To get the sum of the squared deviations for all 100 families (not 7 rows), we add all the values in column (6). This gives a sum of 241.00. We divide by the total number of families, 100.

$$\text{variance} = \frac{\Sigma f\cdot(x - \mu)^2}{n} = \frac{241}{10} = 2.41$$

Step 5. Compute the standard deviation, the square root of the variance.

$$\sigma = \sqrt{\text{variance}} = \sqrt{2.41} = \underline{1.55}$$

Exercises

1. Using the 1979 World Population Data Sheet, find the range for the countries in Asia for each of the following:

   a. mid 1979 population estimate
   b. birth rate
   c. death rate
   d. number of years to double population
   e. infant mortality rate
   f. population under 15 years of age (percent)
   g. population over 64 years of age (percent)
   h. life expectancy at birth (years)

i. urban population (percent)

j. physical quality of life index

k. per capita gross national product (US$)

2. To help you understand the concept of the standard deviation, consider the following five sets of 10 pollution readings which all have an average value of 70 units. Compute the standard deviation for each set of pollution readings and look for patterns.

(1)	(2)	(3)	(4)	(5)
70	65	60	62	54
70	65	65	64	58
70	65	70	65	60
70	65	70	70	70
70	65	70	70	70
70	75	70	70	70
70	75	70	70	70
70	75	70	75	80
70	75	75	76	82
70	75	80	78	86

3. For an industrialized area, the number of days each month that the air quality was found to be unhealthy or unsatisfactory was as follows:

12, 7, 9, 5, 12, 17, 16, 14, 10, 6, 4, 8

Compute the range and the standard deviation.

4. Assume that 10 cars of one type were tested for fuel efficiency and the following results were obtained for mileage per gallon:

18, 17, 15, 23, 25, 26, 19, 25, 20, 22

Compute the range and the standard deviation.

5. A survey of the number of passengers per car coming into a factory parking lot produced the following results:

Number of cars (f)	Number of passengers (x)
40	1
21	2
16	3
7	4
2	5
1	6

Compute the mean, median, and mode, and the standard deviation for the number of passengers.

6. A survey found the following results with regard to the number of children in 100 families. Compute the mean, median, and mode, and the standard deviation for the number of children per family.

Families	Children per family
16	0
18	1
26	2
18	3
6	4
4	5
2	6

7. Find the range and standard deviation for the income changes for country A and country B given in article 11.1.

Chapter 12 Probability

We live in a world of uncertainty. If we toss a coin, we don't know if we'll get a head or a tail. Parents may hope to have a boy and a girl, if they have 2 children, but they can't be sure. Many safeguards are put into nuclear power plants, but we can't be absolutely sure that there will be no accident. As more nations obtain nuclear weapons, there is less certainty that a country won't use them before the year 2000.

Probability theory gives us the means to consider problems such as these. It won't answer all questions, but it will provide helpful insight. In this chapter, we present the basics of probability theory and apply them to current issues.

12.1 Basic Theory

The probability of an event is defined as the number of favorable outcomes divided by the total number of outcomes. This means

$$\boxed{P(\text{event}) = \frac{F}{T}} \qquad \text{Equation 12.1}$$

where P(event) = the probability of a certain event occurring

F = the number of favorable outcomes

T = the total number of possible outcomes

This equation is best understood through several examples:

Example 12.1

A fair coin is tossed. What is the probability that the coin will land with heads facing up?

Here there are two possible outcomes: heads or tails. (For a fair coin, these are equally likely.) T = 2. Only one of the outcomes is favorable (heads).

$$P(\text{heads}) = \frac{F}{T} = \frac{1}{2}$$

Example 12.2

A fair die (singular for dice) is tossed.

a) Find the probability that the die lands on 5.

For a die, T, the total number of possible outcomes is 6; the fair die may, with equal probability, land on 1, 2, 3, 4, 5, or 6. Let x = the number that the die lands on.

$P = F/T$

Here F = 1 since the only favorable case is 5.

$P(x = 5) = \frac{F}{T} = \frac{1}{6}$

b) Find the probability that the die lands on either 2 or 5.

Here F = 2 since there are 2 favorable cases, 2 and 5.

$P(x = 2 \text{ or } x = 5) = F/T = 2/6 = \underline{1/3}$ (always reduce fractions to lowest terms)

c) Find the probability that the die lands on an odd number.

Here F = 3 (x = 1, 3, or 5)

$P = F/T = 3/6 = \underline{1/2}$

d) Find the probability that the die lands on any number except 5.

Here F = 5 (x = 1, 2, 3, 4, or 6)

$P = F/T = \underline{5/6}$

Note that the probability that x = 5 and the probability that x is not 5 add up to 1. $P(x = 5) + P(x \neq 5) = 1/6 + 5/6 = 1$

In general, if p (note the lower case letter p) is the probability that an event will occur and q is the probability that an event will not occur, then

$$\boxed{p + q = 1} \qquad \text{Equation 12.2}$$

In words: the probability that an event will occur plus the probability that the event will not occur always adds up to one.

Example 12.3

If the probability of rain on a given day is 0.2, what is the probability that it will not rain on that day?

$p + q = 1$

Here, p = the probability of rain = 0.2

$0.2 + q = 1 \qquad q = \underline{0.8}$

The probability that it won't rain is 0.8, or 80%.

Example 12.4

A card is chosen from an ordinary deck.

a) Find the probability that the card is the 3 of diamonds.

There are 52 cards in the deck, 4 suits (spades, hearts, diamonds, and clubs) and 13 cards in each suit. T = 52. Again we use the basic equation P(event) = F/T.

Since we succeed only if we get the 3 of diamonds, there is only one favorable card; F = 1.

P = F/T = 1/52

b) Find the probability that the card chosen is a heart.

Here F = 13 since there are 13 cards that are hearts.

P = F/T = 13/52 = 1/4

We could have considered this problem from another point of view. There are a total of 4 suits and only one of them (hearts) is favorable. Hence,

P = F/T = 1/4

It is good to look at problems from more than one point of view to get a check on our thinking.

c) Find the probability that the card chosen is a 3 or a diamond.

Here any three or any diamond is favorable. There are 4 threes and 13 diamonds. So, it appears that F = 4 + 13 = 17. However, note that the three of diamonds appears in the group of threes and the group of diamonds and has been counted twice. Hence F = 4 + 13 - 1 = 16. We must always be careful to make sure that we haven't counted any event twice.

P = F/T = 16/52 = 4/13

d) Find the probability that the card chosen is red.

Half of the cards are red. Hence,

P = F/T = 1/2

Example 12.5

In a certain city, the air quality has been unfavorable on 63 out of the last 257 days. What is the probability that the air quality will be unfavorable tomorrow?

In situations like this, the only way that we can obtain a probability for predicting a future event is to use information from the past. Out of a total (T) of 257 days, there were 63 unfavorable days (note F here will denote the number of unfavorable events, since we are asked to find the probability that the air quality will be unfavorable).

$$P = F/T = 63/257 = \underline{0.245}$$

The probability that the air quality will be unfavorable tomorrow, or on any other specified day is 0.245.

Note that the probability of an event must always be between 0 and 1. If an event is impossible (it can never occur), its probability is 0. Thus, if you roll a die, the probability of getting a number above 6 is zero.

If an event must occur, its probability is 1. Thus the probability that you'll get a head or a tail from a toss of a coin is 1.

12.2 Successive events

Often we have to find a probability related to two events occurring in succession.

Example 12.6

Two coins are tossed. Find the probability that

a) both coins are heads

b) one coin is a head and the other a tail

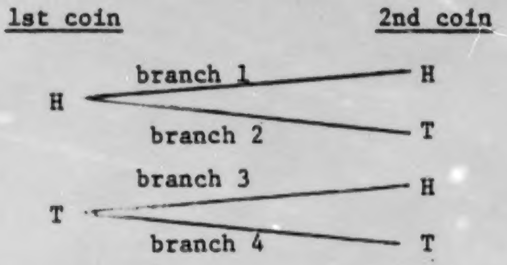
c) the first coin is a head and the second is a tail

d) at least one coin is a head

The basic equation $P = F/T$ must still be used. However, it is a little more difficult to find F and T.

For the first coin we can get a head or a tail. No matter what we got for the first coin, the second coin can also be a head or a tail. This is pictured below in a "tree diagram."

Tree diagram:



There are a total (T) of 4 possibilities, as summarized in the table below.

	First coin	Second coin
case 1.	Head	Head
case 2.	Head	Tail
case 3.	Tail	Head
case 4.	Tail	Tail

With the tree diagram and accompanying table, we are ready to obtain answers.

a) Both coins are heads.

Here, F = 1, only one of the four cases (case 1) involves both coins landing on heads. Hence, P(2 heads) = F/T = 1/4

b) One coin is a head and the other a tail

Here, F = 2; we can get one head and one tail in two different ways. The first can be a head and the second a tail (case 2), or vice versa (case 3); note that the order was not specified in the question. Thus, P(1 head and 1 tail) = F/T = 2/4 = 1/2.

c) The first coin is a head and the second is a tail.

Here, F = 1. The order is specified; we must get a head first and a tail second. Only case 2 in the table above satisfies this criterion. Hence, P(a head, then a tail) = F/T = 1/4.

d) At least one coin is a head

There are three cases in which at least one coin is a head (cases 2, 3, and 4). Thus, P = F/T = 3/4.

Note that we could have found this result in a second way. Recall that p + q = 1, where p = the probability that an event occurs and q is the probability that an event does not occur. Here,

p = probability of getting no heads = 1/4 (from case 4)

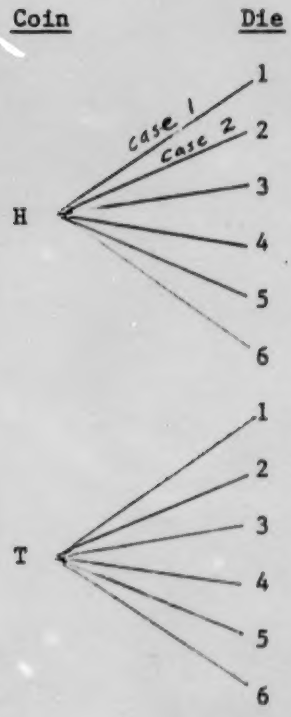
q = probability of getting at least one head

p + q = 1 1/4 + q = 1 q = 3/4

Example 12.7

A coin is tossed and a die is tossed. Find the probability that the coin lands on the head and that the die lands on 5.

Here too a tree diagram can be very valuable. The table shown on the right is also helpful.



Case	Coin	Die
1	Head	1
2	Head	2
3	Head	3
4	Head	4
5	Head	5
6	Head	6
7	Tail	1
8	Tail	2
9	Tail	3
10	Tail	4
11	Tail	5
12	Tail	6

There are 12 possible outcomes. Of these, only one (case 5) is favorable.

Hence, $P = F/T = \underline{1/12}$

There is another way in which we could have obtained this result. Note that the probability of getting a head on the coin is 1/2 and the probability of getting a 5 on the die is 1/6. We can get the probability of getting a head and a 5 by multiplying these two probabilities.

$$P(\text{head and } 5) = P(\text{head on coin}) \cdot P(5 \text{ on die})$$

$$P = P_1 \cdot P_2 = \frac{1}{2} \cdot \frac{1}{6} = \underline{\frac{1}{12}}$$

This procedure can be used for any case involving successive events. We merely multiply the successive probabilities to get the overall probability. Hence,

$$\boxed{P = P_1 \cdot P_2 \cdot P_3 \ldots} \qquad \text{Equation 12.3}$$

where P = probability of success for a number of successive events

P_1 = probability of success in the first event

P_2 = probability of success in the second event

P_3 = probability of success in the third event

The probability of success in several successive events equals the probability of success in the first event times the probability of success in the second event ...

Example 12.8

Find the probability of getting a royal flush (Ace, King, Queen, Jack, ten in one suit) when 5 cards are selected at random from a deck of cards.

In order to get a royal flush, the first card selected must be a ten, jack, queen, king, or ace in any one of the four suits. There are just 20 (5 cards times 4 suits) favorable cards out of the 52 cards in the deck. Hence

$P_1 = 20/52$.

For the second card there are only 4 favorable cards out of 51 remaining cards. For example, if our first card was a jack of hearts, the only favorable cards remaining are the ten, queen, king, and ace of hearts. Thus, $P_2 = 4/51$.

In a similar manner, we note that $P_3 = 3/50$, $P_4 = 2/49$, and $P_5 = 1/48$. The probability of getting a royal flush in any suit is obtained by multiplying the 5 probabilities:

$$P = P_1 \cdot P_2 \cdot P_3 \cdot P_4 \cdot P_5$$

$$P = \frac{20}{52} \cdot \frac{4}{51} \cdot \frac{3}{50} \cdot \frac{2}{49} \cdot \frac{1}{48}$$

$$P = 0.0000015 \text{ (using a hand calculator)}$$

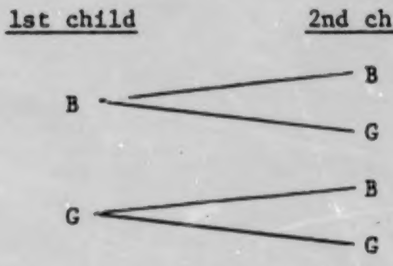
The probability of being dealt a royal flush is extremely small.

12.3 Pascal's Triangle

Suppose we wish to know the probability that, of the next ten births in a hospital, exactly 6 will be girls. To help solve such problems, we will start with simpler cases and try to establish a pattern that will help in more advanced cases.

Start with 1 child, the easiest case. There is a total of 2 possibilities: either a boy or a girl.

Next consider two children. Refer to the tree-diagram and table below.



Case	1st child	2nd child
1	boy	boy
2	boy	girl
3	girl	boy
4	girl	girl

There are 4 possibilities. Note that one possibility involves 2 girls (case 4), two (cases 2 and 3) involve one girl (and one boy), and one (case 1) involves no girls (and two boys). Our analysis, for simplicity, will focus only on the number of girls.

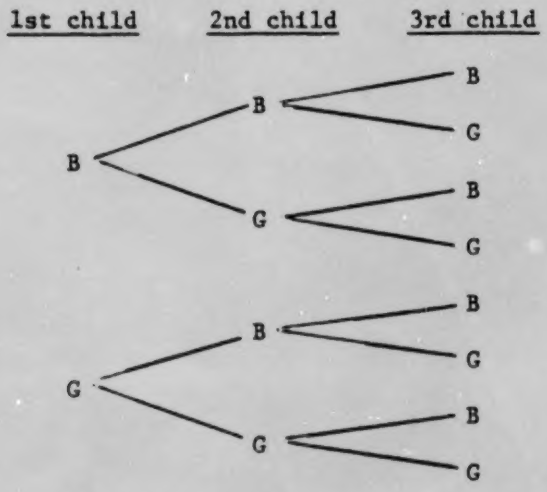
Let us summarize what we have found so far in the table below (the beginning of Pascal's triangle):

	Children	0	1	2	3	4	5	Total	
		Number of girls							
row 1	1	1	1					2	→ B or G
row 2	2	1	2	1				4	→ B-B B-G G-B G-G

Note: For one child, there are 2 possibilities; one possibility is no girls and the other is 1 girl. Hence in row 1, we put a 1 in the 0 girls column and a 1 in the 1 girl column. For two children, there are a total of 4 possibilities; 1 involves no girls, 2 involve exactly 1 girl and 1 involves 2 girls. Thus, for row 2 (2 children), we put a 1 in the 0 girls column, a 2 in the 1 girl column, and a 1 in the 2 girls column. The same results would have been obtained if we considered the number of boys instead of the number of girls.

Look at the table and see if you notice any patterns. Try to guess the numbers you think we will get for 3 children. Check your results against those from the following analysis.

Let us draw the tree diagram for 3 children, and an accompanying table.



Case	1st child	2nd child	3rd child
1	boy	boy	boy
2	boy	boy	girl
3	boy	girl	boy
4	boy	girl	girl
5	girl	boy	boy
6	girl	boy	girl
7	girl	girl	boy
8	girl	girl	girl

From the tree diagram and accompanying table, we can find that:

1. There are a total of 8 cases.
2. 1 case involves 3 girls (case 8).
3. 3 cases involve exactly 2 girls (cases 4, 6, and 7)
4. 3 cases involve exactly 1 girl (cases 2, 3, and 5)
5. 1 case involves no girls (case 1)

Using this information, we can continue our Pascal's triangle table. We add line 3: a 1 in the 0 girls column, a 3 in the 1 girl column, a 3 in the 2 girls column, and a 1 in the 3 girls column.

Number of girls

	Children	0	1	2	3	4	5	Total
row 1	1	1	1					2
row 2	2	1	2	1				4
row 3	3	1	3	③	1			8

The 3 circled indicates that when there are 3 children, of the 8 possible combinations, exactly 3 involve 2 girls.

Now you should be able to observe some patterns.

Look at the totals. We have 2, 4, 8, so far. What would you guess for the total for 4 children, for 5 children, etc?

Note [redacted] the column for no girls. We have 1 in each box. What do you guess for the next boxes below in this column?

See if you can work out the results for 4 children. Compare it with the results below.

		Number of girls						
	Children	0	1	2	3	4	5	Total
row 1	1	1	1					2
row 2	2	1	2	1				4
row 3	3	1	③	③	1			8
row 4	4	1	4	⑥	4	1		16

Note the many patterns. The totals give us a geometric sequence (2, 4, 8, 16). The column for one girl gives us an arithmetic sequence (1, 2, 3, 4 ...).

Observe the symmetry; going from left to right or right to left, we get exactly the same numbers (for example, for row 4, we have 1, 4, 6, 4, 1).

How do we get the 6 in row 4? There are several ways:

1. We can set up the tree diagram or related table, as before, for 4 children and count the cases involving exactly 2 girls.

2. We can note that we need a 6 to give a total of 16 combinations.

3. We can observe another pattern. Any number on Pascal's triangle involves the sum of the number immediately above it plus the number above and to the left. Here, 3 + 3 = 6 (note the circled numbers).

Now, you should be ready to continue the table (Pascal's triangle). Please do so for up to 10 children and compare with the results below.

		Number of girls											
Row	Children	0	1	2	3	4	5	6	7	8	9	10	Total
1	1	1	1										2
2	2	1	2	1									4
3	3	1	3	3	1								8
4	4	1	4	6	4	1							16
5	5	1	5	10	10	5	1						32
6	6	1	6	15	20	15	6	1					64
7	7	1	7	21	35	35	21	7	1				128
8	8	1	8	28	56	70	56	28	8	1			256
9	9	1	9	36	84	126	126	84	36	9	1		512
10	10	1	10	45	120	210	252	210	120	45	10	1	1024

Again, observe symmetry and the many patterns formed. Also note that we have relatively easily obtained results for 10 children by using patterns. This would have been very difficult with just the use of a tree diagram, as this would involve considering 1024 combinations (see the total column for 10 children).

Now let us apply Pascal's triangle to some examples.

Example 12.8

Of the next 10 births in a certain hospital, what is the probability that exactly 6 will be girls?

Since we are considering 10 births, we use row 10 of Pascal's triangle. P = F/T is still the key equation. Here T = 1024 (the total number of possibilities). To find F, we find where the column for 6 girls meets the row for 10 children. This gives us 210 (circled above); of the 1024 combinations, 210 will have exactly 6 girls. Hence,

$$P = \frac{F}{T} = \frac{210}{1024} = \underline{\frac{105}{512}}$$

Example 12.9

For the next 8 births, what is the probability of getting: a) all girls; b) exactly 5 girls; c) exactly 5 boys; d) 5 or more girls; e) at least 1 girl?

For each question, we use row 8 of Pascal's triangle, corresponding to 8 children. There are a total of 256 possibilities for the 8 births; one example would be B-G-B-B-G-B-G-G.

a) All 8 babies are girls:

There is only one way of getting 8 girls in 8 births. This would require that the first birth be a girl, the second a girl, and each succeeding birth a girl, or G-G-G-G-G-G-G-G. Note the 1 for row 8, corresponding to 8 girls. Thus,

$$P = F/T = \underline{1/256}$$

b) Exactly 5 babies are girls:

There are 56 combinations which involve exactly 5 girls (see row 8 and column 5). Hence,

$$P = \frac{F}{T} = \frac{56}{256} = \frac{28}{128} = \frac{14}{64} = \underline{\frac{7}{32}}$$

c) Exactly 5 babies are boys:

If there are 5 boys in 8 births, there must be 3 girls. Pascal's triangle gives F = 56 (row 8, column 3); note the symmetry - there are just as many ways of having 3 girls in 8 births as 5 girls in 8 births.

We may get F = 56 from another point of view. As mentioned previously, Pascal's triangle could have been set up in terms of boys rather than girls. Hence, if we had "number of boys" written across the top of Pascal's triangle, rather than "number of girls", the numbers would be identical. Hence F = 56 for 5 boys in 8 births, as well as 5 girls in 8 births.

$$P = \frac{F}{T} = \frac{56}{256} = \underline{\frac{7}{32}}$$

d) Five or more girls means 5 girls or 6 girls or 7 girls or 8 girls.

To find F in this case, we must add the results for these possibilities

$F = 56 + 28 + 8 + 1 = 93$ (row 8, columns 5, 6, 7, and 8)

$P = F/T = \underline{93/256}$

e) At least one baby is a girl:

At least one girl means one or more girls. Hence, we can add the values for 1 through 8 girls. This gives

$F = 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 255$ (row 8, columns 1 through 8)

$P = F/T = \underline{255/256}$

There is a shortcut we can use. Either we have no girls or we have at least one girl. Hence,

$p + q = 1$

P(no girls) + P(at least one girl) = 1

P(no girls) = 1/256 (row 8, column 0)

Hence, P(at least one girl) = 1 - 1/256 = $\underline{255/256}$

Pascal's triangle may be used not only for probabilities involving boys and girls, but for any case where there are equal probabilities. (In the next article, we will see that it is helpful even in cases where the probabilities for two events are not equal.) For example, it may be applied to cases involving the tossing of coins, and guessing on a true-false test.

Example 12.10

In a certain town there is a 50% chance that air pollution readings will exceed an acceptable level on any given day. What is the probability that the acceptable level will be exceeded exactly 4 times in 7 days?

Pascal's triangle may be used here because we have two equally likely possibilities: the air quality will be acceptable or it won't.

From row 7, $T = 128$ (totals column)

From row 7, column 4, $F = 35$ (for 4 "successes")

Thus, $P = F/T =$ 35/128

12.4 Binomial Probability

In many cases, there are two things that can occur. We win or we lose at a game. We get heads or tails when we toss a coin. We pick an ace from a deck of cards, or we don't. We succeed or we fail at some endeavor, such as scoring a foul shot. In general, either we succeed or we fail in some attempt.

Such cases are known as binomial probability cases

Let p = the probability of success in one trial

q = the probability of failure in one trial

If we want to investigate probabilities with regard to getting heads on several tosses of a coin,

p = probability of getting a head in one toss = 1/2

q = probability of not getting a head = probability of getting a tail = 1/2

Note that $p + q = 1/2 + 1/2 = 1$. This will always be true, since either we succeed or we don't succeed.

To use the theory in this section, p and q must be constants. When drawing cards from a deck of cards, the total number of cards would keep decreasing; hence probabilities would change and we could not apply the theory which follows. Another way of stating this is that each event must be independent of previous events.

Consider the following situation. You toss a fair die 3 times. You want to know your probability of winning two times if winning means you roll a 3.

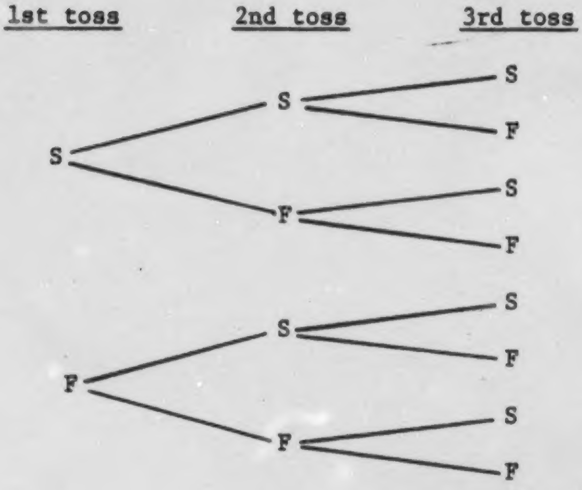
Here, p = probability of getting a 3 on one roll of the die = 1/6.

q = probability of not getting a 3 on one roll of the die = 5/6.

$p + q = 1/6 + 5/6 = 1$ (it checks)

Let S stand for success and F for failure. Hence $P(S) = 1/6$ and $P(F) = 5/6$.

Consider the tree diagram below.



case	1st toss	2nd toss	3rd toss
1	S	S	S
2	S	S	F
3	S	F	S
4	S	F	F
5	F	S	S
6	F	S	F
7	F	F	S
8	F	F	F

We cannot use Pascal's triangle directly in this case because p and q are not equal; we don't have two equally likely cases.

One way we can have two successes out of 3 is to succeed on the 1st two trials and fail on the 3rd (case 2). Recall that to obtain the probability in successive events, we multiply the successive probabilities (Equation 12.3).

Hence, the probability of succeeding on the 1st two trials and failing on the 3rd is

$$P(S) \cdot P(S) \cdot P(F)$$

$$= p \cdot p \cdot q$$

$$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{5}{216}$$

But, this is not the total answer. We can get 2 successes out of 3 in 2 other ways; success-failure-success (case 3) and failure-success-success (case 5). Each of the 3 ways is equally likely. Hence, we must multiply the results for success-success-failure (5/216) by 3.

$$P(2 \text{ successes in } 3 \text{ trials}) = 3 \cdot p \cdot p \cdot q = 3\ (1/6)(1/6)(5/6)$$

$$= 3\ (5/216) = \underline{15/216}$$

The 3 means that there are just 3 different ways of obtaining 2 successes in 3 trials. This value, 3, can be obtained from Pascal's triangle (3 trials, 2 successes), from row 3, column 2. All we have to do is change the former heading, "number of girls" to "number of successes."

Example 12.11

What is the probability of 4 successes in 6 trials if you win when you get a 3 or a 5 on a toss of a fair die?

Here, p = probability of getting a 3 or a 5 on one toss of a fair die

= 2/6 = 1/3

q = probability of not getting a 3 or a 5 on one toss of a fair die

= 4/6 = 2/3

Again, note that p + q = 1/3 + 2/3 = 3/3 = 1 (it checks)

Refer to Pascal's triangle, row 6, column 4. There are 15 ways of having 4 successes in 6 trials. (These include S-S-S-S-F-F, S-S-F-F-S-S, etc.)

Hence, the probability of 4 successes out of 6 is $15 \cdot p \cdot p \cdot p \cdot p \cdot q \cdot q =$

$$15 \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \underline{\frac{60}{729}}$$

The 60 is obtained from $15 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2$; and the 729 is $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

Example 12.12

Assume that the probability that air pollution readings in a certain town exceed an allowable value on any given day is 1 out of 3. You'll be in that town for 5 days. Let x = the number of days that the air is polluted that week. Find the probabilities that x = a) 0, b) 1, c) 2, d) 3, e) 4, f) 5

Here, p = probability that the air pollution reading exceeds the allowable value on a given day = 1/3

q = 2/3, since p + q must = 1.

C

Using coefficients from row 5 of Pascal's triangle, we get the following tabulation of results.

a) $P(x=0) = 1 \cdot q \cdot q \cdot q \cdot q \cdot q = 1 \cdot q^5 = 1 \cdot (2/3)^5 = 32/243$

b) $P(x=1) = 5 \cdot p \cdot q \cdot q \cdot q \cdot q = 5pq^4 = 5(1/3)(2/3)^4 = 80/243$

c) $P(x=2) = 10 \cdot p \cdot p \cdot q \cdot q \cdot q = 10 \cdot p^2q^3 = 10(1/3)^2(2/3)^3 = 80/243$

d) $P(x=3) = 10 \cdot p \cdot p \cdot p \cdot q \cdot q = 10 \cdot p^3q^2 = 10(1/3)^3(2/3)^2 = 40/243$

e) $P(x=4) = 5 \cdot p \cdot p \cdot p \cdot p \cdot q = 5 \cdot p^4q = 5(1/3)^4(2/3) = 10/243$

f) $P(x=5) = 1 \cdot p \cdot p \cdot p \cdot p \cdot p = 1 \cdot p^5 = 1(1/3)^5 = 1/243$

Comments on the above computations:

1. For $x = 0$, no polluted days, we use $q \cdot q \cdot q \cdot q \cdot q$, since there are no "successes."
2. The probabilities all add up to 243/243 or 1. This is a very valuable check.
3. Note the patterns. The exponents of q decrease from 5 to 4 to 3 to 2 to 1 as the exponents of p increase. The exponents of p and q add up to 5 in every case.
4. As a review, for $P(x=3) = 10p^3q^2$, this means that there are 10 different ways of having 3 successes in 5 tries (row 5, column 3 of Pascal's triangle), and we are considering 3 wins (hence, p^3) and two losses (hence, q^2).

Example 12.13

Assume that air quality is unhealthy in a given town on an average of 1 out of 4 days. What is the probability that the air quality will be unhealthy on exactly 4 out of 10 days.

$p = 1/4$

$q = 1 - 1/4 = 3/4 \qquad (p + q = 1)$

From Pascal's Triangle (row 10, column 4), we find that there are 210 different ways in which there can be 4 unhealthy days out of 10.

We have 4 days when the air quality is unhealthy; use p^4.

We have $10 - 4 = 6$ days when the air quality is not unhealthy; use q^6.

Hence, P(4 unhealthy days out of 10) = $210p^4q^6$

$$= \underline{210(1/4)^4(3/4)^6}$$

As we are just trying to illustrate a technique, we will leave the answer in this form.

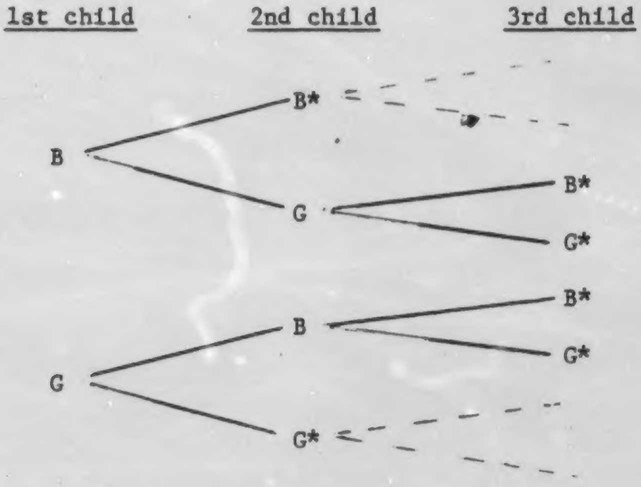
12.5 Probability Related to Population Growth

The tree-diagram can be very valuable in computing the average number of children that will occur under various conditions.

Example 12.14

Imagine a society where each couple will have children until they have either 2 boys or 2 girls. What will be the average number of children per couple?

Consider the following tree diagram.



* means that there are 2 boys or 2 girls, and hence no more children in that family.

Let us analyze the results. For the first child, half of the couples will have a boy and half will have a girl. No couple has 2 children of one sex so they'll all continue.

After the second child, 1/4 of the couples will have two boys and 1/4 of the couples will have two girls. They've met the criteria for the society, hence they'll stop. Thus, 1/2 of the couples will stop at two children.

The remaining couples will have one additional child which will bring them all to either 2 boys or 2 girls. Note that of the 8 potential couples having a third child, 4 would stop there (the other half of the couples have previously stopped). This means 1/2 the couples stop at 3 children.

Since half of the couples stop at 2 children and half stop at 3 children, the average number of children per couple is 2.5.

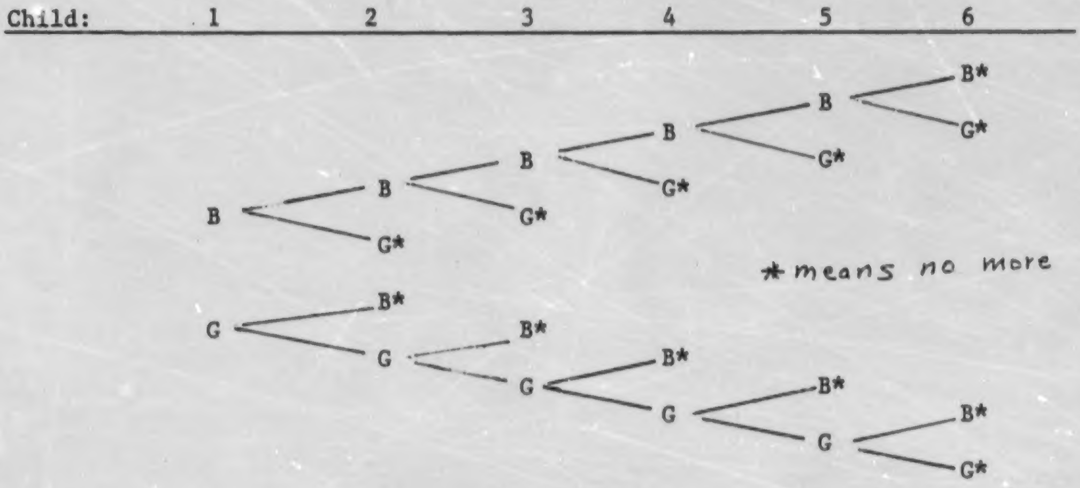
Or, using numbers directly from the tree-diagram,

$$\frac{2}{4}(2) + \frac{4}{8}(3) = 1 + 1.5 = \underline{2.5}$$

Example 12.15

Imagine a society where each couple will have children until they have a boy and a girl or 6 children total. That is, if the first six children are all of the same sex, they'll stop having children.

Again, we first draw the tree diagram, and indicate by an asterisk when various couples stop, because they've met the stated criterion.



*means no more children.

Observe:

1. No couples stop after one child.

2. Two out of 4 couples stop after 2 children since they have a boy then a girl or a girl and then a boy. This gives us (2/4)· 2, towards the average.

3. Two couples also stop after 3 children. But this must be considered as 2 out of 8 since there would normally be 8 couples at column 3, if no couples stopped at 2 children. This gives us (2/8)(3), towards the average.

4. Two out of 16 potential couples stop at 4 children. (2/16)(4) towards the average.

5. Two out of 32 potential couples stop at 5 children. (2/32)(5) towards the average.

6. Everyone left must stop after having 6 children, since this was the preset limit. Hence, the fraction to use here is 4 out of 64 potential couples. (4/64)(6) towards the average.

The average may now be found by adding the various terms:

$$\frac{2}{4}(2) + \frac{2}{8}(3) + \frac{2}{16}(4) + \frac{2}{32}(5) + \frac{4}{64}(6)$$

$$= \frac{4}{4} + \frac{6}{8} + \frac{8}{16} + \frac{10}{32} + \frac{24}{64} = \frac{64 + 48 + 32 + 20 + 24}{64} = \frac{188}{64} = 2\frac{60}{64} = 2\frac{15}{16}$$

Hence, under the conditions stated, the average number of children would be slightly below 3. Since the replacement level is 2.1, this population will not reach equilibrium.

Note that all of the fractions in the first equation on this page add up to 1. That is, $\frac{2}{4} + \frac{2}{8} + \frac{2}{16} + \frac{2}{32} + \frac{4}{64} = 1$. Why is this important?

The results of this problem are very significant since for many couples the ideal family involves a boy and a girl. However, in the underdeveloped world where children are valued in economic terms, a family involving more boys than girls is preferred (see exercise 11).

Exercises

1. Out of every 10 days in a certain town, the air quality is good on 5 days, satisfactory on 3, and poor on 2. If you are in this town for one day, what is the probability that the air quality will be

a) good; b) satisfactory; c) poor.

If you are in this town for 2 days, what is the probability that the air quality will be d) good on both days;

e) good or satisfactory on both days.

2. What is the probability that of the next eight births in a hospital,

a) all will be boys

b) 5 will be girls

c) 6 or more will be boys

d) no more than 3 will be girls

3. Assume that for an accident involving a nuclear power plant, 5 systems must fail. Also assume that the probabilities of failure of the 5 systems in any one year are 0.01, 0.02, 0.004, 0.012, and 0.005, respectively. Find the probability of an accident at a nuclear power plant, in one year, using these given probabilities.

4. If the air quality in a certain area is unfavorable 50% of the time, what is the probability that in one six-day period there will be

a) 4 unfavorable days

b) 4 or more unfavorable days

c) no more than 4 unfavorable days.

5. The air in a certain town is polluted one day in three, on the average. You'll be in that town for a week. Find the probabilities that the air will be polluted no days, one day, two days, three days, four days, five days, six days, and seven days. Put your calculations and results in a tabular form and show a check.

6. Suppose that an oil tanker breaks up off the coast of New England, leaving a tremendous oil slick on the ocean. Suppose further that this oil spill will result in a very bad ecological (and economic) disaster, unless the oil is blown by a west wind out to the deeper waters before it sinks. If the probability of a west wind on any day is 1/3, find the probability of getting more than 4 days of west wind in one week.

7. Suppose that an electric company claims that its smokestacks pollute the air only one out of ten days. The air from the stacks is tested on five days and is dirty twice. Find the probability of this happening if the company was correct.

8. Assume that in a certain town, daily water quality tests have shown that there are equal probabilities that the water quality will be good, fair, or poor. If you will be in that town on two days, what are the probabilities that the water quality will be good a) both days? b) just one day? c) at least one day?

A tree-diagram will be very helpful.

If you were in this town for 3 days, what are the probabilities that the water quality will be good d) on all three days? e) on just one day? f) on none of the days?

9. In a certain town, the air quality is unfavorable one day out of five, on the average. Assume that you will be in that town on two consecutive days for a meeting. Estimate the probability that the air quality will be

a) unfavorable both days

b) favorable on the first day and unfavorable on the second day

c) unfavorable on just one of the two days

d) unfavorable at least one day (make sure that you consider all the combinations that will give unfavorable conditions on at least one of the two days).

10. In a certain society, couples have children until they have either 2 boys or 3 girls. Calculate the average number of children per couple under these conditions.

11. Imagine a society where each couple will have children until they have one son (then they'll stop). Compute the average number of children they'll have per couple,

a) assuming that couples will stop if they have 7 consecutive girls,

b) couples will continue trying until they have a girl. The successive sums after each birth get closer and closer to a limiting value. What is this value?

Chapter 13 The Normal Curve

13.1 The Normal Curve

Imagine that a very large group of people each tossed 1000 coins to find out how many heads they will get. More people would get 500 heads than any other amount. Many people would get a number of heads not too far above or below 500. The number of people getting more than 500 heads would be very close to the number getting less than 500, on a percentage basis. The number of people getting values far above or far below 500 heads would be relatively small.

If we plotted a histogram (see article 4.4) of our results and smoothed it off, we would get a curve similar to the one shown below.

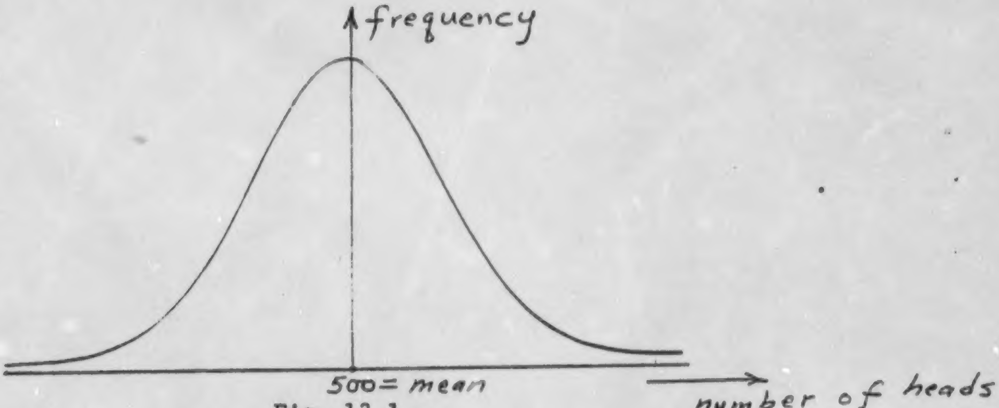


Fig. 13.1

This type of curve is known as the bell-shaped or normal curve. The heights of the curve corresponding to various numbers of heads indicate the frequency of getting that number of heads. Note the following characteristics of this theoretical curve:

a. It is symmetrical. There is just as much probability of getting a value above the mean as there is of getting a value below the mean.

b. The mean, median, and mode are all equal, in this case to 500 heads.

c. Most values are close to the mean; the frequencies decrease for numbers of heads far from the mean.

Consider another example. Suppose we test for the strength of many samples of steel which will be used for nuclear power plants. This is similar to the coin-tossing case in that the overall strength of a steel sample is dependent on the strengths and orientations of the many crystals (internal structures) which make up the sample. In most cases we will have crystal strengths and orientations which will lead to an average strength or a strength not too far from the average. The probability of a strength 5% above the average, for example, will be equal to that of a strength 5% below the average. Frequencies for strengths far from the average will be low. Again, we'll get a curve such as that shown below, the normal curve.

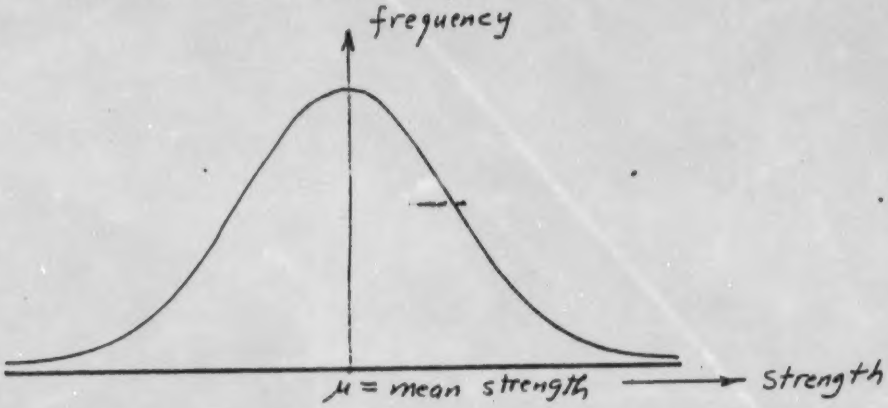


Fig. 13.2

The normal curve approximates the frequency histogram for many situations related to global issues as well as many other types of problems. Therefore, calculations have been made to make the use of a normal curve easier. Some important results are shown below.

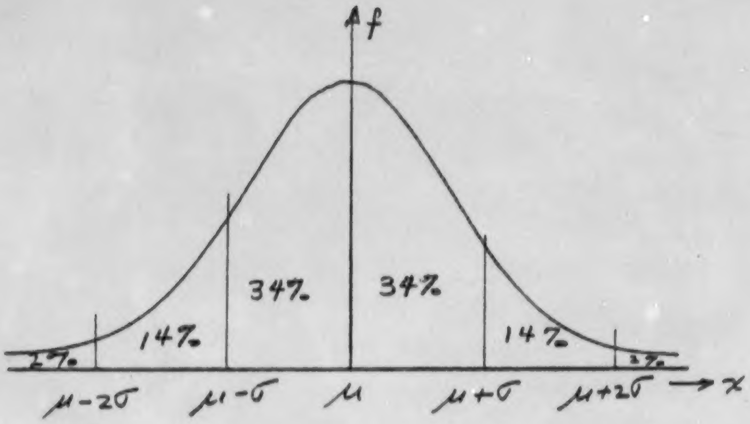


Fig. 13.3

As indicated, 34% of all values fall between the mean (μ) and one standard deviation above the mean (μ + σ), 14% of all values fall between one standard deviation above the mean and two standard deviations above the mean (μ + 2σ), and so on. Note that all percentages add up to 100%.

It is important to realize that not all distributions have normal curves. For example, in a country where most of the people have low incomes and a small percent have very high incomes, the incomes will not be normally distributed.

Example 13.1

Air pollution readings in a certain city are normally distributed (fall on a normal curve) with a mean reading of 50 units and a standard deviation of 5 units. Find the probability that the reading on any given day will be

a) between 50 and 55

b) between 55 and 60

c) between 40 and 55

d) below 40

It is often best to list the given information on a normal curve, as shown.

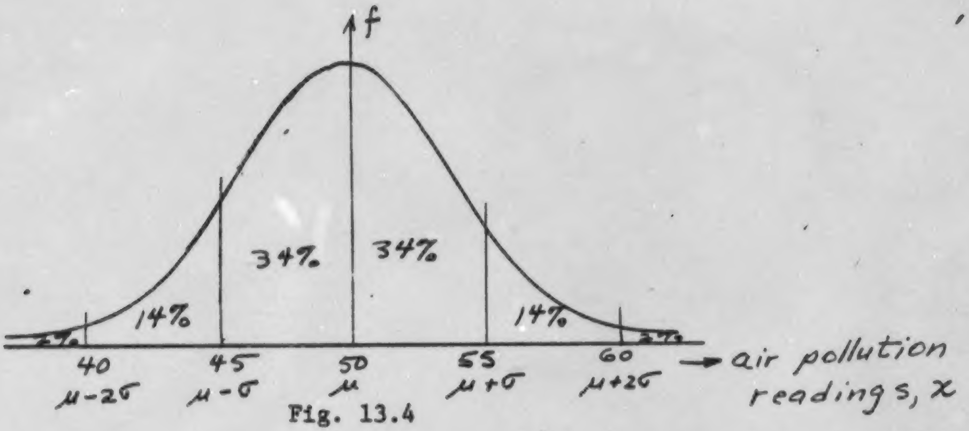


Fig. 13.4

The probabilities that are required are directly proportional to the percentages between given values of air pollution readings on the normal curve. This is based on the equation for probability, P = F/T, where T is the total of all percentages, or 100%, and F is the percentage for the region being considered.

We can get the probabilities needed directly from supplementary normal curves which spotlight the regions considered.

a) readings between 50 and 55.

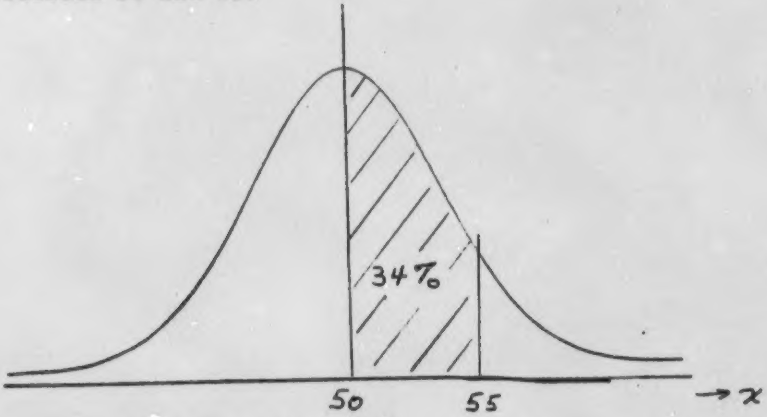


Fig. 13.5

The probability of getting an air pollution reading between 50 and 55 is 34%.

b) readings between 55 and 60

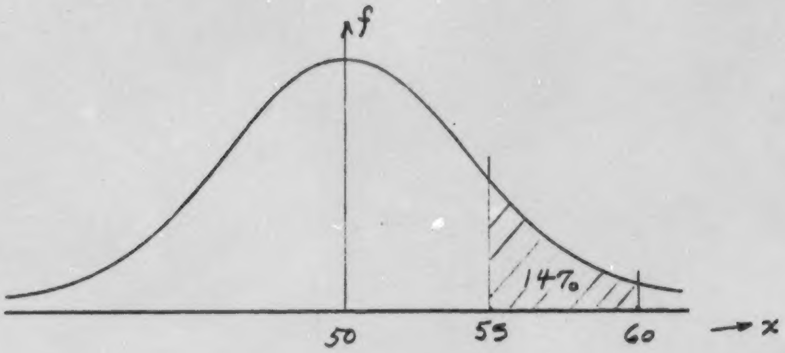


Fig. 13.6

The probability of getting an air pollution reading between 55 and 60 is 14%.

c) readings between 40 and 55.

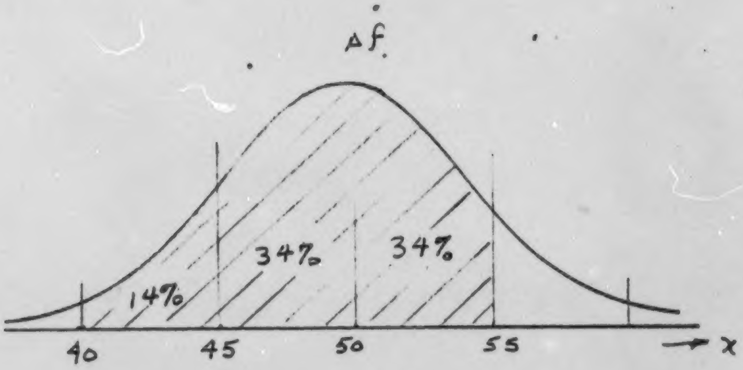


Fig. 13.7

The probability of getting an air pollution reading between 40 and 55 is the sum of the values between 40 and 45, 45 and 50, and 50 and 55, or 14% + 34% + 34% = 82%.

d) readings below 40

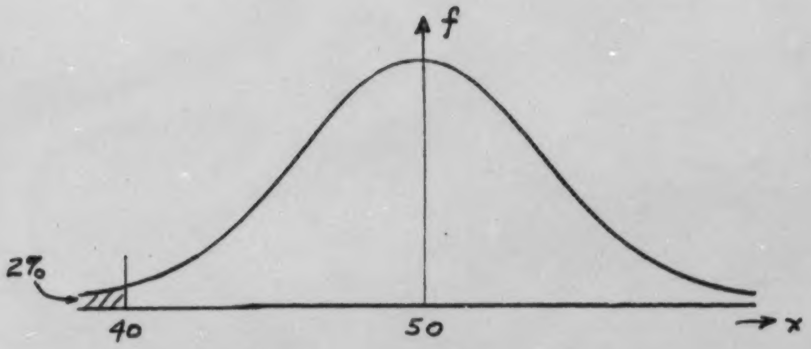


Fig. 13.8

The probability of getting an air pollution reading below 40 is 2%.

Suppose we wish to know the probability of getting a result between two values on the normal curve, at least one of which is not an integral number of standard deviations from the mean. We need to define a new term, the z score.

$$\boxed{\text{z score} = \frac{x - \mu}{\sigma}} \qquad \text{Eq. 13.1}$$

where x is the value of the distribution being considered

μ is the mean of the distribution

σ is the standard deviation

The z score tells us how many standard deviations we are above (positive z score) or below (negative z score) the mean.

Example 13.2.

Using the data of example 13.1 (mean = 50, σ = 5), find the z score for (a) x = 50; (b) x = 55; (c) x = 58; and (d) x = 43.

a) $$z = \frac{x - \mu}{\sigma} = \frac{50 - 50}{5} = 0$$

As expected, the z score for the mean is zero, since the z score represents the number of standard deviations above or below the mean.

b) $$z = \frac{x - \mu}{\sigma} = \frac{55 - 50}{5} = \frac{5}{5} = 1$$

This agrees with the results of example 13.1 since 55 was one standard deviation above the mean.

c) $$z = \frac{x - \mu}{\sigma} = \frac{58 - 50}{5} = \frac{8}{5} = 1.6$$

d) $$z = \frac{x - \mu}{\sigma} = \frac{43 - 50}{5} = \frac{-7}{5} = -1.4$$

Note again that values of z are negative for values of x below the mean.

We also need to be able to find areas under the normal curve between various z scores. These areas are given to us in table 13.1 on the next page.

Note the following about the table of areas under a standard normal curve (table 13.1):

1. Areas are given between the mean and the given z score.

2. Areas are given only for positive values of z scores. Due to symmetry, the area between the mean and a negative z score is the same as the area between the mean and the corresponding (same absolute value) positive z score. That is, to find the area between the mean, $z = 0$, and $z = -1.2$, we can look up the area next to $z = 1.2$, on the table, and get 0.3849 for the required area.

3. To find the area between $z = 0$ and $z = 0.83$, we go down to 0.8 under the z column and then over to the box under 0.03, and read an area of 0.2967.

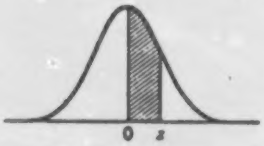
4. For $z = 1.0$, the area under the curve is 0.3413. This checks with the approximate value of 34% that we have been using.

5. As z increases, the area under the normal curve between the mean and the

z value increases. For z = 3.00, the area is 0.4987, or nearly 0.5000, the total area to the right of the mean (the area under the complete normal curve is 1.0). Hence there is very little area to the right of a z score of 3; that is there is a very small probability that a value will fall more than 3 standard deviations from the mean of a normal distribution.

Areas of a Standard Normal Distribution

An entry in the table is the proportion under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Table 13.1

Example 13.3

Using the data of example 1 ($\mu = 50$ and $\sigma = 5$), find the probability of getting an air pollution reading on any given day

a) between 50 and 58

b) between 50 and 43

c) between 43 and 58

d) below 43

3) above 47

f) between 53 and 58

g) equal to 55

We draw the normal curve again for reference purposes and to be used as a check.

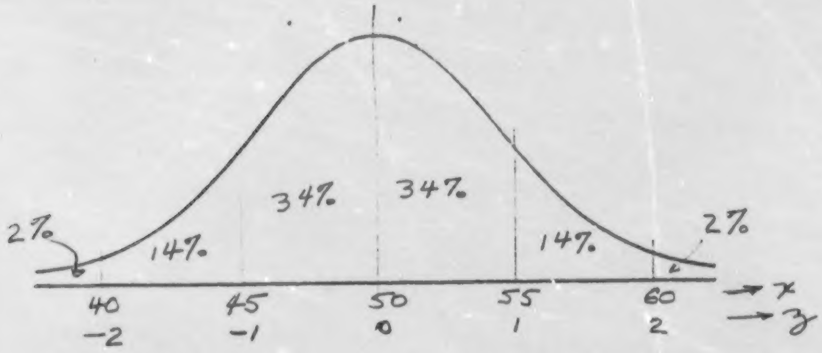


Fig. 13.9

Each part of the problem can be solved using the values for areas under a normal curve (table 13.1). Normal curve sketches are shown for each part to help in the analysis.

a) between 50 and 58

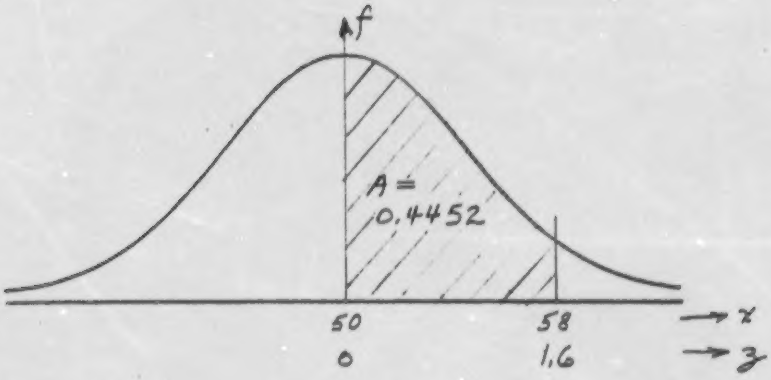


Fig. 13.10

For x = 58, $z = \frac{58 - 50}{5} = \frac{8}{5} = 1.6$.

From table 13.1, we find that the area of the region between z = 0 and z = 1.6 (shaded above) is 0.4452. We can convert this to a percent by multiplying by 100%. Hence the required probability is 44.5% (to the nearest tenth).

b) between 50 and 43

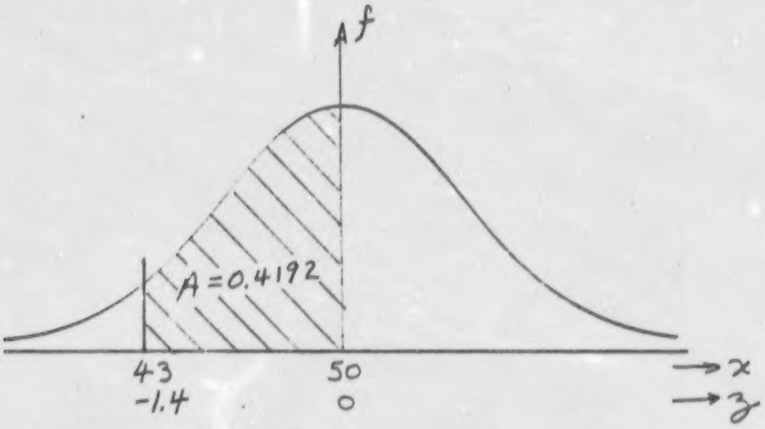


Fig. 13.11

As indicated, areas in table 13.1 are given only for positive values of z. However, because of symmetry, the results are the same for negative values of z.

For $x = 43$, the z score is $\frac{43 - 50}{5} = -1.4$. The area between $z = 0$ and $z = +1.4$ is 0.4192.

The probability that an air pollution reading will be between 43 and 50 is 41.9%.

c) between 43 and 58

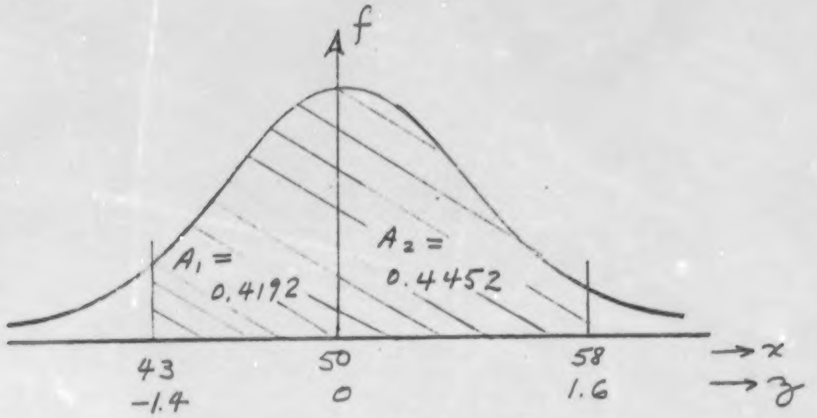

f
$A_1 =$ 0.4192
$A_2 =$ 0.4452
43
-1.4
50
0
58
1.6
→ x
→ z

Fig. 13.12

Results in the table are for regions between the mean and a given z score. Hence we must look up two values (for $z = 1.4$ and for $z = 1.6$) and add. The required probability is $0.4192 + 0.4452 = 0.8644$, or 86.4%.

d) below 43

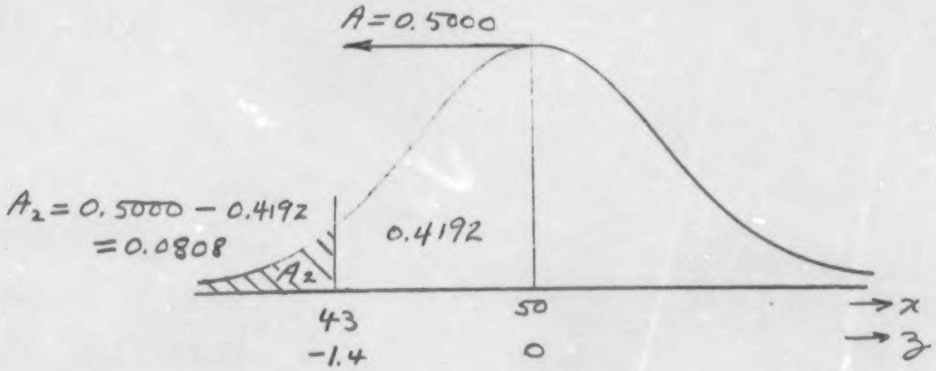

A=0.5000
$A_2 = 0.5000 - 0.4192$
$= 0.0808$
A_2
0.4192
43
-1.4
50
0
→ x
→ z

Fig. 13.13

We require the area shaded in the diagram above. However the table only gives areas between the mean and a given z score. Between z = 0 and z = -1.4, the area is 0.4192. Hence, the required area is 0.500 - 0.4192 = 0.0808. We use 0.5000 since the total area under the normal curve is 1.0000 and half of the area is to the left of the mean. The probability of getting a reading below 43 is 0.0808, or 8.1%.

e) above 47

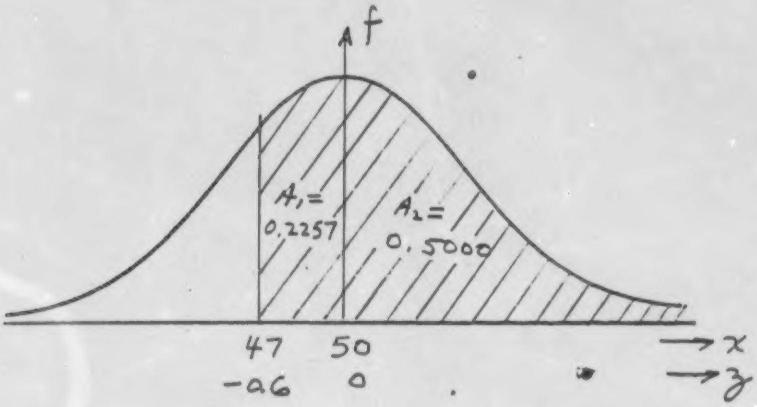


Fig. 13.14

The table gives us the area between z = 0 and z = -0.6: 0.2257. Half of the total area is to the right of the mean (z = 0). Hence, the total area to the right of x = 47 is 0.2257 + 0.5000 or 0.7257. The probability that an air pollution reading will be above 47 is 72.6%.

f) between 53 and 58

The required area is between x = 53 and x = 58 (corresponding to z = 0.6 and z = 1.6). We show both areas below and subtract to get the desired result, 0.4452 - 0.2257 = 0.2195, or 22%.

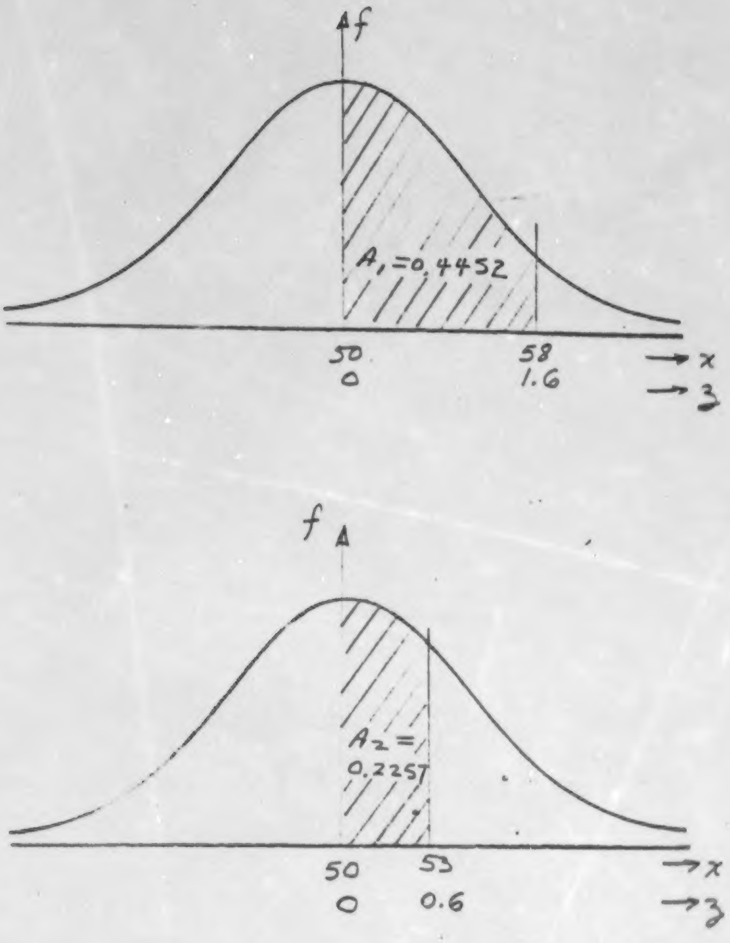


Fig. 13-15

g) equal to 55

A value of x is considered to be 55 if it is between 54.5 and 55.5, that is, no more than a half unit from 55.

$$\text{For } x = 54.5, \quad z = \frac{54.5 - 50}{5} = \frac{4.5}{5} = 0.9$$

$$\text{For } x = 55.5, \quad z = \frac{55.5 - 50}{5} = \frac{5.5}{5} = 1.1$$

The appropriate normal curves are shown below. Areas are obtained from table 13.1, corresponding to the above z scores.

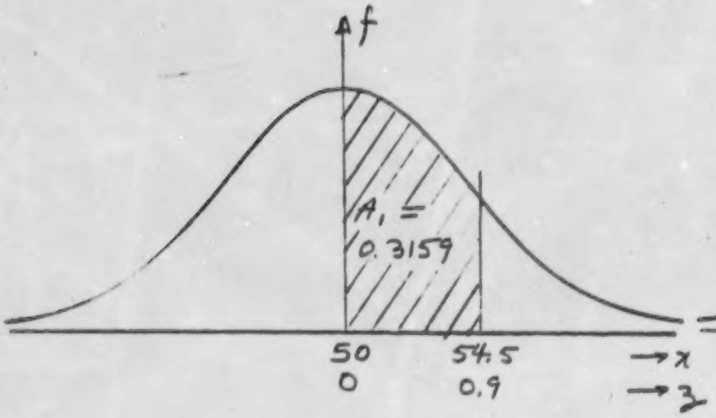

f
A₁ = 0.3159
50
0
54.5
0.9
→ x
→ z

Fig. 13.16(a)

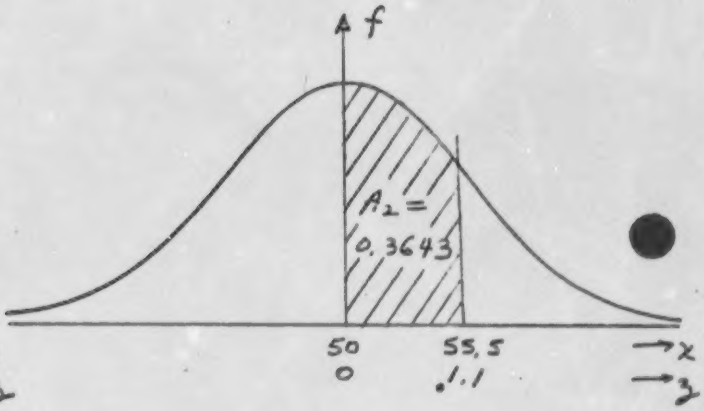

f
A₂ = 0.3643
50
0
55.5
1.1
→ x
→ z

Fig. 13.16(b)

The required area is $A_2 - A_1$, or 0.3643 - 0.3159 = 0.0484. The probability of getting an air pollution reading of 55 is 4.8%.

h) What air pollution reading is exceeded on 20% of the days?

Here we have to work backwards from a knowledge of areas under the normal curve to a z score ($z_?$) and finally to an air pollution reading.

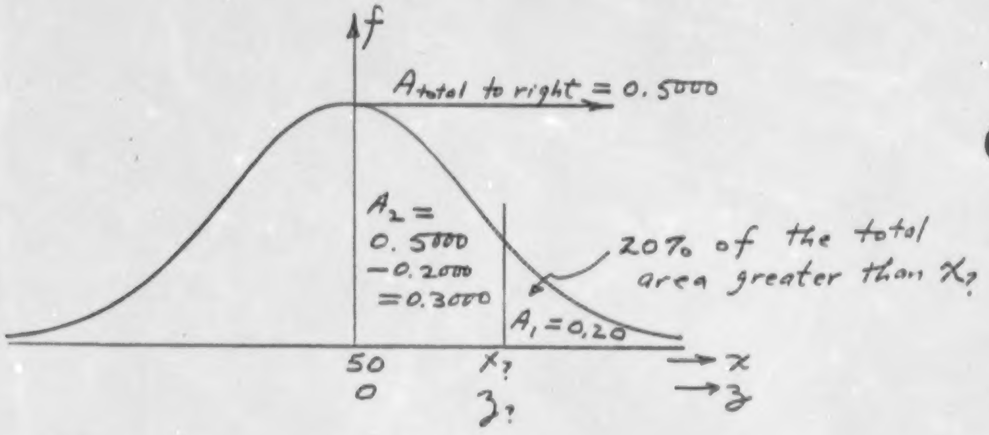

f
A total to right = 0.5000
A₂ = 0.5000 − 0.2000 = 0.3000
20% of the total area greater than x?
A₁ = 0.20
50
0
x?
z?
→ x
→ z

Fig. 13.17

The area to the right of our unknown z score ($z_?$) is 0.20. Hence, the area between z = 0 and $z_?$ is 0.50 - 0.20, or 0.30.

We now look in table 13.1 for the z score corresponding most closely to an area of 0.30. Note that this is the reverse of the process we have been using up to this point of looking up an area corresponding to a given z score.

From table 13.1, the area closest to 0.3000 is 0.2995, corresponding to a z score of 0.84. Hence, our required z score is approximately 0.84.

We next have to convert this z score to an air pollution reading. We start with our definition of a z score

$$z = \frac{x - \mu}{\sigma}$$

and solve for x.

$$z \cdot \sigma = x - \mu \qquad \text{(multiply both sides by } \sigma\text{)}$$

$$\boxed{x = \mu + z \cdot \sigma} \qquad \text{(add } \mu \text{ to both sides)} \qquad \text{Eq. 13.2}$$

$$x_? = 50 + 0.84(5) = 50 \pm 4.2 = 54.2$$

Hence, 20% of the air pollution readings exceed a value of 54.2.

The use of areas under the normal curve is very valuable in checking hypotheses (guesses) about a given distribution. This is not considered here, but I hope to include it in future editions of this book.

Exercises

1. Assume that water pollution readings taken on various days for a stream have a mean value of 100 units and a standard deviation of 10 units. Compute the probability that the water pollution reading on any given day will be

   a) between 100 and 105;

   b) above 112;

   c) between 88 and 98;

   d) equal to 107;

   e) above 112 or below 108.

   f) what water pollution reading will be exceeded on 30% of the days?

2. Assume that the amount of garbage collected daily in a town has a mean of 300 tons with a standard deviation of 40 tons. Find the probability that the amount of garbage collected on any one day will be

a) between 300 tons and 332 tons;

b) below 318 tons;

c) equal to 330 tons;

d) between 284 tons and 324 tons.

3. Assume that a 1979 model of a car gets an average (mean) of 20 miles per gallon for highway driving with a standard deviation of 2 miles per gallon. What percent of this model of car will get

a) between 20 and 23 miles per gallon?

b) over 22 miles per gallon?

c) less than 21 miles per gallon?

d) between 18 and 23 miles per gallon?

e) What value of miles per gallon will be exceeded by 60% of the cars?

4. Assume that the mean tensile (pulling) strength of samples of the steel used in a nuclear power plant is 70,000 psi (pounds per square inch) with a standard deviation of 4,000 psi. What is the probability that a steel sample chosen at random will have a tensile strength

a) over 75,000 psi?

b) below 60,000 psi?

c) between 62,000 psi and 74,000 psi?

d) What tensile strength will be exceeded by 80% of the samples?

5. Assume that the mean sulfur content of a certain type of coal is 2% with a standard deviation of 0.2%. Find the probability that a random sample of this coal will have a sulfur content

a) above 2%

b) above 3%

c) above 2.4%

d) below 1.6%

e) What sulfur content will be exceeded by 10% of the coal?

6. A certain type of bomb has a mean explosive capacity of 1,000,000 tons of TNT with a standard deviation of 50,000 tons of TNT. Find the probability that one of these bombs chosen at random has an explosive capacity

a) over 1,100,000 tons of TNT

b) between 1,000,000 and 1,050,000 tons of TNT

c) below 1,020,000 tons of TNT

d) 20% of the bombs of this type have an explosive capacity greater than what amount of explosive power in tons of TNT.

7. Assume that the mean literacy rate for various communities of a developing country is 30% with a standard deviation of 5%. Compute the probability that a community in that country chosen at random will have a literacy rate

a) above 36%

b) between 32% and 34%

c) below 33%

d) What literacy rate is exceeded by 15% of the communities?

Chapter 14 Chi-Square Analysis

Often we are interested in knowing whether or not there is a statistical relationship between two variables. For example, we may wish to know if the time for a country's population to double is dependent on its average per capita gross national product. Or if there is a relationship between a person's income and his or her view on some environmental issue. The chi-square test enables us to investigate such situations. The analysis is best illustrated through examples.

Example 14-1

Assume that a survey was conducted by asking 200 randomly selected people their income and their belief that either jobs or environmental pollution is the more important issue. The data obtained is summarized below:

	jobs	pollution	totals
less than \$12,000	#1 40	#2 30	70
\$12,000 to \$20,000	#3 30	#4 50	80
above \$20,000	#5 10	#6 40	50
totals	80	120	200

Table 14.1

We wish to determine whether a person's opinion is independent of his or her income. We will compare the given (observed) frequencies from the random sample with the frequencies we would expect if the two variables are independent. The chi-square (χ^2) statistic will be used to tell if the differences between the observed and expected frequencies is large enough to state that the two variables are not independent.

We start by writing down a statement called the null hypothesis, H_0, which is to be tested. The null hypothesis assumes that there is independence between the two variables, that one variable's value does not affect that of the other variable. In this example,

H_o: A person's view on jobs vs. pollution is independent of income.

We next state an alternate hypothesis, H_a, which assumes dependence between the variables, that the value of one variable does depend on that of the 2nd variable.

H_a: A person's view on jobs vs. pollution is dependent on income.

The two hypotheses are always stated in this general form in performing a χ^2 test. The null hypothesis always assumes independence between the variables, while the alternate hypothesis always assumes dependence between the variables.

We next assume the null hypotheses, H_o, to be correct, in order to obtain expected frequencies based on the assumption of independence. If the actual frequencies give results that differ greatly from those based on the null hypothesis, we will have to reject it and accept H_a, the alternate hypothesis.

Table 14.1 shows that 80 of the 200 people surveyed, or 40%, feel that jobs is the more important issues, and 60% (120 of the 200 people surveyed) feel that pollution is the more important issue. _If_ opinion is independent of income level (if H_o is correct), we would expect 40% of the people in each income category to believe that jobs is the more important issue, and 60% of the people in each income category to believe that pollution is the more important issue. We start with cells 1, 3, and 5, and use 40%:

cell #1 $\frac{40}{100} \times 70 = 28$ (40% means 40/100)

(There are a total of 70 people in the income category of less than $12,000.)

cell #3 $\frac{40}{100} \times 80 = 32$ (80 people have incomes from $12,000 to $20,000).

cell #5 $\frac{40}{100} \times 50 = 20$ (50 people have incomes above $20,000)

Similarly, (using 60% for cells #2, #4, and #6, since 60% of the entire survey think that pollution is the more important issue).

cell #2 $\frac{60}{100} \times 70 = 42$

cell #4 $\frac{60}{100} \times 80 = 48$

cell #6 $\frac{60}{100} \times 50 = 30$

An alternate way of getting the values in each cell is from the equation

$$\boxed{\text{expected frequency} = \frac{(\text{row total}) \times (\text{column total})}{\text{sample size}}} \qquad \text{Eq. 14.1}$$

Let us use this equation to check our result for cell #4.

row total = 80 (80 is in totals column to right of cell #4)

column total = 120 (120 is in totals column below cell #4)

sample size = 200 (200 people in the survey.)

$$\text{expected frequency} = \frac{(80)(120)}{200} = 48 \quad \text{(it checks)}$$

Now we can fill in the table of expected values, as shown below.

	jobs	pollution	totals
less than $12,000	#1 28	#2 42	70
$12,000 to $20,000	#3 32	#4 48	80
above $20,000	#5 20	#6 30	50
totals	80	120	200

Table 14.2

Note that the totals for the observed and expected frequency tables are the same, but the values in each cell are different.

We must now investigate the differences between the observed (table 14.1) and expected (table 14.2) values. We repeat the observed and expected values and compute their differences, as shown below.

Cell #	Category	O Observed	E Expected	O - E Difference
1	low income - jobs	40	28	+12
2	low income - pollution	30	42	-12
3	middle income - jobs	30	32	- 2
4	middle income - pollution	50	48	+ 2
5	high income - jobs	10	20	-10
6	high income - pollution	40	30	+10
			sum	= 0

Table 14.3

Note that the sum of differences $\Sigma(O - E)$ is equal to zero. This provides a check on the calculated expected values.

The question now is whether these differences in table 14.3 are large or small. If they are large, we can reject the null hypothesis of independence and claim that there is a dependence of a person's income on opinions on the jobs vs. environment issue. If they are small, we will be unable to reject the null hypothesis. We would like to have one number as a measure of the differences which will enable us to judge whether the differences are large or small.

Note that we can't just add up the differences, because we get zero; the negative differences exactly balance the positive differences. To get around this difficulty, we will work with squares of the differences, just as we did when we had a similar problem with regard to computing the standard deviation. (Chapter 11)

Squaring the differences gives us the following table:

Cell	Observed O	Expected E	Difference O - E	Difference squared $(O - E)^2$
1	40	28	+12	144
2	30	42	-12	144
3	30	32	- 2	4
4	50	48	+ 2	4
5	10	20	-10	100
6	40	30	+10	100

Table 14.4

Note that we got a squared difference of 100 for both cells 5 and 6. However, this squared difference is more important for cell 5 where we have an expected value of 20 than for cell 6 where the expected value is 30. Put another way, a difference of 5, for example, is much more significant when the expected value is 10 than when the expected value is 100. So we must take the expected values as well as the difference into account. To do this, we divide each squared difference by the corresponding expected value for its cell. The sum of these numbers is called χ^2 (chi-square). In symbols,

$$\boxed{\chi^2 = \Sigma \frac{(O - E)^2}{E}} \qquad \text{Eq. 14.2}$$

We can now compute χ^2 for our problem.

cell	O	E	O - E	$(O - E)^2$	$\frac{(O - E)^2}{E}$
1	40	28	+12	144	144/28 = 5.14
2	30	42	-12	144	144/42 = 3.43
3	30	32	- 2	4	4/32 = 0.13
4	50	48	+ 2	4	4/48 = 0.08
5	10	20	-10	100	100/20 = 5.00
6	40	30	+10	100	100/30 = 3.33
					17.11

Table 14.5

$$\chi^2 = \Sigma\left(\frac{(O - E)^2}{E}\right) = 17.11$$

If the two variables (income level and opinion) were independent, χ^2 would be zero or near zero. If χ^2 is far from zero, there is a high probability that the variables are not independent and that there is a statistical relationship between them. In that case we could reject the null hypothesis of independence and accept the alternate hypothesis.

So we have to determine if our χ^2 value is significantly above zero. To do this, we compare our χ^2 with a critical χ^2 value from table 14.6 on the next page. This table lists critical values of chi square for different degrees of freedom and significance levels, α. We will discuss degrees of freedom and significance levels after this problem. As we shall soon see, for a table with 3 rows and 2 columns, there are two degrees of freedom. We will use a significance level of 0.01. From table 14.1, we find that the critical chi-square value for 2 degrees of freedom and a significance level, α, of 0.01 (1%) is 9.21.

Since our computed value for χ^2, 17.11, is greater than the critical value, 9.21, we conclude that we can reject the null hypothesis. We accept the alternative hypothesis. There is a statistical relationship between income level and a person's opinion as to whether jobs or environment is the more important issue.

Table 14.6

Critical Chi Square (χ_{cr}^2)

	significance level, α	
d.o.f.	(.05) 5%	(.01) 1%
1	3.84	6.64
2	5.99	9.21
3	7.82	11.34
4	9.49	13.28
5	11.07	15.09
6	12.59	16.81
7	14.07	18.48
8	15.51	20.09
9	16.92	21.67
10	18.31	23.21
11	19.68	24.72
12	21.03	26.22
13	22.36	27.69
14	23.68	29.14
15	25.00	30.58
16	26.30	32.00
17	27.59	33.41
18	28.87	34.80
19	30.14	36.19
20	31.41	37.57

Degrees of freedom

We have stated that when we have 3 rows and 2 columns, there are two degrees of freedom. Let us see why this is so.

Consider the tabulation of values below, which is part of table 14.2. Expected frequencies are given for cells 1 and 3.

	jobs	pollution	total
less than $12,000	#1 28	#2	70
$12,000 to $20,000	#3 32	#4	80
above $20,000	#5	#6	50
	80	120	200

Table 14.7

Notice that with just the values in cells 1 and 3, we can get values for each of the other cells, as follows:

#2 70 - 28 = 42 (value in cell #2 = total for top row - value in cell #1)

#4 80 - 32 = 48

#5 80 - (32 + 28) = 20 (value in cell #5 = total for jobs column - value in cells #1 and #3)

#6 50 - 20 = 30

Note that these results agree with those in table 14.2. Hence, if we know frequencies in just 2 cells, we can find frequencies in all the other cells. This is why the degrees of frequency is 2 in this case.

Consider the table with two rows and two columns, shown below. The totals are known from the results of a survey.

			totals
	#1	#2	60
	#3	#4	40
totals	30	70	100

Table 14.8

How many expected frequencies do we need in order to be able to compute all the others?

If you said "one," you are correct. We can compute the expected frequency for cell #1, $\frac{60 \times 30}{100} = 18$. We can then get all the other expected frequencies, as follows:

#2 $60 - 18 = 42$
#3 $30 - 18 = 12$
#4 $70 - 42 = 28$

We get the following table of expected results.

#1 18	#2 42	60
#3 12	#4 28	40
30	70	100

Table 14.9

We can check these values in several ways. For example, adding up all the expected frequencies gives 100, which is the total sample size. Or we can use the equation:

$$\text{expected frequency} = \frac{(\text{row total}) \times (\text{column total})}{\text{sample size}}$$

Now that we have explained the rationale for values of the degrees of freedom (d.o.f.) we present a formula:

$$\boxed{\text{d.o.f.} = (\text{rows} - 1) \times (\text{columns} - 1)} \qquad \text{Eq. 14.3}$$

Let us apply this to the above cases:

a) 3 rows and 2 columns

$\text{d.o.f.} = (3 - 1)(2 - 1) = 2 \cdot 1 = 2$ (it checks)

b) 2 rows and 2 columns

$\text{d.o.f.} = (2 - 1)(2 - 1) = 1 \cdot 1 = 1$ (it checks)

Significance level

The two values of the significance level, α, that are generally used in statistical studies are 0.01 and 0.05, as shown on table 14.6. If we use $\alpha = 0.01$, we are saying that we wish to be 99% ($1.00 - 0.01 = 0.99$) sure that we will be correct if we reject the null hypothesis. If we use $\alpha = 0.05$, we are saying that we are willing to accept 95% certainty in rejecting a null hypothesis. Note the values of χ^2 in table 14.6 are greater for $\alpha = 0.01$ than they are for $\alpha = 0.05$, for any value for the degrees of freedom. Thus, we are less likely to reject a null hypothesis when we use $\alpha = 0.01$ than we are when we use $\alpha = 0.05$.

Outline for a chi-square test

There are two conditions which must be met in order to carry out a chi-square test.

(1) Every observation in the sample must fall into one and only one cell.

(2) The expected frequency of at least eighty percent of the cells must be at least 5.

Here is a summary of the chi-square testing procedure.

1. State the two hypotheses. They will always have the same general form:

   H_o: the two variables are independent

   H_a: the two variables are dependent

   Assume H_o is valid for testing purposes.

2. Fill in the table for the expected frequencies, using the equation

   $$\text{expected frequency} = \frac{(\text{row total}) \times (\text{column total})}{\text{sample size}}$$

   Check the expected frequencies against the required totals for each row and column.

3. Compute chi-square using

   $$\chi^2 = \Sigma\left(\frac{(O - E)^2}{E}\right)$$

4. Use Table 14.6 to find the critical chi-square value (χ_{cr}^2), taking the significance level and the degrees of freedom into account.

5. Compare χ^2 with χ_{cr}^2

If χ^2 is greater than χ_{cr}^2, we reject the null hypothesis and accept the alternative hypothesis that there is a dependence between the two variables.

If χ^2 is less than χ_{cr}^2, we cannot reject the null hypothesis. This does not mean that the null hypothesis is correct. It indicates that we lack sufficient evidence to claim that the two variables are not independent.

Example 14.2

Table 14.10 gives information on birth rates and per capita GNP for each country from the 1979 World Population Data Sheet. Using $\alpha = 0.01$, determine if the two variables are independent.

		GNP per capita			
		≤ \$500	\$500-1199	≥ \$1200	Totals
Birth Rate	≤ 20	#1 1	#2 1	#3 36	38
	21-39	#4 12	#5 16	#6 19	47
	≥ 40	#7 51	#8 15	#9 10	76
	Totals	64	32	65	161

Table 14.10

As an example of how data was obtained for the table, consider Algeria, the first country on the Data Sheet.

Birth rate = 48 (births per 1000 population)

Per capita GNP = \$1,100

Algeria would be placed in cell #8 since its birth rate is above 40 and its per capita GNP is between \$500 and \$1,199

This example is very significant because many population experts have claimed that high birth rates are related to poor economic conditions, and that birth control approaches won't succeed, unless social and economic conditions improve.

We will use the chi-square analysis even though two of the nine cells have frequencies of only one; about 78% (almost 80%) of the cells have frequencies of 5 or more. If χ^2 is far from the critical χ^2 value, we can still use the analysis since we only fail by about 2% of meeting the requirement of 80% of the cells having frequencies of 5 or more.

Step 1. We state the two hypotheses

H_o: Birth rates are independent of the average GNP per capita

H_a: Birth rates are dependent on the average GNP per capita

We assume that H_o is valid; the following calculations will test the correctness of this assumption.

Step 2. We obtain the value for expected frequencies, using the equation

$$\text{expected frequency} = \frac{\text{(row total) x (column total)}}{\text{sample size}}$$

We put the expected frequencies in the table below.

GNP per capita

		≤ $500	$500-1199	≥ $1200	Totals
Birth Rates	≤ 20	$\frac{64 \times 38}{161} = 15.1$	$\frac{32 \times 38}{161} = 7.6$	$\frac{65 \times 38}{161} = 15.3$	38.0
	21-39	$\frac{64 \times 47}{161} = 18.7$	$\frac{32 \times 47}{161} = 9.3$	$\frac{65 \times 47}{161} = 19.0$	47.0
	≥ 40	$\frac{64 \times 76}{161} = 30.2$	$\frac{32 \times 76}{161} = 15.1$	$\frac{65 \times 76}{161} = 30.7$	76.0
	Totals	64.0	32.0	65.0	161.0

Table 14.11

Note that the totals in table 14.11 (the expected frequencies) are equal to the totals in table 14.10 (the observed frequencies). This provides a check on the computation of frequencies in each cell in table 14.11. Usually, slight discrepancies occur due to round off.

Step 3. We compute chi-square using the tabulation below.

cell	O	E	O - E	$(O - E)^2$	$\frac{(O - E)^2}{E}$
1	1	15.1	-14.1	198.8	13.2
2	1	7.6	-6.6	43.6	5.7
3	36	15.3	20.7	428.5	28.0
4	12	18.7	-6.7	44.9	2.4
5	16	9.3	6.7	44.9	4.8
6	19	19.0	0	0	0.0
7	51	30.2	20.8	432.6	14.3
8	15	15.1	-0.1	0.0	0.0
9	10	30.7	-20.7	428.5	14.0
sums			0 (this checks)		82.4

Table 14.12

Note that the sum of the (O - E) values equals zero, providing a partical check.

$$\chi^2 = \Sigma\left(\frac{(O - E)^2}{E}\right) = 82.4$$

Step 4. We obtain χ_{cr}^2. For 3 rows and 3 columns, the degrees of freedom is

$$\text{d.o.f.} = (\text{rows} - 1)(\text{columns} - 1) = (3 - 1)(3 - 1) = 4$$

From table 14.6, with d.o.f. = 4 and $\alpha = 0.01$, we find that $\chi_{cr}^2 = 13.28$

Step 5. We compare χ^2 with χ_{cr}^2. Since χ^2, 82.4, is greater than χ_{cr}^2, we may reject the null hypothesis of independence and say that there is a relationship between the birth rate of a country and its average per capita GNP.

Since several countries which have high birth rates with high average GNP per capita have very great inequality in distribution of income and much poverty, the relationship is even stronger than that indicated by this analysis.

We will look at this relationship further in the next chapter on correlation.

Exercises

1. Assume that a survey was taken to find out if the number of children that young people want is related to the number of children, including themselves, in their family. The results are shown below.

		children in family	
		1 to 3	more than 3
children wanted	0 to 2	25	25
	more than 2	20	30

Does the survey show a dependency between the number of children wanted and the number of children in a family? Use $\alpha = 0.05$.

2. Obtain data from the 1979 World Population Data Sheet to complete the following table and test the hypothesis that infant mortality rates are independent of the average GNP per capita in a country. Use $\alpha = 0.01$.

		per capita GNP		
		$\leq$ \$500	\$501 - \$1100	$\geq$ \$1200
Infant Mortality	$\leq$ 40			
	41-49			
	$\geq$ 100			

3. Obtain data from the 1979 World Population Data Sheet to complete the following table and test the hypothesis that the percentage of a population under 15 years of age is independent of the average GNP per capita in a country. Use $\alpha = 0.01$

	per capita GNP		
	≤ \$500	\$501 - \$1199	≤ \$1200
% under 15 < 40%			
% under 15 ≥ 40%			

4. Use the 1979 World Population Data Sheet to fill in the table below. Determine whether there is a dependency between the doubling time of a country and the average per capita GNP. Use $\alpha = 0.01$.

		Average per capita GNP		
		≤ \$500	\$501 - \$1199	≥ \$1200
Doubling Time	less than 30 years			
	30 - 50 years			
	over 50 years			

5. Assume that a random sample of 150 people were asked their age and their view on whether or not the arms budget should be reduced. Using the results indicated below, is a person's opinion on this issue dependent on the person's age. Use $\alpha = 0.05$.

	under 25 years	25 to 45 years	over 45 years
cut arms budget	28	36	11
maintain arms budget	22	44	9

6. Use the 1979 World Population Data Sheet to fill in the table below. Test the hypothesis that the Physical Quality of Life Index (PQLI) is independent of the per capita GNP. Use $\alpha = 0.01$.

		Average per capita GNP		
		≤ \$500	\$501 - \$1199	≥ \$1200
	≤ 40			
PQLI	41 - 75			
	≥ 76			

Chapter 15 Linear Correlation Analysis

In the previous chapter, we showed how the chi-square analysis could test whether a relationship existed between two variables. In this chapter, we will discuss methods which will enable us to find the form and strength of a relationship between two variables.

There are several types of correlation, but we will only be concerned with linear correlation in this book.

15.1 Linear Correlation Analysis

There are two ways in which we can tell if there is a linear correlation between two variables. We can plot the variables in a graph which is called a scattergram. Also, we can compute a coefficient which will measure the correlation between the two variables. Such a coefficient is Pearson's correlation coefficient, which is symbolized by the letter r, and is obtained from the following equation.

$$\boxed{r = \frac{n\Sigma(xy) - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2}\ \sqrt{n(\Sigma y^2) - (\Sigma y)^2}}} \qquad \text{Eq. 15.1}$$

where n = the number of pairs of data

x = the first variable

y = the second variable

The values for the summations required are best found by using a tabulation of x and y values. We must be careful to distinguish between (Σx^2) and $(\Sigma x)^2$.

Σx^2 is the sum of the squares of all the individual x's. That is, we square each value of x and add all the values of x^2.

$(\Sigma x)^2$ is found by first adding all the x values and then squaring the sum.

An alternate form for Pearson's correlation coefficient is

$$r = \frac{\Sigma x \cdot y - \left(\frac{\Sigma x}{n}\right) \cdot \left(\frac{\Sigma y}{n}\right)}{\sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}}$$

or

$$\boxed{r = \frac{\frac{\Sigma x \cdot y}{n} - \mu_x \mu_y}{\sigma_x \cdot \sigma_y}} \qquad \text{Eq. 15.2}$$

where μ_x = the mean of the x values

μ_y = the mean of the y values

σ_x = the standard deviation of the x values

σ_y = the standard deviation of the y values

This equation for Pearson's correlation coefficient is easier to remember and is in a convenient form for computations on many types of hand calculators. However, in this chapter, we will utilize Equation 15.1, as this is the form of the equation presented in most textbooks.

Possible results of a correlation analysis are indicated below in terms of scattergrams and Pearson's correlation coefficient, r.

1. Perfect positive linear correlation

Scattergram: plotted points all lie on a straight line sloping up to the right, Figure 15.1a.

Pearson's correlation coefficient: $r = +1.0$

2. Positive linear correlation

Scattergram: general trend of plotted points is a straight line sloping up to the right, Figure 15.1b.

Pearson's correlation coefficient: r is close to +1.0.

3. Little Linear Correlation

Scattergram: no general pattern to the plotted points, Figure 15.1c

Pearson's correlation coefficient: r is close to zero

4. Negative linear correlation

Scattergram: general trend of the plotted points is a line sloping down to the right, Figure 15.1d.

Pearson's correlation coefficient: r is close to -1.0

5. Perfect negative linear correlation

Scattergram: plotted points all lie on a straight line sloping down to the right, Figure 15.1e.

Pearson's correlation coefficient: r = -1.0

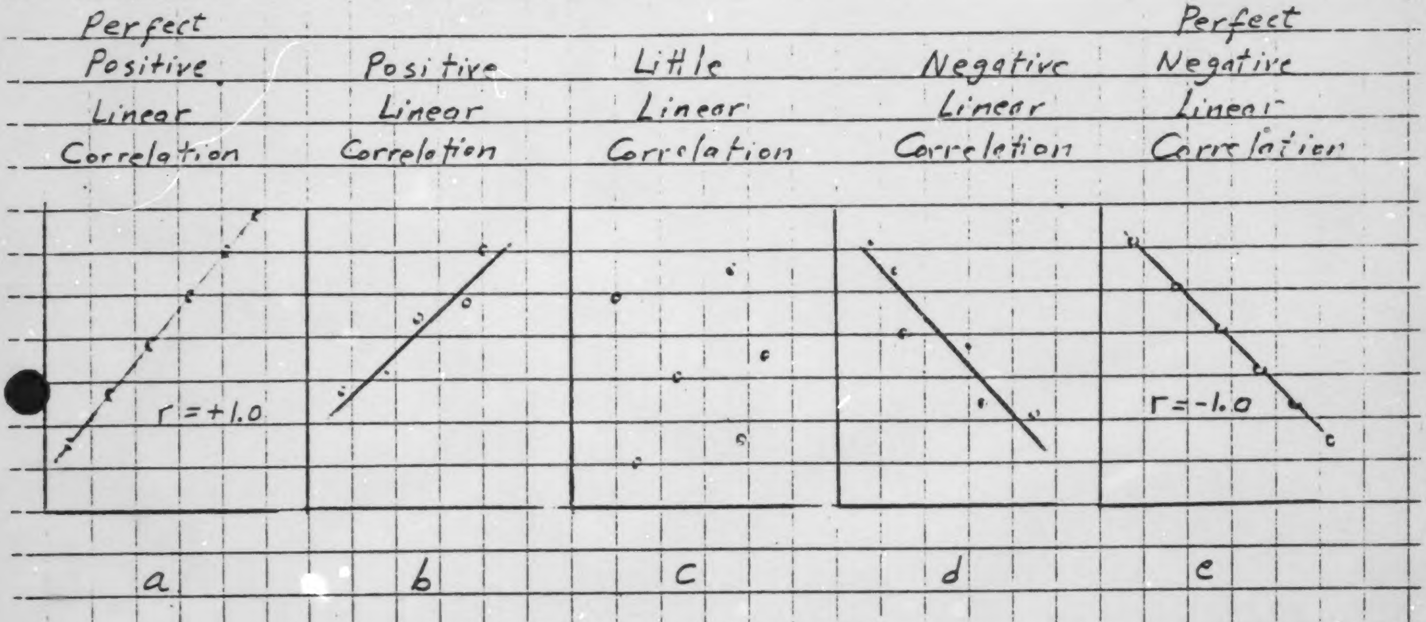


Figure 15.1

Note that Pearson's correlation coefficient is limited to values from -1.0 to +1.0, inclusive.

We will illustrate the use of the scattergram and Peason's correlation coefficient using 2 examples with relatively easy, made-up data before applying them to actual data.

Example 15.1. Examine the relationship between the sets of data shown below for the variables x and y by drawing a scattergram and computing Pearson's correlation coefficient.

x	y
0	1
1	3
2	4
3	5
4	7
5	8
6	10

The scattergram is constructed below.

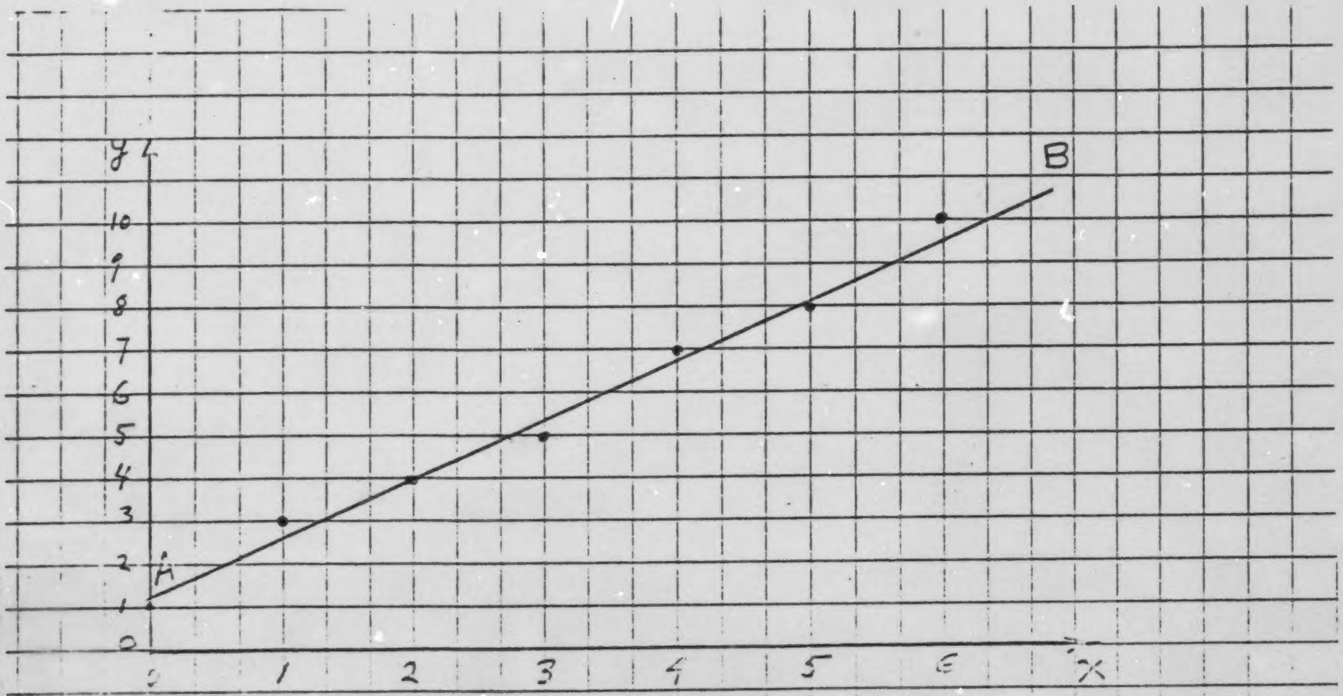


Figure 15.2

We see that the general trend of the points is a straight line, AB, that slopes up to the right. In other words, as x increases, y increases. This indicates positive linear correlation.

Next we set up a table to help us compute the correlation coefficient, r.

x	y	xy	x^2	y^2
0	1	0	0	1
1	3	3	1	9
2	4	8	4	16
3	5	15	9	25
4	7	28	16	49
5	8	40	25	64
6	10	60	36	100
$\Sigma x = 21$	$\Sigma y = 38$	$\Sigma xy = 154$	$\Sigma x^2 = 91$	$\Sigma y^2 = 264$

Table 15.1

Pearson's correlation coefficient, r, can now be computed. Note that there are 7 pairs of x and y values; n = 7.

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2}\ \sqrt{n(\Sigma y^2) - (\Sigma y)^2}}$$

$$r = \frac{7(154) - (21)(38)}{\sqrt{7(91) - (21)^2}\ \sqrt{7(264) - (38)^2}}$$

$$r = \frac{1078 - 798}{\sqrt{637 - 441}\ \sqrt{1848 - 1444}}$$

$$r = \frac{280}{\sqrt{196}\ \sqrt{404}} = \frac{280}{(14)(20.1)} = \frac{280}{281.4} = \underline{0.995}$$

r is very close to 1, again indicating a strong positive linear correlation.

Example 15.2. Examine the relationship between the sets of data shown below for the variables x and y by drawing a scattergram and computing Pearson's correlation coefficient.

x	y
1	3
2	9
3	5
4	2
5	8
6	4

The scattergram is constructed below.

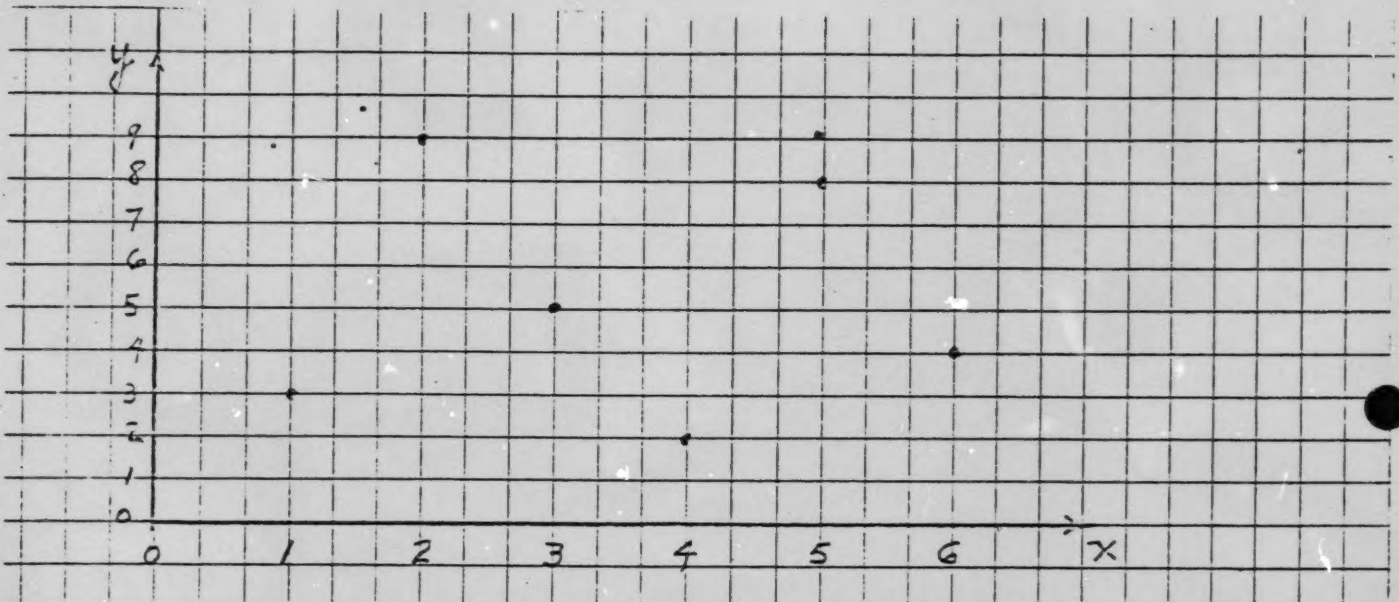


Figure 15.3

The points are widely scattered. There is no suggestion of a straight line at all. We can say that there is no (or very little) linear correlation between the variables x and y.

To compute r, we set up a tabulation of values.

x	y	xy	x^2	y^2
1	3	3	1	9
2	9	18	4	81
3	5	15	9	25
4	2	8	16	4
5	8	40	25	64
6	4	24	36	16
$\Sigma x = 21$	$\Sigma y = 31$	$\Sigma xy = 108$	$\Sigma x^2 = 91$	$\Sigma y^2 = 199$

Table 15.2

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2}\ \sqrt{n(\Sigma y^2) - (\Sigma y)^2}}\ .$$

Here, there are 6 pairs of x and y values; n = 6.

$$r = \frac{6(108) - (21)(31)}{\sqrt{6(91) - (21)^2}\ \sqrt{6(199) - (31)^2}}$$

$$r = \frac{648 - 651}{\sqrt{546 - 441}\ \sqrt{1194 - 961}} = \frac{-3}{\sqrt{105}\ \sqrt{233}} = \frac{-3}{(10.25)(15.26)} = \frac{-3}{156.3}$$

$$r = -0.019$$

Since r is close to zero, we say that there is very little linear correlation between x and y.

In the previous chapter, we used the chi-square analysis to show that there is a relationship between a country's birth rate and its average GNP per capita. We will pursue this relationship further in the following example.

Example 15.3. Values from the 1979 World Population Data Sheet for per capita GNP and birth rate are given for 27 countries in table 15.3. These countries were chosen for geographic balance. Draw a scattergram and calculate Pearson's correlation coefficient, r.

Country	Per capita GNP (x)	Birth rate (y)
Egypt	310	38
Mali	110	49
Nigeria	420	50
Somailia	110	48
Zambia	450	49
Zaire	130	46
Namibia	1,000	44
Israel	2,920	26
Saudi Arabia	4,980	49
India	150	34
Pakistan	190	44
Burma	140	39
China, Peoples Rep. of	410	20
Japan	5,640	15
United States	8,640	15
Mexico	1,110	41
Cuba	900	20
Haiti	230	42
Brazil	1,390	36
Argentina	1,730	22
Sweden	9,250	12
United Kingdom	4,430	12
France	7,290	14
Poland	3,150	19
Italy	3,450	13
USSR	3,010	18
Australia	7,340	16

Table 15.3

The scattergram is shown on the next page.

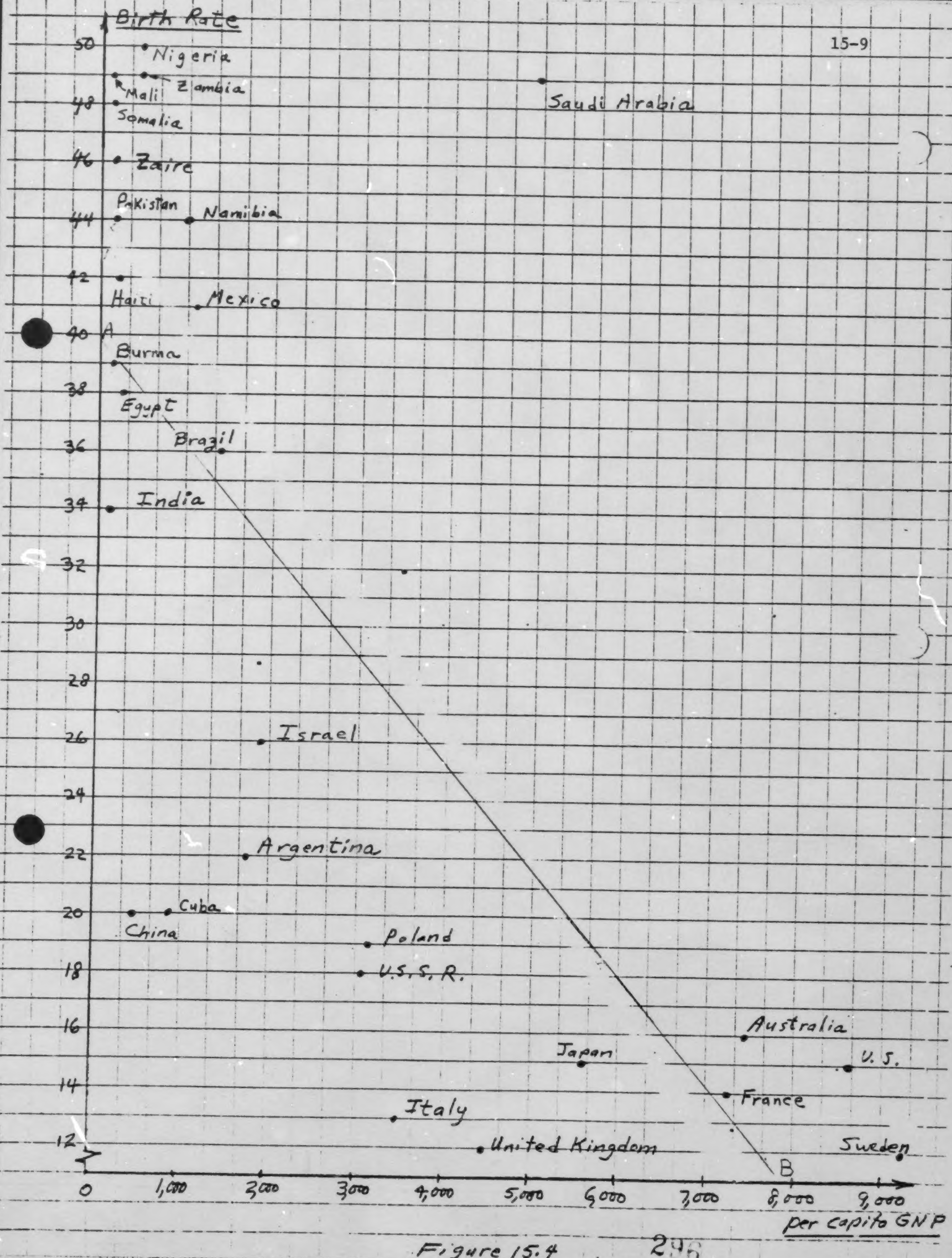

Birth Rate
50
Nigeria
Zambia
Mali
48
Somalia
Saudi Arabia
46
Zaire
Pakistan
Namibia
44
42
Haiti
Mexico
40
A
Burma
38
Egypt
Brazil
36
34
India
32
30
28
26
Israel
24
22
Argentina
20
Cuba
China
Poland
18
U.S.S.R.
16
Australia
Japan
U.S.
14
France
Italy
12
United Kingdom
Sweden
B
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
per capita GNP

Figure 15.4

As expected, most countries with high birth rates have very low average GNP per capita and vice versa. There are some exceptions. Most notable is Saudi Arabia which has a very high birth rate even though it has a very high average GNP per capita. This is due to a very uneven distribution of wealth; the median income is relatively low. The same is true to a lesser extent of Brazil and Mexico where relatively high GNP per capita are not shared equitably by most of the populations.

There are many factors other than average GNP per capita which affect birth rate. Hence we wouldn't expect all the points to lie on or close to a straight line. However, there is a general trend as shown by line AB (judged by eye to be the line of best fit) which is down to the right. Hence there appears to be some degree of negative correlation. There is a method known as the theory of least squares which enables statisticians to draw the "line of best fit" for problems such as this. This method is beyond the scope of this book.

Next, we will calculate Pearson's correlation coefficient to give us more information about the relationship between the variables.

First we calculate values for xy, x^2, and y^2 and tabulate them as shown below.

Country	x per capita GNP	y Birth rate	xy	x^2	y^2
Egypt	310	38	11,780	96,100	1,444
Mali	110	49	5,390	12,100	2,401
Nigeria	420	50	21,000	176,400	2,500
Somalia	110	48	5,280	12,100	2,304
Zambia	450	49	22,050	202,500	2,401
Zaire	130	46	5,980	16,900	2,116
Namibia	1,000	44	44,000	1,000,000	1,936
Israel	2,920	26	75,920	8,526,400	676
Saudi Arabia	4,980	49	244,020	24,800,400	2,401
India	150	34	5,100	22,500	1,156
Pakistan	190	44	8,360	36,100	1,936
Burma	140	39	5,460	19,600	1,521
China, People's Rep. of	410	20	8,200	168,100	400
Japan	5,640	15	84,600	31,809,600	225
United States	8,640	15	129,600	74,649,600	225
Mexico	1,110	41	45,510	1,232,100	1,681
Cuba	900	20	18,000	810,000	400
Haiti	230	42	9,660	52,900	1,764
Brazil	1,390	36	50,040	1,932,100	1,296
Argentina	1,730	22	38,060	2,992,900	484
Sweden	9,250	12	111,000	85,562,500	144
United Kingdom	4,430	12	53,160	19,624,900	144
France	7,290	14	102,060	53,144,100	196
Poland	3,150	19	59,850	9,922,500	361
Italy	3,450	13	44,850	11,902,500	169
USSR	3,010	18	54,180	9,060,100	324
Australia	7,340	16	117,440	53,875,600	256
Sums	68,880	831	1,380,550	391,660,600	30,861

Table 15.4

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x)^2 - (\Sigma x)^2}\ \sqrt{n(\Sigma y^2) - (\Sigma y)^2}}$$

$$r = \frac{27(1,380,550) - 68,880(831)}{\sqrt{27(391,660,600) - 68,880^2}\ \sqrt{27(30,861) - 831^2}}$$

$$r = \frac{37,274,850 - 57,239,280}{\sqrt{5,830,381,800}\ \sqrt{142,686}}$$

$$r = \frac{-19,964,430}{(76,356.9)(377.74)} = \frac{-19,964,430}{28,843,055} = \underline{-0.692}$$

Use of a calculator with statistical functions in conjunction with equation 15.2 would decrease the amount of calculations needed to get this result.

We next have to test the significance of the correlation coefficient. Is it sufficiently far from zero to indicate a correlation between the variables? To answer this question we have to compare our value of r with the critical value from table 15.4, on the next page.

We find the degrees of freedom from the equation, d.o.f. = n - 2, where n is the number of pairs of x, y values. The reason that we subtract two from n is that if we only had two points, they would have to fall on a straight line.

Our approach here is similar to that for the chi-square test. If the computed correlation coefficient is more than the critical value, we can reject a null hypothesis which says that the two variables are independent. If the correlation coefficient is less than the critical value, we cannot reject the null hypothesis and thus we cannot claim that there is a relationship between the variables.

Our computed r is -0.692. Using $\alpha = 0.01$, and d.o.f. = 27 - 2 = 25, the critical r value is 0.45.

Table 15.4

Critical Values of the Correlation Coefficient

d.o.f.	$\alpha = 5\%$	$\alpha = 1\%$
1	0.99	1.00
2	0.90	0.98
3	0.81	0.93
4	0.73	0.88
5	0.67	0.83
6	0.62	0.79
7	0.58	0.75
8	0.54	0.72
9	0.52	0.69
10	0.50	0.66
11	0.48	0.63
12	0.46	0.61
13	0.44	0.59
14	0.42	0.57
15	0.41	0.56
16	0.40	0.54
17	0.39	0.53
18	0.38	0.52
19	0.37	0.50
20	0.36	0.49
21	0.35	0.48
22	0.34	0.47
23	0.34	0.46
25	0.32	0.45
27	0.31	0.43

When comparing a computed r value which is negative, we ignore its sign. Thus we compare an r of 0.692 to the critical value, 0.45. Hence, r is greater than the critical value of r, and we can say that the value of r is significant and that there is a relationship between the two variables; the birth rate does depend on the average GNP per capita.

Note two further things about table 15.4.

1) Critical values of r for $\alpha = 1\%$ are greater than for $\alpha = 5\%$. This is similar to the case of critical chi-square values in table 14.6. It is reasonable because when $\alpha = 1\%$ we can state that there is a correlation between variables with a 99% certainty, as compared to 95% certainty when $\alpha = 5\%$.

2) As the number of points increases, the critical values of r decrease. This makes sense because the more points we have the more deviations from a linear relationship we would expect.

Exercises

The use of a hand calculator will be very helpful in many of the following problems.

In exercises 1, 2, and 3, draw a scattergram and compute Pearson's correlation coefficient, r. What conclusions can you make with regard to correlation?

1.

x	y
0	9
1	8
2	6
3	4
4	3
5	2
6	2
7	0

2.

x	y
1	9
3	3
5	7
6	8
7	2

3.

x	y
0	3
2	5
3	6
5	8
5	9
6	11

4. Using the data in table 15.5, draw a scattergram and compute Pearson's correlation coefficient r for rank of per capita military expenditures vs. rank of per capita public education expenditures. Test for the significance of r.

5. Repeat exercise 4 for rank of per capita military expenditures vs. rank of per capita public health expenditures.

6. Repeat exercise 4 for rank in literacy vs. rank in life expectancy.

Performance of Major Military Powers in Education and Health, 1975

In 1975, thirty-four countries had military expenditures of $1 billion or more. Israel—with per capita military expenditures $1,166—ranked second highest in such expenditures (after Qatar with $1,260), although many other countries had per ca military expenditures that exceeded per capita public expenditures for education and health. Total U.S. military expenditures 1975 amounted to $90.9 billion (ranking it second largest military spender after the U.S.S.R.). U.S. per capita military expenditu of $425 exceeded per capita public expenditures on education ($415) and on health ($218).

Countries by Rank in Total Military Expenditures[1]	Total Military Expenditures	Per Capita Military Expenditures		Per Capita Public Education Expenditures		Rank in Literacy[3]	Per Capita Public Health Expenditures		Rank in Life Expectanc
	($ millions)	($)	(rank)[2]	($)	(rank)[2]		($)	(rank)[2]	
U.S.S.R.	115,000	453	4	181	25	1	70	27	29
United States	90,900	425	5	415	7	1	218	13	7
China, People's Rep.	32,700	35	53	11	97	69	3	97	56
Germany, Fed. Rep.	15,300	247	10	308	15	1	424	3	18
France	13,100	248	9	308	15	1	340	6	[illegible]
United Kingdom	11,500	205	16	271	18	16	193	15	10
Iran	7,770	223	13	54	46	85	24	46	77
Poland	5,010	147	23	92	35	16	75	26	29
Italy	4,750	85	33	134	29	30	137	18	10
Japan	4,670	42	49	250	21	1	173	16	3
Israel	4,010	1,166	2	189	24	37	101	24	10
Germany, Dem. Rep.	3,950	234	11	168	26	1	110	23	18
Czechoslovakia	3,200	216	14	136	28	1	133	19	29
Canada	3,070	135	24	506	4	26	413	4	7
Spain	3,010	85	33	50	48	32	83	25	10
Netherlands	2,887	211	15	494	5	16	336	7	1
India	2,857	5	110	4	121	86	1	118	88
Australia	2,430	180	20	336	13	16	293	11	10
Brazil	2,380	22	64	35	61	62	9	68	58
Romania	2,240	106	29	59	44	16	40	41	35
Sweden	2,220	271	8	625	2	1	582	1	1
Saudi Arabia	1,940	311	7	512	3	114	61	31	104
Belgium	1,900	194	18	430	6	16	301	10	18
Bulgaria	1,700	195	17	93	34	26	49	35	18
Yugoslavia	1,650	77	37	77	38	40	11	63	42
Turkey	1,640	41	51	23	72	72	6	79	[illegible]
South Africa	1,450	58	40	22	73	88	5	85	[illegible]
Hungary	1,420	135	24	95	33	16	63	30	29
Iraq	1,390	125	27	33	64	94	6	79	75
Greece	1,360	151	22	43	54	45	47	37	18
Nigeria	1,170	19	67	17	86	96	1	118	123
Indonesia	1,110	8	89	6	116	69	1	118	93
Switzerland	1,090	168	21	414	8	1	318	8	7
Egypt	1,060	29	59	18	82	80	7	76	84

[1] All countries with military expenditures of $1 billion or more.
[2] Rank according to per capita expenditures among 140 countries.
[3] Rank among 140 countries.

NOTES: The rank order number is repeated if more than one country has the same level of expenditure. In 1975, world military expenditures of $370.8 billio were slightly more than the total amount of $322.6 billion spent by governments on education and more than one and a half times the $207.5 billion spent by governments on medical care and other health services. In the developing countries as a whole, more public revenue was devoted to military programs ($80.6 billion than to education and health care combined ($63.7 billion).

SOURCES: Adapted from Ruth Leger Sivard, *World Military and Social Expenditures, 1978*, WMSE Publications, Box 1003, Leesburg, Virginia 22075, pp. 15 an 24-29; military expenditures figures and rankings are based on U.S. Arms Control and Disarmament Agency, *World Military Expenditures and Arms Transfers, 1967 1976* (Washington, D.C.: U.S. Government Printing Office, 1978), Publication No. 98, Tables II and IV, pp. 33-69 and 76-113

Table 15.5

7. For the following exercises, consider the countries listed below and data from the 1979 World Population Data Sheet. The countries have been selected so that each area of the world would have approximately equal representation and so that most countries would be familiar to you so that you could see how they fit into the overall pattern.

Egypt
Mali
Nigeria
Somalia
Zambia
Zaire

Namibia
Israel
Saudi Arabia
India
Pakistan
Burma

China, People's Rep. of
Japan
United States
Mexico
Cuba

Haiti
Brazil
Argentina
Sweden
United Kingdom

France
Poland
Italy
USSR
Australia

Draw a scattergram and compute r for birth rate vs. infant mortality. Also test for the significance of r.

8. Repeat exercise 7 for doubling time vs. % under 15 years of age.

9. Repeat exercise 7 for doubling time vs. per capita GNP.

10. Repeat exercise 7 for infant mortality rate vs. doubling time.

11. Repeat exercise 7 for Physical Quality of Life Index vs. per capita GNP.

15.2 Prediction Based on Linear Correlation

When there is high linear correlation (r close to +1.0 or -1.0) between two variables, the correspondences can be represented by an ideal line of best fit. The formula which determines this line can be used to predict the value of the y variable which corresponds to any given value of the x variable.

The formula for the best fitting line is

$$\boxed{y_{\text{predicted}} = \mu_y + b(x - \mu_x)} \qquad \text{Eq. 15.3}$$

where μ_x is the arithmetic mean of all the x values; $\mu_x = \Sigma x/n$.

μ_y is the arithmetic mean of all the y values; $\mu_y = \Sigma y/n$.

b is the slope of the line of best fit.

We can find b from the formula

$$\boxed{b = \frac{n\Sigma(xy) - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}} \qquad \text{Eq. 15.4}$$

For the data of Example 15.3, we obtain the following (see table 15.4).

$\Sigma x = 68,880$

$\Sigma y = 831$

$\Sigma xy = 1,380,550$

$\Sigma x^2 = 391,660,600$

Hence,

$$\mu_x = \frac{\Sigma x}{n} = \frac{68,880}{27} = 2551.1$$

$$\mu_y = \frac{\Sigma y}{n} = \frac{831}{27} = 30.78$$

$$b = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b = \frac{27(1,380,550) - (68,880)(831)}{27(391,660,600) - (68,880)^2}$$

$$b = \frac{37,274,850 - 57,239,280}{10,574,836,000 - 4,744,454,400}$$

$$b = \frac{-19,964,430}{5,830,382,000} = -0.00342$$

Hence,

$$y_{predicted} = 30.78 - 0.00342(x - 2551.1)$$

This is the formula for the best fitting line.

Let us calculate a predicted value of y (birth rate) corresponding to a value of x (per capita GNP) of \$3000.

$$
\begin{aligned}
y_{\text{predicted}} &= 30.78 - 0.00342(3000 - 2551.1) \\
&= 30.78 - 0.00342(448.9) \\
&= 30.78 - 1.54 = \underline{29.24}
\end{aligned}
$$

For a value of x (per capita GNP) equal to \$200, we get

$$
\begin{aligned}
y_{\text{predicted}} &= 30.78 - 0.00342(200 - 2551.1) \\
&= 30.78 - 0.00342(-2351.1) \\
&= 30.78 + 8.04 = \underline{38.82}
\end{aligned}
$$

For a value of x (per capita GNP) equal to \$7500, we get

$$
\begin{aligned}
y_{\text{predicted}} &= 30.78 - 0.00342(7500 - 2551.1) \\
&= 30.78 - 0.00342(4948.9) \\
&= 30.78 - 16.93 = \underline{13.85}
\end{aligned}
$$

Exercises

The following exercises are related to the exercises after article 15.1.

1. For exercise 1, find the equation of the line of best fit. Find $y_{\text{predicted}}$ for $x = 5$.

2. For exercise 3, find the equation of the line of best fit. Find $y_{\text{predicted}}$ for $x = 4$.

3. For exercise 7, find the equation of the line of best fit. Compute the predicted doubling time corresponding to 35% of the population under 15 years of age.

4. For exercise 9, find the equation of the line of best fit. Calculate the predicted doubling time for a per capita GNP of \$1500.

5. For exercise 10, find the equation of the line of best fit. Find the infant mortality rate predicted for a doubling time of 27 years.

6. For exercise 11, find the equation of the line of best fit. Compute the PQLI corresponding to a per capita GNP of $1200.

Note:

1. Reprinted from Michael M. McLaughlin and the Staff of the Overseas Development Council, The United States and World Development: Agenda 1979. Published for the Council by Praeger Publishers (New York, 1979), pp. 228,9. Based on data from sources indicated below the table.

Chapter 16 Mathematics and Smoking

"The sum total of human disease, disability, death and lost productivity directly attributable to cigarette smoking is so staggering that a reduction in cigarette smoking may be the single most important health measure open to us for the foreseeable future. In the United States, one-third of all the deaths for men aged 35-59 would not have occurred if cigarette smokers had the same death rates as nonsmokers." This statement by Dr. Jerome H. Jaffe, Dept. of Psychiatry, Columbia University in the Sept. 20, 1974 issue of Science magazine indicates the importance of the smoking issue.

In its pamphlet, "Breathing...What You Need to Know," the National Tuberculosis and Respiratory Disease Association states, "New and much more effective ways are needed to counter the persuasive aura of sociability that cigarette smoking has acquired in our society. Here every teacher and school administrator can play an effective role." The mathematics problems in this chapter, many of which spotlight the dangers of cigarette smoking, can be used by mathematics educators to keep students away from the nicotine habit.

Today, much valuable land throughout the world which could be used to grow nutritious crops is being used to grow tobacco. The use of nicotine has been spreading in recent years to many people in underdeveloped countries.

The mathematics problems that follow cover many of the topics in this book. Hence, they may be used as a valuable term-ending review.

Basic Calculations

1. Assume that a person smokes two packs of cigarettes a day and that each pack costs 70 cents. How much money does this person spend on cigarettes per year?

2. A national health survey of June, 1967, gave the following breakdown of people who suffer from conditions that may be aggravated by cigarette smoke:

Condition	Number of people affected
chronic bronchitis	400,000
emphysema	726,000
chronic sinusitus	16,818,000
asthma or hay fever	16,099,000

How many people are estimated to have a condition that may be aggravated by cigarette smoke? (The report also stated that additional millions of people have some sensitivity to smoke.)

Percent

1. The yearly per capita consumption of manufactured cigarettes increased from 1,285 in the period 1925-29 to 3,985 in 1970 in the United States.[2] What was the percent increase?

2. The American Medical Association estimates that at least 34 million Americans have respiratory conditions which are made worse, often dangerously so, by tobacco fumes.[1] What percent of the American people does this represent?

3. The American Lung Association estimated several years ago that over 30 million adult Americans have stopped smoking.[3] Approximately what percent of Americans does this represent?

4. Between 1968 and 1974, the number of teen-age smokers increased from 3 million to 4.5 million.[4] What was the percent increase?

5. Over 37 million Americans will die years earlier because of smoking.[5] What percent of the American people does this represent?

6. More than 53 million Americans smoke cigarettes.[6] This represents about what percent of all Americans?

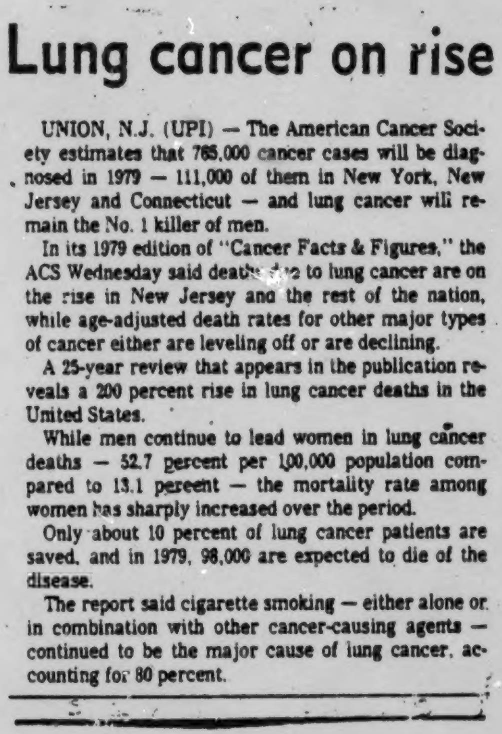
Ratio

1. Carbon monoxide in most industrial situations is limited to 50 parts per million parts of air.[7] Investigators have found the carbon monoxide level in a room full of smokers to reach 80 parts per million.[7] What is the ratio of carbon monoxide level in a smoke-filled room to that generally allowed in industry?

2. Long term exposure to levels of about 100 parts per million (ppm) of hydrogen cyanide is considered dangerous.[8] Concentration of this chemical in cigarette smoke is 1600 ppm.[8] What is the ratio of the level of hydorgen cyanide in cigarette smoke to that considered dangerous in long term exposures?

3. A Ralph Nader Health Research Group reported that the Federal Government spends almost $1,000 in subsidies to tobacco growers per each smoking-caused cancer death in the U. S. but only $14.30 per such death on-anti-smoking education. What is the ratio of money spent on tobacco subsidies to money spent on anti-smoking education?

General Problems



Lung cancer on rise

UNION, N.J. (UPI) — The American Cancer Society estimates that 765,000 cancer cases will be diagnosed in 1979 — 111,000 of them in New York, New Jersey and Connecticut — and lung cancer will remain the No. 1 killer of men.

In its 1979 edition of "Cancer Facts & Figures," the ACS Wednesday said deaths due to lung cancer are on the rise in New Jersey and the rest of the nation, while age-adjusted death rates for other major types of cancer either are leveling off or are declining.

A 25-year review that appears in the publication reveals a 200 percent rise in lung cancer deaths in the United States.

While men continue to lead women in lung cancer deaths — 52.7 percent per 100,000 population compared to 13.1 percent — the mortality rate among women has sharply increased over the period.

Only about 10 percent of lung cancer patients are saved, and in 1979, 98,000 are expected to die of the disease.

The report said cigarette smoking — either alone or in combination with other cancer-causing agents — continued to be the major cause of lung cancer, accounting for 80 percent.

1. For the article above, answer the following questions:

a) What percent of lung cancer cases are not related to smoking?

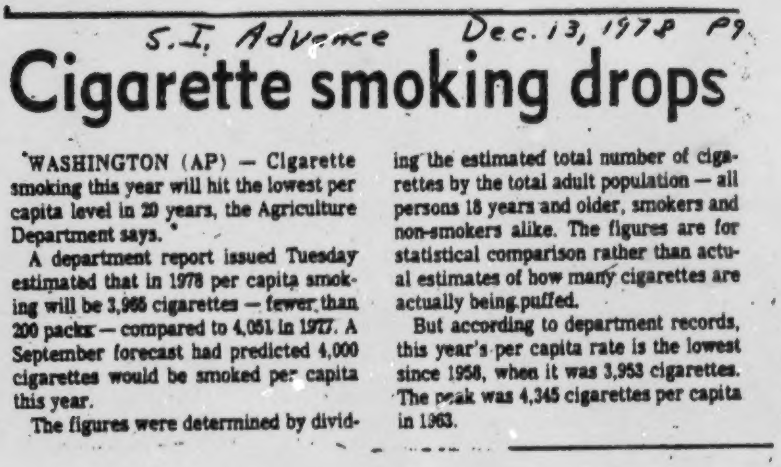
b) For every lung cancer death 25 years ago how many are there today?

c) Assuming 70 million adult males in the U. S., how many can be expected to die of lung cancer this year? Repeat for 70 million women.

2. For the article below,[10]

a) What was the decrease in cigarettes smoked per capita from 1963 to 1978?

b) What was the percent decrease in per capita smoking from 1977 to 1978?



S.I. Advance Dec. 13, 1978 p9

Cigarette smoking drops

WASHINGTON (AP) — Cigarette smoking this year will hit the lowest per capita level in 20 years, the Agriculture Department says.

A department report issued Tuesday estimated that in 1978 per capita smoking will be 3,965 cigarettes — fewer than 200 packs — compared to 4,051 in 1977. A September forecast had predicted 4,000 cigarettes would be smoked per capita this year.

The figures were determined by dividing the estimated total number of cigarettes by the total adult population — all persons 18 years and older, smokers and non-smokers alike. The figures are for statistical comparison rather than actual estimates of how many cigarettes are actually being puffed.

But according to department records, this year's per capita rate is the lowest since 1958, when it was 3,953 cigarettes. The peak was 4,345 cigarettes per capita in 1963.

3. Using the table below,[11]

a) How many years of life, on the average, will be lost by a 30 year old who smokes 15 cigarettes a day? Hint: compare the life expectancy of a 30 year old non-smoker to one who smokes 15 cigarettes a day.

b) Repeat for a 25 year old who smokes 25 cigarettes a day.

c) About how many minutes of life per cigarette will be lost by a 25 year old who smokes 15 cigarettes a day? (Hint: compute the total number of minutes of life lost and the total number of cigarettes smoked.)

..—Estimated years of life expectancy at various ages for males in the United States, by daily cigarette consumption

Age	Never smoked regularly	Number of cigarettes smoked per day			
		1–9	10–19	20–39	40 and over
25 years	48.6	44.0	43.1	42.4	40.3
30 years	43.9	39.3	38.4	37.8	35.8
35 years	39.2	34.7	33.8	33.2	31.3
40 years	34.5	30.2	29.3	28.7	26.9
45 years	30.0	25.9	25.0	24.4	23.0
50 years	25.6	21.8	21.0	20.5	19.3
55 years	21.4	17.9	17.4	17.0	16.0
60 years	17.6	14.5	14.1	13.7	13.2
65 years	14.1	11.3	11.2	11.0	10.7

Table 16.1

Circle Diagrams

1. Among the general public, 31.5% of the adult males never smoked cigarettes, 29.2% are former smokers, and 39.3% are currently smokers of cigarettes (survey conducted by the U. S. Public Health Service in 1975).[12] Draw a circle diagram to illustrate this data.

Graphs

1. Plot a graph using the following data for deaths per 100,000 population due to lung cancer.[13]

Year	1955	1960	1965	1967	1969	1974
Deaths	16.3	20.3	25.0	27.5	30.6	39.1

2. Plot a graph using the following data for deaths per 100,000 people due to emphysema.[14]

Year	1955	1960	1965	1967	1969
Deaths	2.2	5.2	9.6	10.6	11.4

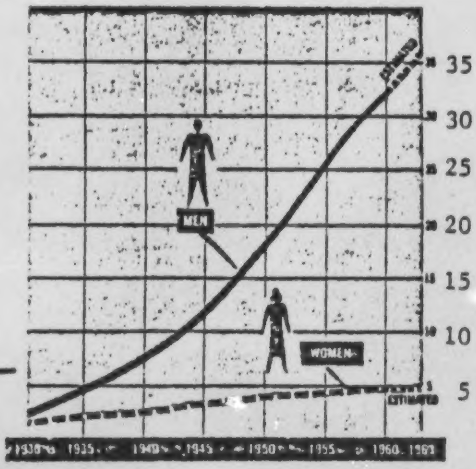
3. Using the graphs[15] shown below:

a) What were the death rates per 100,000 men due to lung cancer in 1930, 1935, 1940, 1945, 1950, 1955, and 1960?

b) Estimate the doubling time for lung cancer death rates for men.

c) Estimate (by extrapolation) the lung cancer death rates for men in 1970 and 1980.

c) Repeat parts a, b, and c for women.



Deaths attributed to lung cancer per 100,000 men and women in the U.S. from 1930 to 1963 (the figures for 1960 to 1963 are estimated). The figures are weighted to allow for the gradual aging of the population

Fig. 16.1

4. Using the graph shown below:

a) What were the death rates per 100,000 people due to cancer of the respiratory system in 1930, 1940, 1950, 1960, and 1970?

b) Estimate the doubling time for the death rate.

c) Estimate (by extrapolation) the death rates in 1980 and 2000.

CIGARETTES AND CANCER IN THE U.S.[16]
1900-1975

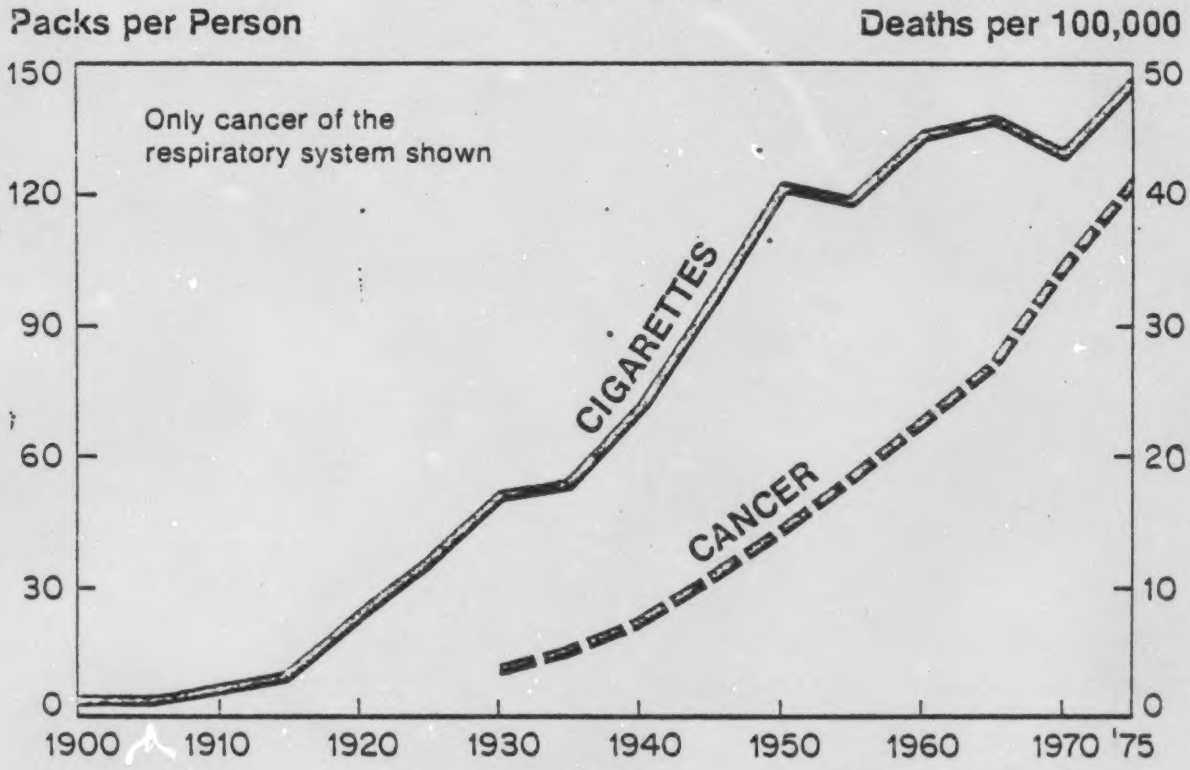


Fig. 16.2

Bar Charts

1. Draw a bar chart for the following information on work days lost per year due to cigarette smoking:[17]

Cigarettes smoked per day	none	under 11	11-20	21-40	over 40
Number of work days lost per year (days/person)	4.6	5.5	6.1	6.5	7.6

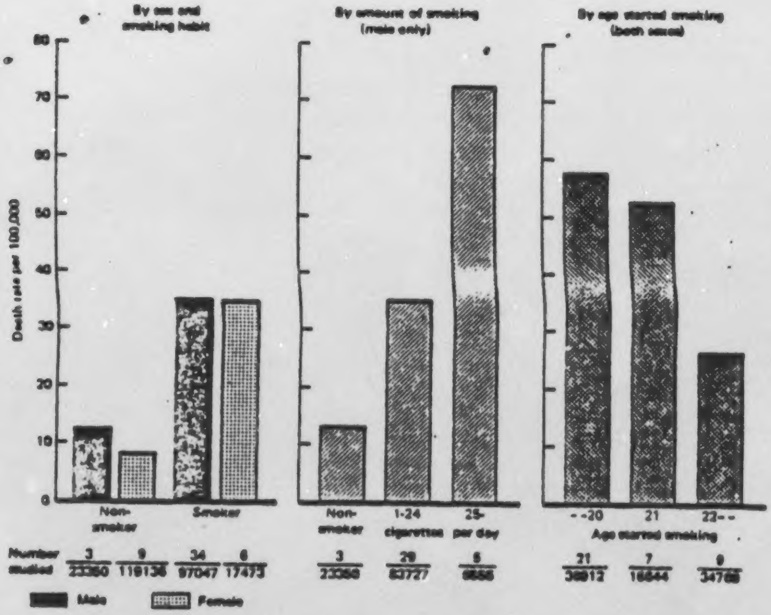
2. Use the bar charts shown below[18] to answer the following questions:

a) What is the ratio of deaths per 100,000 males for smokers compared to non-smokers?

b) Repeat for females.

c) What is the ratio of deaths per 100,000 for those who smoke 1 to 24 cigarettes per day compared to non-smokers?

d) Repeat for those who smoke 25 or more cigarettes per day.



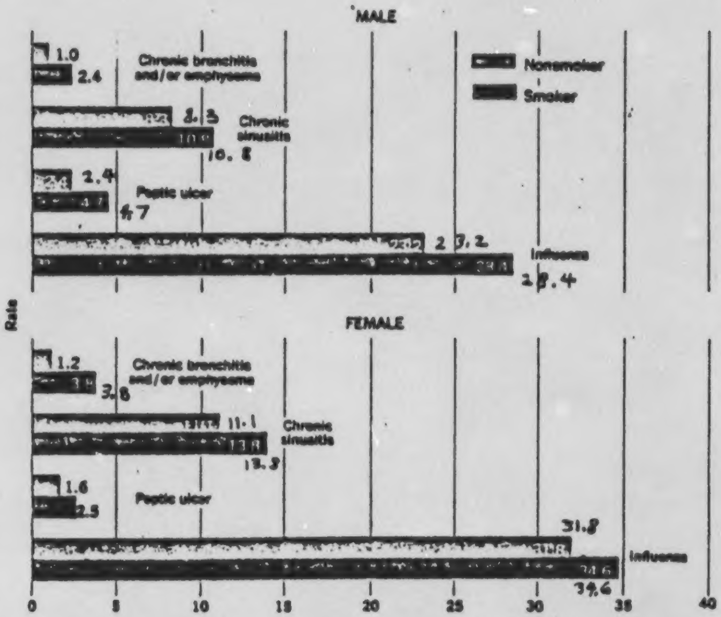
. —Death rates for lung cancer, among persons age over 40 years, classified by sex and extent of cigarette smoking, and by age smoking began: Study of 29 Health Center Districts in Japan, January 1966 to March 1967.

Source: Hirayama, T.

Fig. 16.3

3. Compute ratios for illnesses of smokers compared to non-smokers for each of the bar-charts shown below.

TOBACCO—ILLNESS AND DISABILITY



Illness from bronchitis, emphysema, sinusitis, peptic ulcer, and influenza: smokers versus nonsmokers, males and females, fiscal year 1965. (Rates per 100 persons per year.) (Source: U.S. Public Health Service Publication No. 1,000, May 1967.)

Fig. 16.4

Sampling

Here are some questions that may be used to generate data for mathematical problems involving such topics as percents, averages, variability, graphs, bar and circle charts, correlation, chi-square analysis, and hypothesis testing.

1. Do you smoke cigarettes?
2. About how many cigarettes per day?
3. How many of your parents smoke?
4. Do you feel that smoking should be permitted in classrooms?
5. Do you favor legislation limiting smoking in public places?

6. What percent of the students in your school do you think smoke cigarettes?

7. Do you feel that cigarette smoking is enough of a health hazard for government to do something about it?

8. Do you find it annoying to be near a person who is smoking a cigarette?

Averages and variability

Assume that a survey of smokers revealed the data shown below. Compute the mean number of cigarettes smoked by the smokers. Also compute the range and standard deviation.

Number of smokers	Cigarettes smoked
6	5
11	10
14	15
12	20
9	25
5	30
6	35
4	40
3	45

Normal Curve

Assume that a survey of smokers in one college indicated that the mean number of cigarettes smoked per day was 24, with a standard deviation of 8. Assume a normal distribution. What is the probability that a smoker chosen at random smokes:

a) more than 32 cigarettes

b) between 16 and 40 cigarettes

c) between 32 and 40 cigarettes

Logarithms and Geometric Sequences

1. Assume that a pack of cigarettes costs 70 cents in 1979 and the price increases by 5% per year.

   a) How long would it take for the price to double?

   b) What would be the price for a pack of cigarettes in the year 2000?

Chi-Square analysis

1. The following information is based on the American Cancer Society Publication, "Cancer Facts and Figures," 1965.

		both parents smoke	one parent smokes	neither parent smokes
boys	smokers	44	37	29
	non-smokers	56	63	71

		both parents smoke	one parent smokes	neither parent smokes
girls	smokers	37	29	16
	non-smokers	63	71	81

Use the chi-square analysis to determine if there is a dependency between the smoking of (a) boys and (b) girls and the smoking or non-smoking of parents. Use $\alpha = 0.01$.

The following "smoking quiz" can be used as a pre-test, a post-test, the basis of a class discussion, or as a motivation device for a talk to some group on the issue of smoking.

SMOKING QUIZ

The following questions are based on information from various pamphlets of the American Lung Association and the American Cancer Society.

1. Each year the number of Americans that die prematurely from the effects of smoking is

   a. 25,000 b. 100,000 c. 300,000 d. 500,000

2. An inbalance of air flow and blood flow in a smoker's body can be measured immediately after one cigarette. T or F

3. Smoking is the most important cause of chronic bronchitis and also of lung cancer. T or F

4. The risk of death from lung cancer is ________ times greater for smokers than for non-smokers.

   a. 2 b. 5 c. 10 d. 20

5. The chances of being cured of lung cancer are one in

   a. 2 b. 5 c. 10 d. 20

6. The number of Americans that die annually of lung cancer is about

   a. 10,000 b. 50,000 c. 100,000 d. 150,000

7. The chances of dying from chronic bronchitis or emphysema are about ___________ times greater for smokers than for non-smokers.

   a. 3 b. 6 c. 9 d. 12

8. A 25 year old man who never smoked regularly can expect to live, on the average, _______________ years longer than a man who smokes a pack a day.

   a. 2 b. 4 c. 6 d. 8

9. On the average, every cigarette that a smoker smokes cost him or her____________minutes of life

   a. 2 b. 5 c. 10 d. 20

10. A pregnant woman who smokes two packs a day blocks off the equivalent of ___________ per cent of the oxygen supply of the fetus.

    a. 10 b. 20 c. 30 d. 40

11. About ______________ Americans are very sensitive to tobacco smoke and suffer smoke-caused asthma attacks.

a. 500,000 b. 1,000,000 c. 2,000,000 d. 4,000,000

12. Smoke from an idling cigarette contains even more tar and nicotine than an inhaled one. T or F.

13. Carbon monoxide levels from cigarette smoke can exceed the Federal Air Quality Standards. T or F.

14. Inhaling second-hand smoke makes the heart beat faster, the blood pressure rise, and the level of carbon monoxide in the blood increase. T or F

15. After _______ years without smoking, the death rates of ex-smokers approach that of non-smokers.

a. 2 b. 5 c. 10 d. 20

16. The groups that show the largest increase in the number of smokers in recent years are _______ and _________.

17. About _________ teenagers take up smoking each year

a. 250,000 b. 500,000 c. 1,000,000 d. 2,000,000

18. There are now more than ____________ million Americans who are ex-smokers

a. 10 b. 20 c. 30 d. 40

19. Among adults the per cent of smokers is about

a. 25 b. 35 c. 45 d. 55

20. The smoke from cigars and pipes is more offensive to nonsmokers than cigarette smoke. T or F

R9: AMC

1. (c) 2. (T) 3. (T) 4. (c) 5. (d) 6. (c) 7. (b) 8. (c)
9. (b or c)* 10. (d) 11. (c) 12. (T) 13. (T) 14. (T)
15. (c) 16. (Teens, women) 17. (c) 18. (c) 19. (b) 20. (T)

* depends on conditions

Notes

1. Quoted in testimony by John F. Benzhaf III on behalf of ASH (Action on Smoking and Health) before a Senate Committee on transportation (studying the effects of cigarette smoke on airline passengers), Vol. 115, No. 213, Dept. 20, 1969.

2. Chart Book on Smoking Tobacco and Health, U. S. Department of Health, Education, and Welfare, Public Health Service, National Clearinghouse for Smoking and Health, Rockville, Maryland 20852.

3. "Cigarette Smoking, The Facts About Your Lungs," a pamphlet of the American Lung Association.

4. Dept. of Health, Education, and Welfare, quoted in Dear Abby column, "Sorry Statistics" in the S. I. Advance.

5. Ibid

6. Ibid

7. "Tobacco Smoke Emissions Fact Sheet," American Cancer Society.

8. Ibid

9. S. I. Advance, Dec. 1978 (UPI)

10. Ibid, Dec. 13, 1978, p. 9 (AP)

11. Hammond, E. C. "Life Expectancy of American Men in relation to their smoking habits," presented at the World Conference on Smoking and Health, New York City, chart reprinted in Chart Book on Smoking, op cit., p. 12.

12. U. S. Public Health Service Survey, reported in ASH (Action on Smoking and Health) Newsletter, Vol. VI, number 5, Sept.-Oct., 1976, p. 2.

13. National Center for Health Statistics

14. Ibid

15. The Consumers Union Report on Smoking and the Public Interest, Consumers Union, Mount Vernon, N. Y., p. 19.

16. "Mortality American Style: A Tale of Two States," a teaching module of the Population Reference Bureau, 1337 Connecticut Ave., N. W., Washington, D. C., Data Sheet 9.

17. Chart Book on Smoking, op. cit., p. 19.

18. Hirayam, T., "Smoking in Relation ot the death rates of 265,118 men and women in Japan." Tokyo, National Cancer Center Research Institute, Epidemiology Division, Sept. 1967.

Chapter 17 Mathematics and The Arms Race

The current arms race is the most critical issue facing mankind today. The sheer survival of the human race is at stake.

"Mankind must put an end to war or war will put an end to mankind."

John F. Kennedy

"There was a time when only G-d could end the world."

World Federalist Association flyer

As more and more nations obtain modern sophisticated weapons and as the power and accuracy of the weapons increase, the security of all the world's people is decreased.

"The unrestrained competition among nations to build more arms, to sell more weapons, to deploy more forces is senseless, wasteful, but, above all, dangerous to our security."

Vice President Walter Mondale, 1976

"The belief that it is possible to achieve security through armaments on a national scale is, in the present state of military technology, a disastrous illusion."

Albert Einstein

The world's expenditures on armaments is scandalous, especially at a time when so many human needs are unmet and there are so many critical problems facing humanity.

"Every gun that is made, every warship launched, every rocket fired signifies, in the final sense, a theft from those who hunger and are not fed, those who are cold and are not clothed."

Dwight D. Eisenhower

"It always comes down to a question of priorities: A new generation of fighters for the air force or a new generation of infants who will live beyond their fifth birthday.

Former U.S. Secretary of Defense, Robert S. McNamara, 1977

The following mathematical problems reinforce and quantify the above statements. As they require the use of many of the techniques covered in this book, they can serve as an effective review.

Basic Computations

1. The 1978 U. S. defense budget of about \$126 billion[1] is approximately equal to the total income of the poorest billion people on earth.[2] What is their average per capita income?

2. The 1978 world defense budget of about \$400 billion[3] is approximately equal to the total income of the poorest two billion people on earth[4] (nearly half of the world's population). What is their average per capita income?

3. In 1974 the U. S. military spent more in 16 hours than the World Health Organization and the Food and Agriculture Organization spent in the whole year.[5] About how many times that year did the U. S. military spend an amount equal to the budget of these two organizations?

4. If the world's 1978 defense budget of \$400 billion[3] was counted at the rate of \$10 per second, how many years would it take to count it? If it were stacked up in a single column of silver dollars (each about one tenth of an inch thick), how high would it reach?

Ratios

1. According to Bread For The World, a Christian ecumenical group devoted to affecting public policy on hunger issues, the U. S. 1976 budget asked every man, woman, and child in this country to pay the federal government, on average, about:[6]

\$450 for military costs

\$35 for education

\$18 for housing

\$37 for natural resources and the environment

\$6 for economic assistance to hungry nations.

Find the ratio of military costs to each of the other expenditures.

2. In 1978 the world spent about $135 million for international peacekeeping forces and about $400 billion for military uses.[7] Find the ratio of total military spending to spending for international peacekeeping forces.

3. Since 1960, donor nations have spent a yearly average of $5 per capita to help the development of poorer nations, and $95 per capita for their own military forces.[8] Find the ratio of military expenditures to development aid money.

4. Using the chart[9] on the next page find the ratio of per capita military expenditures to per capita public education expenditures and to per capita public health expenditures for a) U.S.S.R.; b) United States; c) Germany (FRG); d) Israel; e) Japan; f) Canada; g) India; h) Netherlands; i) Sweden

5. William Winpisinger, internation president of the International Association of Machinists and Aerospace Workers, has stated that $1 billion spent on capital intensive defense industries would create 45,800 jobs, but the same amount of money spent on public service employment would create 98,000 jobs.[10] Find the ratio of public service to defense jobs for a given amount of money.

6. The blockbuster bomb of World War II had a power of 10 tons of TNT.[11] The atomic bomb that fell on Hiroshima had 13,000 tons of TNT.[11] A nuclear warhead today has a power of up to 25,000,000 tons of TNT.[11] Find the ratio of the explosive force of today's largest nuclear warhead to a) the Hiroshima atomic bomb, and b) the blockbuster bomb of World War II.

Percents

1. In 1973 the Pakistani government allocated $405 million for defense from a total national budget of $608 million.[12] What percent of the budget did the Pakistani government allocate for defense?

2. U. S. arms sales jumped from $1 billion in 1970 to over $8 billion in 1974.[13] Compute the approximate percent increase in arms sales.

Performance of Major Military Powers in Education and Health, 1975

In 1975, thirty-four countries had military expenditures of $1 billion or more. Israel—with per capita military expenditures of $1,166—ranked second highest in such expenditures (after Qatar with $1,260), although many other countries had per capita military expenditures that exceeded per capita public expenditures for education and health. Total U.S. military expenditures in 1975 amounted to $90.9 billion (ranking it second largest military spender after the U.S.S.R.). U.S. per capita military expenditures of $425 exceeded per capita public expenditures on education ($415) and on health ($218).

Countries by Rank in Total Military Expenditures[1]	Total Military Expenditures	Per Capita Military Expenditures		Per Capita Public Education Expenditures		Rank in Literacy[3]	Per Capita Public Health Expenditures		Rank in Life Expectancy[3]
	($ millions)	*($)*	*(rank)*[2]	*($)*	*(rank)*[2]		*($)*	*(rank)*[2]	
U.S.S.R.	115,000	453	4	181	25	1	70	27	29
United States	90,900	425	5	415	7	1	218	13	7
China, People's Rep.	32,700	35	53	11	97	69	3	97	56
Germany, Fed. Rep.	15,300	247	10	308	15	1	424	3	18
France	13,100	248	9	308	15	1	340	6	10
United Kingdom	11,500	205	16	271	18	16	193	15	10
Iran	7,770	223	13	54	46	85	24	46	77
Poland	5,010	147	23	92	35	16	75	26	29
Italy	4,750	85	33	134	29	30	137	18	10
Japan	4,670	42	49	250	21	1	173	16	3
Israel	4,010	1,166	2	189	24	37	101	24	10
Germany, Dem. Rep.	3,950	234	11	168	26	1	110	23	18
Czechoslovakia	3,200	216	14	136	28	1	133	19	29
Canada	3,070	135	24	506	4	26	413	4	7
Spain	3,010	85	33	50	48	32	83	25	10
Netherlands	2,887	211	15	494	5	16	336	7	1
India	2,857	5	110	4	121	86	1	118	88
Australia	2,430	180	20	336	13	16	293	11	10
Brazil	2,380	22	64	35	61	62	9	68	58
Romania	2,240	106	29	59	44	16	40	41	35
Sweden	2,220	271	8	625	2	1	582	1	1
Saudi Arabia	1,940	311	7	512	3	114	61	31	104
Belgium	1,900	194	18	430	6	16	301	10	18
Bulgaria	1,700	195	17	93	34	26	49	35	18
Yugoslavia	1,650	77	37	77	38	40	11	63	42
Turkey	1,640	41	51	23	72	72	6	79	70
South Africa	1,450	58	40	22	73	88	5	85	84
Hungary	1,420	135	24	95	33	16	63	30	29
Iraq	1,390	125	27	33	64	94	6	79	75
Greece	1,360	151	22	43	54	45	47	37	18
Nigeria	1,170	19	67	17	86	96	1	118	123
Indonesia	1,110	8	89	6	116	69	1	118	93
Switzerland	1,090	168	21	414	8	1	318	8	7
Egypt	1,060	29	59	18	82	80	7	76	84

[1] All countries with military expenditures of $1 billion or more.
[2] Rank according to per capita expenditures among 140 countries.
[3] Rank among 140 countries.

NOTES: The rank order number is repeated if more than one country has the same level of expenditure. In 1975, world military expenditures of $370.8 billion were slightly more than the total amount of $322.6 billion spent by governments on education and more than one and a half times the $207.5 billion spent by governments on medical care and other health services. In the developing countries as a whole, more public revenue was devoted to military programs ($80.6 billion) than to education and health care combined ($63.7 billion).

SOURCES: Adapted from Ruth Leger Sivard, *World Military and Social Expenditures, 1978*, WMSE Publications, Box 1003, Leesburg, Virginia 22075, pp. 15 and 24-29; military expenditures figures and rankings are based on U.S. Arms Control and Disarmament Agency, *World Military Expenditures and Arms Transfers, 1967-1976* (Washington, D.C.: U.S. Government Printing Office, 1978), Publication No. 98, Tables II and IV, pp. 33-69 and 76-113.

3. World military spending went from $200 billion in 1970 to the current (1978) $400 billion.[14] What percent increase does this represent?

4. Strategic nuclear stockpiles of the U. S. and U.S.S.R. rose from 6000 in 1970 to about 14,000 in 1977.[15] What was the percent increase during this period?

5. In 1977 the U. S. and U.S.S.R. combined spent about $200 billion of the global military total of $400 billion.[16] What percent of the total was spent by the U. S. and U.S.S.R.?

6. If 5 percent of the world's $400 billion[3] military budget was converted into alternate programs to meet human needs, how much money would this represent? Could this amount of money have a significant impact?

7. Many military analysts contend that $30 billion could be cut from the $126 billion US defense budget without risk to United States security.[17] What percent saving does this represent?

Sequences

1. Total world military expenditures rose from $107 billion in 1960 to $200 billion in 1970 to $400 billion in 1978.[18] What is the approximate doubling time? By extrapolation, roughly when would total world military expenditures reach $800 billion, if the average increase per year stayed about the same?

Graphs

1. For the four graphs shown below,[19]

   a) estimate the value of each variable in 1970;

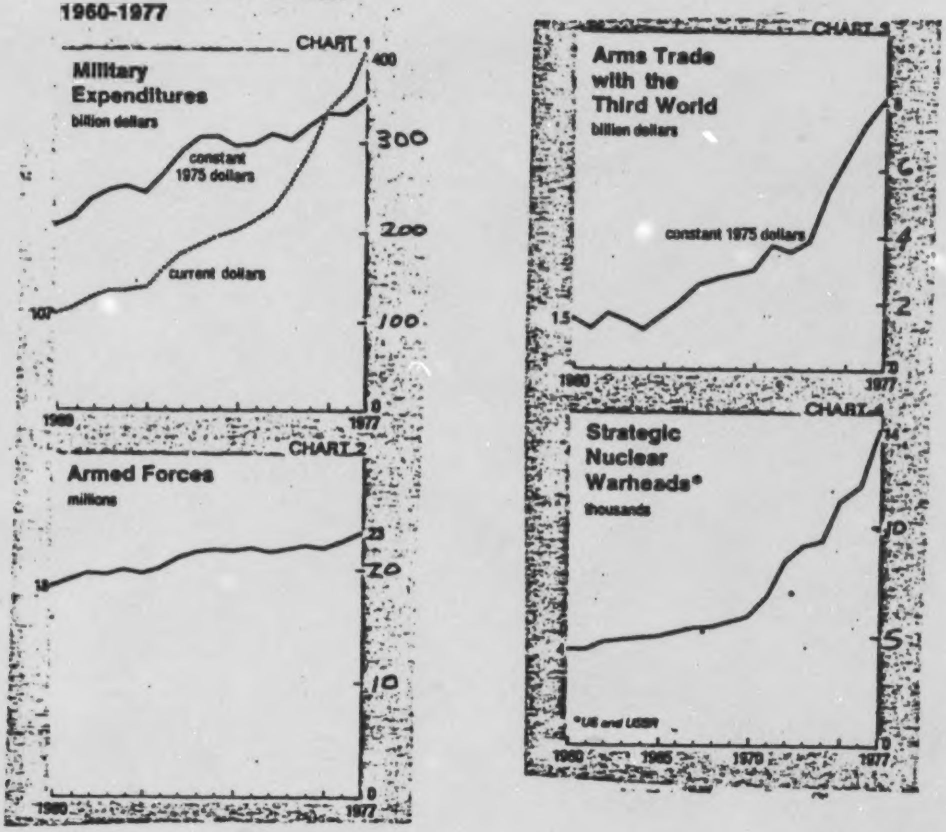
   b) extrapolate to predict the value of each variable in 1980;

   c) estimate the doubling times for

      1) arms trade with the 3rd world, in constant 1975 dollars;

      2) number of strategic nuclear weapons.

GROWTH OF WORLD MILITARY POWER 1960-1977



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Circle Charts

1. Use the circle charts shown below[20] to answer the following questions:

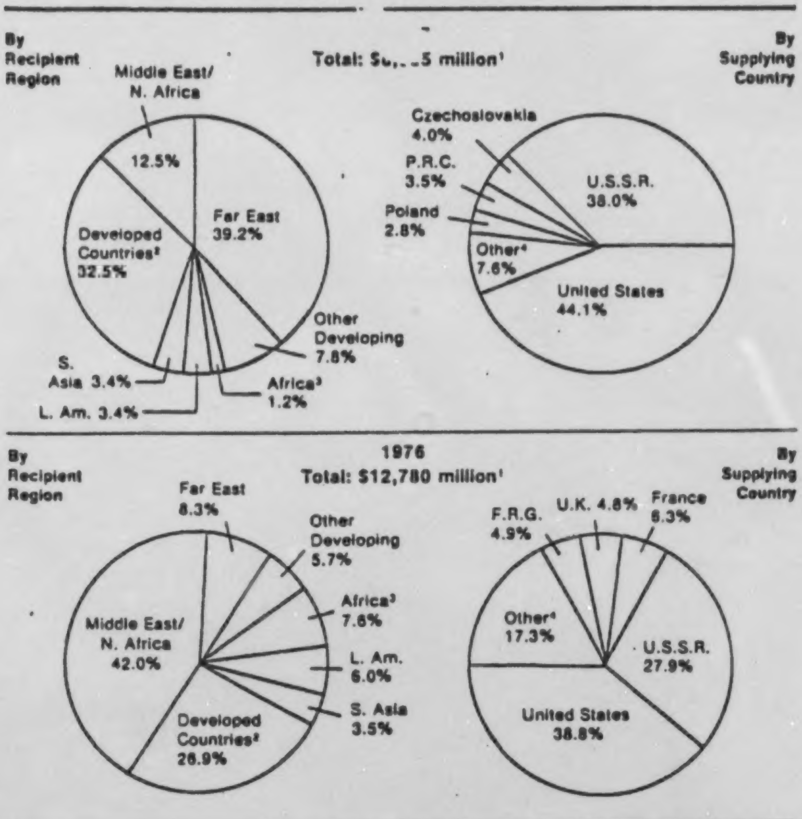
a) What was the cash value of arms supplied by the United States to other countries in 1967 and in 1976?

b) Repeat part a for the U.S.S.R.

c) What was the increase in U. S. arms sales (in $) from 1967 to 1976?

d) How many degrees on the circle correspond to the U.S.S.R. in 1976?

Supplying Countries, 1967 and 1976
($ millions and percentages)



[1] All data in constant 1975 dollars.

[2] Those countries identified as "developed" in Table A-4 of this volume.

[3] Black Africa (including South Africa, which accounted for 0.3 per cent in 1967 and 1.3 per cent in 1976).

[4] The major "other" suppliers in 1967 were the United Kingdom, France, Canada, the Federal Republic of Germany, and Italy; in 1976, they were Czechoslovakia, Italy, Switzerland, Poland, and Belgium.

NOTES: "Arms transfers" indicates actual *deliveries* (through either sales or aid), as distinguished from sales orders, contracts, or agreements, or the value of aid programs that may result in a future transfer of goods. The data include deliveries of military equipment only; they exclude the value of training, technical and other services, and consumables.

World arms transfers, which in constant dollars amounted to $8,005 million in 1967 and $12,780 million in 1976, in current dollars equalled $5,055 million in 1967 and $13,370 million in 1976.

SOURCE: Based on data from U.S. Arms Control and Disarmament Agency, *World Military Expenditures and Arms Transfers, 1967-1976* (Washington, D.C.: U.S. Government Printing Office, 1978), Publication No. 98, Tables V and VI, pp. 115-156.

2. According to the National Science Foundation, the 1976 U. S. Federal Research and Development budget of $21.7 billion was divided as follows:[21]

Military and Space	$13.4 billion
Energy	2.8 billion
Health, Education & Welfare	2.6 billion
All other	2.9 billion

Draw a circle diagram.

3. In his book, Toward a Dependable Peace,[22] Robert Johansen, President of the Institute for World Order, proposes that the U. S. cut military spending by 10% for the next 3 years. He suggests that the money saved be used as shown below. Draw a circle diagram showing his proposed distribution of expenditures from the savings.

Item	Percent
Rebate to taxpayers	35
Economic Conversion and Job retraining	15
Global fund for economic development	25
Global fund for environmental protection	10
Operation of a world security system	15

4. According to SANE (A Citizen's Organization For a Sane World), the Fiscal Year 1979 budget proposed by the administration has the following breakdown[23] (This considers only money controllable by the President and Congress):

Military and past wars	53%
Human resources	24%
Physical resources	15%
All other costs	8%

Draw a circle diagram.

Bar Charts

1. Draw a bar chart using the data below which represents cumulative 1970-76 arms exports for major arms exporters, in billions of dollars.[24]

Country	Arms exports
United States	25.5
USSR	12.2
France	2.4
United Kingdom	2.1
West Germany	1.6
Italy	0.7
China	0.4
All others	3.2

2. The amount of jobs that could be created by $1 billion of spending in various fields is shown below:[25]

Nurses	85,000	Firefighters	70,000
Teachers	76,000	Military personnel	58,000
Police	73,000	Military industry	45,000

Draw a bar chart using this data.

Correlation

1. Refer to the chart on page 17-4. Draw a scattergram for the first 12 countries on the list for per capita military expenditures vs. per capita public education expenditures. Compute Pearson's correlation coefficient .

2. Repeat for per capita military expenditures versus per capita public health expenditures.

Logarithms

1. World arms expenditures increased from \$107 billion in 1960 to \$400 billion in 1977. Compute the average rate of increase during this period. If that average rate continued, what would world arms expenditures be by 1990?

Chi Square Analysis

1. Complete the table below using values for countries on the chart on page 17-4.

		per capita military expenditures	
		$< \$125$	$\geq \$125$
per capita public education expenditures	$< \$150$		
	$\geq \$150$		

Is there a relationship between per capita military expenditures and per capita public education expenditures. Use $\alpha = 0.05$.

2. Repeat problem 1 but substitute per capita public health expenditures for per capita public education expenditures. Use $\alpha = 0.05$.

Notes

1. Disarmament Action Guide, 1979, a pamphlet of the Coalition for a new Foreign and Military Policy, 120 Maryland Avenue, N. E., Washington, D. C. 20002, 1979, p. 10.

2. Arthur, Simon, Bread For the World, New York, Paulist Press, 1975, p. 123.

3. Ruth Leger Sivard, World Military and Social Expenditures, 1978, WMSE Publications, Box 1003, Leesburg, Virginia 22075, p. 5.

4. Ibid, p. 6, and Simon, Bread, op. cit., p. 123

5. Simon, Bread, op. cit., p. 123

6. Ibid, p. 124

7. Sivard, World, op. cit., p. 5

8. Ibid

9. Martin M. McLaughlin and the Staff of the Overseas Development Council, The United States and World Development, Agenda 1979, New York, Praeger, 1979, pp. 228,229.

10. William Winpisinger, "Defense boondoggles tolerated too long," op-ed article in the S. I. Advance, Nov. 17, 1978.

11. Sivard, World, op. cit. (1977 edition), p. 9.

12. Simon, Bread, op. cit., p. 128.

13. Ibid

14. Sivard, World, op. cit., p. 6.

15. Ibid

16. Ibid, p. 7

17. Norman Cousins, op. ed. article in the S. I. Advance.

18. Sivard, World, op. cit., p. 6.

19. Ibid

20. McLaughlin, The United States, op. cit., p. 233.

21. Data shown on a circle diagram in "More Jobs: Converting To a Peaceful, Productive, Economy," a pamphlet of The Coalition For a New Foreign and Military Policy.

22. Robert C. Jo$\overset{a}{\text{h}}$nsen, Toward a Dependable Peace, Institute for World Order, 777 United Nations Plaza, New York, N. Y. 10017, 1978, p. 40.

23. Information compiled by the Center for Defense Information, Washington, D. C.

24. Sivard, World, op. cit. p. 9.

25. Anderson, The Empty Porkbarrel, 1978, Figures shown on a bar chart in pamphlet, "Your Taxes - Your Choice," a pamphlet of the Coalition for a New Foreign and Military Policy.

Chapter 18 Mathematics and Staten Island

Most of the problems in this book are concerned with issues of national and global significance. This chapter has a variety of problems related to a local area, Staten Island, a borough of New York City. By using Chamber of Commerce sutdies, local news reports, etc., similar problems can be found for other local communities.

Staten Island, on a smaller scale, is faced with many problems like those faced by the United States and the world:

Rapid population growth, especially after the opening in 1964 of the Verrazanno Narrows Bridge which connects Staten Island to Brooklyn, has led to strain on transportation, medical, and educational facilities.

Staten Island suffers from various forms of pollution. Air pollution blown in on westerly winds from petro-chemical plants in New Jersey contributes to some areas of the borough having the highest respiratory cancer rate in New York City. Although surrounded by water, there are very few beaches considered clean enough for swimming.

Although Staten Islanders have a median income higher than any other borough, there are several poverty areas with housing and crime problems.

The largest landfill in the world is at Fresh Kills in Staten Island. About 30% of the city's garbage is barged there from other parts of New York City.

Staten Island has become increasingly urbanized. Hardly any farms remain and many wooded areas have been destroyed for housing and highways. This plus very rapid development with inadequate sewerage has led to many severe flooding problems.

As in the previous two chapters, the following problems may be used for a review of many of the topics covered in this book.

Basic Computations

1. Staten Island has 6,123 acres of parks in a total acreage of 38,947 acres.[1] What percent of Staten Island is park land?

2. The Franklin D. Roosevelt Boardwalk in South Beach, Staten Island is 7,500 feet long, the 4th largest boardwalk in the world.[2] If 1 mile = 5,280 feet, express the length of the boardwalk in miles

Percents and ratios

1. The annual average age-adjusted respiratory cancer deaths (per 100,000 population) is given below for the 5 boroughs for the 1974-76 period.[3]

Staten Island	38.7	Brooklyn	30.0
Manhattan	30.0	Queens	28.1
Bronx	27.6		

a) Compute the percents by which the Staten Island respiratory cancer deaths exceed those of the other boroughs.

b) Compute the ratios of the Staten Island respiratory cancer deaths to those of each of the other boroughs.

2. In 1967, it was found that men over 45 living in the northern part of Staten Island (near the industrial areas of New Jersey) experienced a higher rate of death from respiratory cancer (55/100,000) than that of men living in the southern part where the rate was 40 per 100,000 deaths.[4] For such men, compute

a) The percent by which respiratory deaths in the northern part of S. I. exceeded those in the southern part.

b) The ratio of respiratory deaths in the northern part of S. I. to those in the southern part.

3. In 1970, of 110,029 Staten Island workers, 57,610 went to work by car.[5] What percent of Staten Islanders got to work by car?

4. In 1970, Staten Island's total population of 295,443 included 277,604 whites.[6] How many non-whites were there? What is the ratio of white people to non-white people?

Graphs

1. Data for population growth on Staten Island are given below:[7]

year	population	year	population
1860	25,492	1930	158,346
1870	33,029	1940	174,441
1880	38,991	1950	191,551
1890	51,693	1960	221,991
1900	67,021	1970	295,443
1910	85,961	1978	345,325
1920	112,000		

a) Draw a graph showing the growth of population on Staten Island.

b) Extrapolate to get estimated populations in 1980 and 1990.

c) Estimate the 1975 population by interpolation.

d) Compute the percent increase in population from 1960 to 1970. Why was the increase so large?

e) Compute the ratio of the 1978 S. I. population to that in 1860.

2. The Port Authority of New York and New Jersey, in May 1974, made the following estimates of future populations of Staten Island.[8] Plot a graph.

Year	Population (thousands)
1970	295
1975	327
1985	415
1990	473

3. The number of respiratory cancer deaths for Staten Island for the years 1970-77 is given below.[9] Plot a graph.

year	1970	1971	1972	1973	1974	1975	1976	1977
deaths	95	134	126	144	116	153	164	143

Circle diagrams

1. The following table shows how Staten Islanders got to work in 1970.[10] Draw a circle diagram.

private auto: driver	48,718
private auto: passenger	8,892
bus or streetcar	22,475
subway, elevated train, or railroad	13,051
walked to work	6,095
worked at home	1,096
other	9,702
all workers	110,029

2. Approximate 1977-78 enrollments in Staten Island Colleges were as follows:[11]

College of Staten Island	11,000
St. John's University	1,873
Wagner College	2,594

Draw a circle diagram using this data.

3. In 1966, the S. I. income distribution was as follows:[12]

income range	percent
under $3,000	11.3
$3,000 - $9,999	55.5
$10,000 and over	33.2

Draw a circle diagram.

Bar charts

1. The following gives estimated populations for various postal regions on Staten Island for 1978.[13] Show the data on a bar chart.

Zone 103	
St. George (01)	43,663
Port Richmond (02)	15,484
Mariners Harbor (03)	14,411
Stapleton (04)	34,462
Rosebank (05)	30,339
New Dorp (06)	50,511
Tottenville (07)	6,469
Great Kills (08)	21,749
Princes Bay (09)	8,363
W. New Brighton (10)	28,081
Eltingville (12)	30,971
Castleton Corners (14)	57,605

2. Staten Island's 1970 population breakdown by age and sex was as follows:[14]

	males	females
under 5 years	13,845	13,222
5 to 9 years	15,642	14,832
10 to 14 years	15,115	14,015
15 to 19 years	12,829	12,259
20 to 24 years	10,273	12,054
25 to 34 years	19,338	20,203
35 to 44 years	17,665	18,021
45 to 54 years	16,675	17,816
55 to 59 years	6,736	7,538
60 to 64 years	5,504	6,239
65 to 74 years	6,765	9,003
75 years and over	3,951	5,903
totals	144,338	151,105

Draw a bar chart. Make the bars proportional to the range in ages. Assume 75 years and over to be from 75 to 85, for the purpose of drawing the bar chart.

AVerages

1. Air pollution is a major problem on Staten Island. The following are arithmetic means for readings of SO_2 (sulfur dioxide) for various seasons taken at a station at the Fresh Kills landfill.[15]

Season	mean (parts per million)
winter	0.021
spring	0.016
summer	0.010
fall	0.015

Use this data to compute the arithmetic mean for the calendar year.

2. Repeat problem 1 using the data below taken at the Borough Hall measuring station.

Season	mean (parts per million)
winter	0.030
spring	0.022
summer	0.018
fall	0.021

3. A 1970 study showed that 57,610 Staten Islanders went to work by car. Of these, 48,718 were drivers and 8,892 were passengers. What was the arithmetic mean and mode for the number of people in each car coming to work.[16]

4. Compute the mean precipitation per month on Staten Island using the data given below for 1977:[17]

month	total precipitation (inches)
January	2.41
February	2.55
March	5.23
April	3.76
May	2.12
June	5.85
July	2.41
August	6.96
September	5.91
October	4.46
November	12.36
December	4.98

5. In 1978, the estimated S. I. population was 345,325 and the estimated number of households was 100,094.[18] What was the average number of people per household?

Notes

1. Statistical Guide, Chamber of Commerce, S.I., p. 1

2. Ibid.

3. Trends in Respiratory Cancer Deaths, New York City - 1960-1977, New York City Department of Health, Office of Biostatistics, 125 Worth Street, New York, N. Y. 10013, 1979, p. 6.

These figures exclude the New York City resident deaths occurring in New York City institutions and those New York City residents who died outside the confines of New York City.

4. "The Pall Above the Victims Below," Medical World News, (Feb. 3, 1967)

5. Statistical Guide, op. cit., p. 7

6. Ibid, p. 3

7. Ibid, p. 2

8. Ibid, p. 2

9. Trends in Respiratory Cancer Deaths, op. cit., p. 10

The number for deaths given exclude the Staten Island residents deaths occurring in New York City institutions and those who died outside the confines of New York City.

10. Statistical Guide, S. I., op. cit., p. 7

11. Ibid., p. 29

12. "Staten Island, A Study of its Resources and their Development," a S.I. Chamber of Commerce Publication, p. 12, using Internal Revenue Service data.

13. Statistical Guide, S. I., op. cit., p. 2

14. Ibid, p. 4

15. New York City, Department of Air Resources

16. Statistical Guide, S. I., op. cit., p. 7

17. Ibid, p. 1

18. Ibid, p. 2

Chapter 19 Mathematics and the Year of the Child

The United Nations General Assembly has proclaimed 1979 as the International Year of the Child (IYC). The emphasis of IYC is to coordinate efforts to improve the conditions of children throughout the world. It is hoped that there will be action in many communities to improve services for children and changes in national laws and policies which have been harmful to children.

Most children in the world are not growing up under satisfactory conditions. Eighty percent live in the less developed regions with insufficient food, education, health care, and sanitary facilities. About one-quarter of the world's children are malnourished; this leads to death, permanent physical and mental retardation, and other diseases for many. In some regions,30 to 40 percent of children do not reach the age of 5.

It is urgent that action be taken to improve the lives of the world's children. As UNICEF leader, Gabriela Mistral stated in 1946:

> We are guilty of many errors and many faults, but our worst crime is abandoning the children, neglecting the fountain of life.
>
> Many things we need can wait. The child cannot. Right now is the time his bones are being formed, his blood is being made, and his senses are being developed. To him we cannot answer 'tomorrow.' His name is 'today.''

The following mathematical problems are designed to provide information and insight on the conditions of the world's children as they serve to help teach basic mathematical concepts. They are based on the following excellent materials prepared especially for the International Year of the Child by The Population Reference Bureau (1337 Connecticut Avenue, N. W., Washington, D. C. 20036).

1. Children in the World by Magda C. McHale and John McHale, with Guy F. Streatfeild.

"A chartbook (over 80 charts and tables) with a brief accompanying text covering child development, numbers and proportions of children in the world, and data on children's health, welfare, and education."

2. World's Children Data Sheet

An 18½" x 24" wall chart which provides much useful information about the world's children. Subjects covered include infant mortality, nutrition, school enrollment, and pupil-teacher ratios.

3. Chart Series on Children

Six charts with bar and circle diagrams and graphs, with essays on the back, on many issues related to children, such as life expectancy at birth, nutritional levels, school enrollment, and infant mortality rates.

4. World of Children by Magda C. McHale, John McHale, and Guy F. Streatfeild.

A description of conditions for children throughout the world and a discussion of the changes necessary to improve their lives.

Basic Calculations

1. In 1975, the U. S. had 15,879,000 children under 5 years of age; 17,325,000 between 5 and 10; and 20,409,000 from 10 to 15 (but not including 15). What was the total number of children in the U. S. under 15 years of age?[2]

2. About 334,000 babies are being born in the world every day in 1979. How many births is this for the entire year?[3]

Percents

1. There are 1,439 million children under the age of 15 in the world. Of these 693 million will live less than 60 years. What percent of the world's children does this represent?[4]

2. What percent of the world's 1,439 million children under the age of 15 are represented by each of the following:[5]

a) 5 million die annually from six major infectious diseases;

b) 230 million are undernourished;

c) 417 million are without adequate housing;

d) 590 million are without access to safe water;

e) 604 million are without access to effective medical care;

f) 250 million are of school age (5-14) but out of school;

g) 173 million need special education or rehabilitation services;

h) 396 million live in countries where average income per capita is less than $200 per year;

i) 156 million live in slums and squatter settlements;

j) 600,000 are refugees.

3. In 1975, 40% of the 2,892 million people in less developed countries were under 15 years of age.[6] Compute the number of children under 15 years.

4. In 1975, 99 of every 1000 children born did not survive their first year.[7] What percent infant mortality rate does this represent?

Ratios

1. In 1975, the infant mortality rate was 158 per 1,000 population in Western Africa and 14 per 1,000 in Western Europe.[8] Compute the ratio of West African to West European infant mortality rates per 1,000 population.

2. In 1975, there were 504 million of the world's children under 15 years of age living in urban areas and 924 million living in rural areas. Compute the ratio of urban children to rural children.[9]

Repeat for the year 2000 when there are projected to be 953 million children under 15 years of age living in urban areas and 1,072 million living in rural areas.[9]

3. In 1975, there were 606 people per physician in Northern America and 9,091 people per physician in Asia.[10] Find the ratio of people per physician in Asia compared to Northern America.

4. The figures below are for representative countries in the four regions listed. Compute the ratio of pupils to teachers for the countries chosen in each region.[11]

region	teachers	pupils enrolled
Africa	2,600	100,500
Asia	10,700	316,000
Latin America	13,500	410,100
Europe	82,800	1,977,900

Graphs

Draw graphs for the following sets of data

1. Growth of the world labor force (1750-2000)[12] (With the rapid growth of population of children, there will be a need for far more jobs in the next 25 years.)

Year	world labor force (millions)
1750	360
1800	440
1850	560
1900	700
1975	1,644
2000	2,545

2. Increase in numbers of children under 15 (1950-2000)[13] (for the world)

Year	number of children under 15
1950	880 million
1979	1,530 million
2000 (est.)	2,030 million

Bar charts

Draw bar charts for the following sets of data:

1. Mean Life Expectancy at Birth [14]

World	60 years
Developed Countries	72 years
Latin America	62 years
Asia	57 years
Africa	46 years

2. Mean Daily Protein Supply per capita (in grams) [15]

World	68 grams
Developed Countries	98 grams
Latin America	65 grams
Asia	57 grams
Africa	55 grams

3. Infant mortality (per 1,000 live births) [16]

World	99	Oceania	41
North America	15	Latin America	84
Europe	20	Asia	105
U.S.S.R.	28	Africa	147

4. Percent of children aged 10-14 in the labor force, selected countries, 1975. [17]

Country	% of children aged 10-14 in the labor force
Mali	45
Bhutan	44
Upper Volta	43
Ivory Coast	37
Bangladesh	32

(contd.)

4. continued

Country	% of children aged 10-14 in the labor force
Haiti	29
India	20
Brazil	11
Mexico	6
Japan	2
U.S.A.	2
Sweden	0.4

Circle Charts

1. In the less developed countries the population - age breakdown is as follows:[18]

under 5 years	439.3 million
5-15 years	377.4 million
over 15 years	341.5 million

Draw a circle diagram.

2. The percent of world population in different stages of the life cycle in 1975 are given below:[19]

Infant and preschool (ages 0-5)	15.7%
Early school and prepubescent (ages 6-13)	18.1%
Adolescent (ages 14-19)	12.1%
Young adults (ages 20-29)	16.1%
Adults (ages 30-59)	29.4%
Older ages (ages 60 up)	8.6%

Notes

1. Quoted in "Children of the World: Rights and Realities" A chart prepared for the International Year of the Child by the Population Reference Bureau.

2. "World's Children Data Sheet," of the Population Reference Bureau, 1979.

3. "Children In the World," Population Reference Bureau, Washington, D. C., 20036, p. 11

4. Ibid, p. 4

5. Ibid, p. 4

6. Ibid, p. 13

7. "World's Children Data Sheet," op. cit.

8. Ibid

9. "Children In the World", op. cit., p. 18

10. Ibid, p. 42

11. Ibid, p. 60

12. Ibid, p. 67

13. Chart: "Children and Population Change," Population Reference Bureau, 1979.

14. Chart: "Children of the World: Rights and Realities," Population Reference Bureau, 1979.

15. Ibid

16. "Children In The World," op. cit. p. 31.

17. Ibid, p. 64

18. "World's Children Data Sheet," op. cit.

19. "Children In The World," op. cit., p. 9.

Chapter 20 Mathematics and Vegetarianism

This book has shown through mathematical problems and related discussions the many interrelated and critical problems faced by the world today.

Much needs to be done to reduce these problems. There is a need for a slowing of the rapid rate of population growth. As indicated previously, one of the ways that this can best be done is to improve the social and economic conditions of the people in the less developed countries.

There is a need for people in the more developed countries to adopt simpler, less wasteful life styles.

The arms race must be halted and reversed before global catastrophe occurs. If even a small percent of the $400 billion now devoted annually to the arms race were devoted to meeting human needs, much progress could be made.

The large and widening social gaps between people in the more developed and less developed countries must be reduced. There is a need for a "new international economic order" which will curb inflation and unemployment, provide fairer conditions of trade, and improve conditions for all of the world's people, the wealthy as well as the poor.

One solution that has been suggested is a switch to vegetarian diets. As the following two articles indicate, a switch away from present meat-centered diets can reduce global hunger, decrease the tremendous amounts of water, fuel, pesticides, and fertilizers presently used to grow food, diminish threats to our ecosystem, improve the health of people as well as the environment, and perhaps even reduce the threats of war.

Questions are provided at the end of both articles to help you focus on the points made in them, as well as to provide further practice in obtaining information from articles and solving mathematical problems.

MONEYSWORTH/MARCH, 1979

20-2

VEGETARIANISM FOR A WORLD OF PLENTY

BY ROBIN HUR -Vegetarian Inform. Service

Our diet is an economic travesty. Those who produce, process and prepare our food do so at the expense of the country's natural resources.

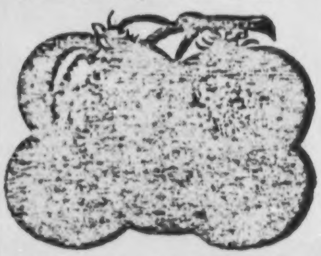
Travesty becomes tragedy when we consider that the very foods which cause our diet to be so expensive are implicated in everything from bowel cancer and coronaries to obesity, "aging," and the desire to smoke. A host of the nation's leading nutritionists are backing recommendations that the nation adopt a prudent diet: that is, a diet which is more vegetarian in nature and which is low in refined foods. We can ill afford to do otherwise.

It takes five acres of farm land to feed just one American. It takes less than two acres to feed the average Briton, only half an acre to supply the food for the average Italian and a mere quarter-acre to feed a single Egyptian.

With a truly prudent diet we, too, could have our food needs met by a quarter-acre of farm land per capita. Were we to adopt such a diet, 95 percent of the nation's farm land—a billion acres in all—could be diverted to tree farming, export crops, spacious living, recreation, etc. There's enough land at stake to build a full-scale 18-hole golf course and country club for every 10 households.

A gargantuan complex of mechanized farms, transportation subsystems, production facilities, *et al.*, is necessary to supply the foods we choose. This complex devours staggering amounts of the nation's dwindling natural resources.

It takes as much fossil fuel energy to supply our food as it does to heat our homes or power our automobiles. It requires as much steel, aluminum, copper, zinc, plastic and glass to get our food on the table as it does to manufacture either automobiles or industrial machinery. The food industry is already the nation's biggest user of tin and quality paper and it may soon become the world's biggest user of plastic.



Those who supply our food use pure water at the incredible rate of a billion gallons every three minutes. That amounts to at least 20 times the volume of water used by households and it underlies the problem of a highly overtaxed national water supply. Compounding the problem are food producers themselves. Within their ranks are many ranchers and cattlemen who have resorted to overgrazing. Overgrazing is not only destroying vast expanses of our once-resplendent grasslands but it is causing water tables to plummet and thereby depleting the nation's water reserves.

Ironically, food producers pollute more water than they consume; they are directly responsible for more than half of all the nation's water pollution. The food they produce is none too clean, either. It arrives in supermarkets tainted with all manner of chemical residues, preservatives and additives. A more distressing problem, however, is the food itself.

Our diet is "aging" us, making us fat, subjecting us to pain, allowing us to become disturbed, causing us to spend $100 billion per year on "health care" and killing us.

Meat and animal products are a major part of the problem.

Rising demand for beef is the underlying cause of overgrazing and the production and processing of meat, especially beef, wins all the prizes for pollution. Feedlot operators and others who fatten livestock are responsible for more of the nation's water pollution than all of industry and all households combined. As if this were not enough, meatpackers are the nation's principal industrial polluter.

The consequences of eating meat and other animal products are as disturbing as the costs of producing them. The consumption of animal products is the most strongly implicated factor in heart disease, strokes, bowel cancer, kidney disorders, obesity, schizophrenia, breast cancer, osteoporosis (which is the basis for so-called "aging") and the desire to smoke. Animal products have also been linked to diabetes, high blood pressure, multiple sclerosis, and arthritis.

Processing exacerbates our dietary problems. Processing, which includes canning and bottling, accounts for most of the petroleum, steel, aluminum, tin, glass, paper and plastics that is expended in getting our food to us.

Processing (in the form of meatpacking) contributes mightily to water pollution also. The most distressing aspect of processing, however, is its effect on our health. Processed foods and beverages, including sugar, refined grains, and alcohol, have been implicated in almost as many degenerative disorders as animal products.

Since much of the foregoing is far from common knowledge it is fitting to examine the basis for at least some of these claims.

The agricrats have been unable to induce cows to give birth to more than one calf at a time; thus, the cost of raising a single animal includes the cost of feeding and maintaining its mother throughout pregnancy and lactation.

Cows are usually grass-fed or grazed until they are ready for the feedlots. In the feedlots they are fed grains and legumes along with protein supplements and other marvels calculated to speed growth and prevent excretion. It takes four to six months in a feedlot plus about 3 tons of grain to transform a 600-pound cow into a 1,200-pound candidate for the slaughterhouse. This 3 tons of grain, along with the many tons of hay and grass it takes to prepare a cow for the feedlot, make raising beef a very expensive operation.

The slaughterhouse operator has his own cost problems. Only 60 percent to 65 percent of the weight of grain-fed cows can be sold as edibles. The slaughterhouse operator's losses would be even greater were he not able to sell bowel, brain, lungs and the like to processors.

Separation of the inedibles, cooking and trimming result in less than half an animal's original mass being eaten, and the meat that survives trimming is mostly water so it's relatively low in calories—so low, in fact, that it could take 3.5 pounds of such beef to provide a single adult with his accustomed caloric intake. It would take about 7.5 pounds of live animal to supply that 3.5 pounds of beef.

The production of 7.5 pounds of live animal per day would require over 4 acres of harvested grain and 7.5 acres of arable land for pasture (most of our pastureland is arable). Thus, it would require up to 11.5 acres of arable land to supply a single adult with a beef diet. That same land could be used to feed 80 adults a nutritionally superior diet of leafy green vegetables and a high-yield staple.

While the U.S. diet is a meat-based fare, meat accounts for only 20 percent of the nation's food intake. Animal products as a whole account for only a third of total food intake. Nevertheless, it requires a billion head of livestock to supply that third of the diet, and the nation's livestock have prodigious appetites.

U.S. livestock consume enough forage each year to mulch a garden the size of England. They devour 97 percent of the nation's legumes and over 90 percent of the nation's grain supply. And believe it or not, they consume 80 percent of the country's fish supply!

Although U.S. grain-growers are heavy users of fossil fuel, they manage to supply more energy in the form of food energy than they consume in the form of fossil fuel energy. To say it another way, our grain producers use less than one calorie of fossil fuel energy to produce one calorie of food energy. Meat producers can make no such claim.

It takes 100 calories of fossil fuel energy to produce enough feedlot beef to supply one calorie of food energy! The production of lobster, shrimp, lamb and rangeland beef requires comparable amounts of fossil fuel energy; the production of milk, pork and eggs requires prodigious amounts of energy also.

It takes the equivalent of 112 gallons of gasoline per year to produce the food of a single American. It takes the equivalent of 224 gallons of gasoline to process, package, transport and prepare that food. Thus, it takes the energy equivalent of 336 gallons of gasoline per year to feed just one of us.

A group of Cornell researchers noted that if the entire world were to adapt to a diet like our own then the world's known oil reserves would be depleted within 12 years. Actually, the time period would be even less, since they made no allowances for non-food-related uses of petroleum.

Adoption of a simple whole-foods vegetarian diet would cut the direct costs of the energy required to supply our food by 80 percent to 95 percent depending on how much cooking was involved. Moreover, the new diet would no doubt be accompanied by a simpler lifestyle, which would entail further energy savings.

Americans are notoriously wasteful. We use over a hundred gallons of fresh water per day on the average. That's probably five times what we need but it's small in comparison to the water it takes to feed us. It takes a hundred gallons of water an hour, 24 hours a day, to supply just one person's food. The big problem, again, is animal products, especially meat; it takes up to 80 times as much water to produce beef as it does to produce tubers and bananas.

The production of meat not only requires vast amounts of water and land, but it leaves water and land in a deplorable state. The nation's feedlots, broiler "hotels" and hog "factories" are responsible for more than half of all the nation's water pollution. The excreta of U.S. livestock contribute much more to water pollution than all of industry and all our sewage combined. And that's not the end of the story. Meatpackers, dairies, and other food processors "dominate the industrial scene as polluters"; so geographer and food scientist George Borstrom tells us.

The nation is spending an estimated $20 billion to combat water pollution this

year. This is expected to double by 1984. The price of the paper on which this is printed reflects the price of anti-water pollution equipment the paper maker was required to install. The same is true of a broad range of manufactured and processed items. If and when we get around to having feedlot operators spend commensurate amounts, it will push the price of beef up by as much as a dollar a pound. Meanwhile, beef-eating environmentalists are an anachronism.

Water pollution is just one of the indirect costs of meat production. Another is the ruination of arable land through overgrazing. Overgrazing tends to defoliate regions and lower water tables so that nothing can be grown or fed in the surrounding area. Overgrazing was responsible for the infamous "dust bowl" of the '30s and in recent years it has taken a heavy toll in Texas. Much of Texas' once-resplendent grasslands are in dire straits; water levels have fallen from a few feet to hundreds of feet.

Overgrazing creates a vicious spiral. As and is used up and abandoned, still less and is available for the herds, so overgrazing becomes even more pronounced. The herders of East Africa are caught in such a cycle now, and it is causing the Sahara to move southward at a rate of 50 miles per year.

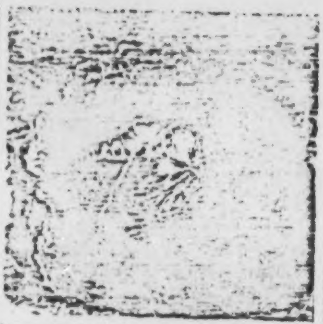
Demand for meat, especially beef, is rising rapidly but neither the United States nor any other beef-producing nation has any new grazing land. Thus, the pressure to overgraze is becoming even stronger. If it is allowed to continue we may end up with little else but desert.

Meat and animal products are by no stretch of the imagination the only economic stigma on our diet. Processing, whether it is meat or wheat, is a problem in its own right. We spend 50 cents for the processed remains of 6 cents' worth of wheat in what passes for bread. Two-thirds of the $1,000 a year per capita we are spending on food goes to processors and handlers, packers, canners, bottlers, freezers, packagers, haulers, distributors, ilers, etc.

Food "containerizing" is one of the biggest businesses in the United States. Containerizers are among the nation's biggest users of steel, aluminum, glass and paper, all of which devour enormous amounts of energy. To make matters worse, cans and bottles are being replaced by plastic containers which are not only made from petroleum, but which cannot be recycled.

With a diet emphasizing greens, sprouts, and fruit or tubers, most of our foods could be grown locally and none of our foods would need any processing beyond possibly washing. Our locally grown foods could be carried in reusable containers (e.g., cloth sacks, wooden boxes, etc.) and gone would be the costly blight of litter.

We regard "good" food as food with a stimulating taste. While this is not necessarily a contradiction in terms, it is short-sighted, if not backward thinking. What our food does for us and to us after cloying the palate is the most important issue of this century.



Cardiovascular diseases (which include heart attacks and strokes) still account for half the nation's death toll. The basis for the problem is "atherosclerosis," which involves the build-up of fat and cholesterol-rich deposits in the arteries. Atherosclerosis is strongly linked to intake of animal fats and cholesterol (which is a constituent of all animal products and no plant foods) and, to some extent, sugar, salt and refined carbohydrates.

The results of numerous long-term studies make it clear that lowering the amount of meat, visible fat and eggs in diets sharply reduces the incidence of heart disease. Recent research indicates a low-fat, whole-foods, strict vegetarian diet will not only prevent atherosclerosis but rid the arteries of existing atherosclerotic deposits.

A Harvard researcher has linked still another cardiovascular disorder, high blood pressure, to animal products, and practitioners report high blood pressure disappears rapidly with a simple vegetarian diet.

Cancer is the nation's second biggest killer, and a number of cancers, including the nation's most widespread malignancy (bowel cancer), the most prevalent female cancer (breast cancer), and the most prevalent, strictly male cancer (cancer of the prostate) have been linked to the ingestion of meat and animal products. These ties are independent of the dangers of nitrites and other chemicals found in foods. They emanate from the growth-promoting capacity of animal products; tumors are in effect pockets of runaway protein synthesis or what is the same thing, runaway growth.

Studies of Seventh-Day Adventists and other such groups indicate vegetarians enjoy lower risks of all forms of cancer. Medical researchers report that the long-living Russian Abkhasians who eat a whole-foods, essentially vegetarian diet are completely free of cancer; similar reports have been made about the even more vegetarian Hunzas.

Available research indicates individuals could reduce the risk of cancer by 90 percent or more simply by adopting a sound, strict vegetarian diet and not smoking. Fortunately, the desire to smoke tends to disappear when individuals switch to simple vegetarian regimes.

The founder of an almost 100 percent effective stop-smoking clinic had participants prepare for the break by partaking of a fruit diet for a few days. A study of a group of adult vegetarians in England revealed only 7 percent of the group smoked. A nationwide survey of subscribers to Vegetarian World indicates fewer than 2 percent of that group smokes. A number were former smokers.

A sound vegetarian diet is possibly the only safe and sure way to overcome obesity. Studies show that lacto-vegetarians (those who eat dairy products) weigh considerably less than meat-eaters, and strict vegetarians weigh a good deal less than lacto-vegetarians. Those who adopt low-fat, whole-foods, strict vegetarian diets tend to take on the physical stature and attributes of long-distance runners; a large number of converts do, in fact, become runners.

Maturity-onset diabetes is closely related to obesity and physicians report the disorder can almost always be corrected with a low-fat, whole-foods vegetarian diet. Clinical research indicates such a diet will also counter most cases of multiple sclerosis, ulcers, psoriasis, and diverticular disease. Low-protein vegetarian diets are proving to be effective in treating kidney disorders, mental disease and osteoporosis. Chronic schizophrenics are being cured by a period of fasting followed by a simple vegetarian diet.

The basis for so-called "aging" is the transfer of calcium from the hard tissues (i.e, bones) to the soft tissues (skin, joints, eyes, arteries, etc.). Researchers have discovered a high-protein diet can and does cause the bones to give up calcium; normal protein intakes are suspect and animal products, especially meat, have a mineral mix which exacerbates the problem. There are population groups who eat whole-foods, essentially vegetarian diets and whose elderly remain erect, healthy and active. There are other groups like the Masai, who eat a great deal more flesh than we do and who become decrepit in their 20s.

We are currently paying some $140 billion per year for medical care, and medical expenses are rising faster than any other component of living costs. It is not only the sick who pay. We all pay. We pay through municipal bond issues for hospitals, the tax deductibility of medical expenses, Medicare and Medicaid taxes, contributions to charities, medical services for the military and veterans, expenditures by HEW, disability payments, and in a myriad of other ways.

Take no solace from the fact that insurance companies pay a big share of the country's medical costs. Medical insurance premiums are, to a large extent, borne by employers, hence passed on to the public in the prices it pays for products and services.

It's immaterial, from an economic point of view, whether a simple vegetarian diet would bring an end to all disease or, say, 70 percent of it. A 70 percent cut in our medical costs would save the country $100 billion annually; that's about 2.5 times the amount we are spending on oil imports and on a par with our mammoth military budget.

A simple, sound vegetarian diet should bring dramatic improvements in the health of the nation's elderly. Decrepitness, senility, loss of sight and hearing, etc., should all but disappear, and with the return to good health, our elderly could care for themselves with a minimum of help. This could bring savings of up to $70 billion in Social Security outlays alone.

Raw, dark green leafy vegetables (e.g., parsley, collard greens, turnip greens, watercress, etc.) and raw sprouts (e.g., mung bean, alfalfa, wheat, soy, etc.) make an excellent foundation for a diet. The combination of greens and sprouts provides the broadest and strongest possible array of necessary vitamins and minerals and other nutrients. Unlike so much of what we are urged to eat, greens and sprouts contain no excesses of acid-forming minerals, sodium, saturated fats and total fat, cholesterol, purines, and other potentially harmful substances.

All that's needed with greens and sprouts is a source of food energy; fresh fruit, potatoes or whole grains will do. There is no need for oil or butter, salt, and overcooking. In addition, one may want to eat a little seaweed once in a while; for those who were still taking as much as a quart of milk per week the seaweed would not be necessary.

If the nation as a whole were to adopt a simple whole-foods vegetarian diet, farmers could abandon the use of noxious farm chemicals and still be able to supply our domestic food needs with only 5 percent of the country's arable land. We could increase the amount of land used to grow crops for export and still have at our disposal 85 percent of the nation's current farm land. Most of this vast expanse of land was formerly forested and by a massive tree planting program we could reforest it.

Our refurbished forests could add as much as $50 billion annually to the value of the nation's future tree harvests. In a few years' time our exports of forest products should exceed our current oil imports. Meanwhile, the new diet would allow us to cut oil imports by a third and reduce our imports of metals and metal ores by 20 percent or more. It would then take only modest increase in our exports of forest products and food crops to bring an end to the balance-of-trade deficit.

The change in diet would bring improvements in the nation's health that could be translated into reductions in federal spending totaling up to $80 billion annually. Savings would come not only in expenditures for health care for the aged, veterans, the military, government employees, the disabled, the needy, etc., but in outlays for Social Security, welfare, agriculture, transportation and "distressed" cities. Spending by state and local governments should also decline because, among other things, most of the roads and highways serving rural areas would no longer be needed.

The new diet would effect big improvements in the quality of the nation's air, eliminate the sources of as much as two-thirds of all water pollution, and bring an end to food pollution in the form of chemical residues and additives. It would facilitate a big increase in the nation's supply of pure water, improve the quality of the grasslands and do away with the sources of most of the nation's litter. Savings in expenditures for water pollution abatement alone would exceed $40 billion annually. An additional $10 billion annually could be saved through lower outlays for solid waste disposal, pure water, etc.

The average family's expenditures for food and health care would drop to a small fraction of current levels. The costs of land and lumber would go way down, thus bringing down the cost of new homes.

Reductions in federal, state and local spending should mean lower tax obligations, and a nation reforested would mean ready access to wilderness, hence lower outlays for leisure-time activities. The nation's overall cost of living should go down at least $250 billion per year.

The switch to a whole-foods vegetarian diet should bring a dramatic improvement in the nation's health. The life expectancy of adults should go up by 10 to 20 years and that of young people 20 to 30 years or more. More important, though, is the fact that most, if not all, individuals could expect to go through life unencumbered by pain, sickness, and emotional disturbances.

Robin Hur is the author of Food Reform: Our Desperate Need. He is presently working on a book about the economics of diet.

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1. What is the ratio of land needed to feed the average American to the land needed to feed the average
    a) Briton?
    b) Italian?
    c) Egyptian?

    Draw a bar chart showing the land needed in the 4 countries.
2. About how many acres are currently being used for farming in the U. S.?
3. What is the approximate ratio of fossil fuel energy needed to supply our food compared to that needed to (a) heat our homes and (b) power our automobiles?
4. About how many gallons of water are used annually in the U. S. to supply our food (this includes water for irrigation, feeding animals, processing food, cleaning food, etc.)?
5. What is the ratio in the U. S. of water used to supply food to water used in all the households?
6. About how much money is spent on the average per person in the U. S. each year on health care?
7. According to this article how many pounds of grain in a feedlot are necessary to produce each pound of weight gained by a cow?
8. How many pounds of live animal are necessary to produce 100 pounds of edible cooked beef?
9. According to the article, about what fraction of an acre would be necessary to provide a "nutritionally superior diet of leafy green vegetables and a high-yield staple" to one person.
10. What fraction of total food intake in the U.S. is from non-animal products?
11. What percents of the following are not consumed by animals?

    a) legumes b) grain c) fish
12. What is the ratio of food energy supplied by feedlot beef to fossil fuel energy input?

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13. What percent of the total gasoline needed for food by the average American is used to process, package, transport, and prepare the food?
14. Of the total energy necessary to supply U. S. food today, about what percent would be required if all Americans adopted a simple whole-foods vegetarian diet?
15. How much water is used annually to supply an average American's food?
16. What is the ratio of water needed to produce beef to water needed to produce bananas (presumably per calorie of each)?
17. What is the estimated annual cost for combatting water pollution in the U. S. by 1984?
18. How much does the average American pay per year for medical care? There are about 220 million Americans.
19. Approximately how much could the cost of living per person in the U. S. go down if a whole-foods vegetarian diet was adopted.
20. By approximately what percents could life expectancies be improved due to a whole-foods vegetarian diet for (a) adults (b) children?

STATEN ISLAND ADVANCE, Friday, April 27, 1979

20-6

Could what we eat change the world?

Switch from meat to vegetarian diet seen solution to social, economic ills

By RICHARD H. SCHWARTZ

The world today is beset by many interrelated problems. These include widespread hunger, ecological destruction, scarcities of resources, a widening rich-poor gap, economic crises and threats of violence and war. A movement toward vegetarianism could play an important, perhaps decisive role in the solution of each of these problems.

At least half of the world's people are either malnourished or undernourished. Between 10 and 20 million people die annually due to hunger and its effects. Many of these are children; in some areas 30 to 40 percent of the children do not reach the age of 5.

One of the factors behind the global hunger crisis is the heavy meat-eating diet of the United States and other industrialized countries. The average person in the United States eats almost five times as much grain (mostly in the form of animal products) as does a person in an undeveloped country; it takes about eight pounds of grain to produce one pound of edible beef in a feedlot. Almost 80 percent of the grain grown in the United States is fed to animals. Half of our agricultural land is devoted to feed crops.

A non-vegetarian diet requires about 3.5 acres per person, whereas a total vegetarian (vegan) diet only requires about 0.25 acres. Hence, a shift to a non-meat diet would free much valuable land that could be used to grow nutritious crops for people.

While the United States is considered the breadbasket of the world and we export much food to other countries, we are one of the world's top food importers. In 1974, only Japan and West Germany imported more food than we did. We are the world's largest importer of beef, importing about 40 percent of all beef in world trade. We also lead the world in the importation of fish, much of which is fed to livestock; food-producing resources in countries where there are many hungry prople are used to produce food for the U.S. and other developed countries.

It is now generally recognized by food experts and U.N. agencies that the only way of feeding the starving millions in the world is by the increased production of plant foods such as cereals, vegetables, fruits, legumes, oil seeds and nuts. An acre of land presently used to produce 183 pounds of beef annually could be used to produce 1,680 pounds of wheat, 4,565 pounds of rice, or 20,160 pounds of potatoes. Under these conditions, it might truly be said that one person's steak is many other persons' starvation.

As Mahatma Gandhi pointed out, "There is enough on this planet for everyone's need but not enough for everyone's greed."

A switch away from a flesh centered diet would not in itself be sufficient to alleviate the plight of the hungry. But it would free valuable agricultural resources which could be used to grow nutritious crops for people. It would then be necessary to see that people in the underdeveloped countries were provided with the proper resources and skills so that they would become self reliant.

Modern agricultural practices are a prime cause of the environmental crisis facing the United States and much of the world today.

In spite of the tremendous amounts of pesticides used in this country the percentages of our crops lost to pests have increased since 1960. Meanwhile, the side effects of the pesticides are extremely damaging to the environment, to wildlife in the air and water and on the earth and finally to people. It has been said that continued use is like playing a giant game of environmental Russian roulette.

Slaughterhouses are also prime sources of pollution. One study revealed that 18 meat packing companies in Omaha, Nebraska discharge over 100,000 pounds of grease, carcass dressing, intestinal waste and other animal residues into its sewer system which empties into the Missouri River.

A shift to vegetarianism would result in far less stress on our ecological systems. It would require less land devoted to feed crops; thus there would be more chance to let land lay fallow and to rotate crops to restore fertility, less need to use marginal land and hence reduced soil erosion. There would be a decreased need for pesticides, fertilizer, and fuel with the many forms of pollution related to them. Also, there would be less animal and slaughterhouse wastes washing into our waters and fouling our air.

Although the United States is generally considered to be the world's most powerful and wealthiest country, in many ways we are a "have-not" nation. We are dependent on other countries (generally the poor countries) for over 90 percent of our tin, nickel, chromium, manganese and many other metals. Our ever-growing demand for these raw materials has led to scarcity and price increases.

One of the prime causes of our rapid use of resources is our wasteful farming methods, again centered on our non-vegetarian diets. Tremendous, ever increasing quantities of pesticides and fertilizer are used on our farms. U.S. food production is dependent on the many farm machines used to grow our food, and the equipment and machines needed to process, transport and store it.

In 1975, the developing countries were short 2 million tons of fertilizer, an amount which could grow food for 100 million Asians or Africans. In that same year, several million tons of fertilizer were used in the U.S. for growing feed-crops for animals.

Experts expect that we will have major shortages of water in the United States by 1985. Many states such as California and Georgia have already experienced severe droughts.

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The largest water-using sector of society is agriculture: it accounts for 86 percent of consumptive water use in the United States. One of the primary reasons for the tremendous quantities of water required is our flesh-centered diet. The standard diet of a person in the United States requires 2,500 gallons of water per day (for animals' drinking water, irrigation of crops, washing, cooking, etc.). A pure-vegetarian diet requires only 300 gallons per person per day. According to the U.S. Department of Agriculture, the cost of water per pound of meat is about 25 times that of the cost of water for a pound of vegetables.

- Food processing requires enormous volumes of water. Just one plant that processes chickens uses 100 million gallons of water daily; this would service a community of 25,000 people.

In addition to concern about the quantity of water available, there is need to consider the quality. As indicated previously, both ground and surface waters have been badly contaminated by fertilizer, pesticides, and livestock wastes.

A switch to a vegetarian diet would greatly reduce the demand for scarce water and also cut many of the sources of pollution.

In recent years, total energy use in the United States has doubled every 20 years; consumption of electrical energy has doubled every 10 years. These tremendous increases have led to many current and potential problems.

We now import nearly half of our petroleum. This has led to much inflation, record balance of payments deficits, and a decrease in the value of the dollar. Our dependence on other countries has placed restrictions on our foreign policy.

Our need for ever increasing quantities of energy has led to many dangers. These include:

- Nuclear power plants: The problem of how to store nuclear wastes which are radioactive for thousands of years is far from solved. There is the danger of proliferation of nuclear weapons made from plutonium produced by the nuclear plants. There is also the possibility of major accidents, such as that at Three Mile Island.
- Large oil tankers and off-shore drilling: Major spills could badly damage the seas.
- Coal-burning power plants: They reject carbon monoxide, solid and liquid particles, sulfur dioxide, and other pollutants into the atmosphere.

About 12 percent of the total U.S. energy budget is accounted for by food-related energy consumption. This includes fuel and electricity used on farms, tremendous energy imputs for the manufacture of fertilizer and pesticides and for irrigation and energy used in the transportation, processing, cooking, and storing (refrigeration) of goods.

Once again, our flesh-centered diets constitute a major factor. It takes about 10 calories of fuel energy to produce and deliver each calorie of food consumed by Americans. In other countries, they get 20 or more food calories for each calorie of fuel energy. This great difference is largely due to the huge amounts of energy related to beef and poultry production and deep-sea fishing. One large slaughterhouse uses as much energy daily as a community of 25,000 people and there are about 7,000 slaughterhouses in the U.S. and Canada.

The world today is faced with a many-faceted economic crisis. Rampant inflation is worldwide. Rapid increases in costs of fuel and fertilizer have helped cause food scarcities and price increases which have led to widespread hunger. There is tremendous unemployment in many countries of the world, especially the poorer countries.

The shift to vegetarian diets would improve the world economic picture. It would free presently scarce resources, help reduce world-wide inflation, and provide more jobs by leading to more labor-intensive activities.

One of the primary causes of war and violence in the world is competition over scarce resources. When one country or group within a country lives luxuriously and utilizes much food and resources, and a neighboring country or group of people within a country has a lack of adequate food, there is much potential for violence.

Hence, our non vegetarian diets with its tremendous waste of land, food, fuel, water, fertilizer and energy is a prime factor in the instability, violence and threats of war in the world today. Those who live in the half of the world which is "underdeveloped" won't be content to starve for long; war is a likely result. Sen. Mark Hatfield has stated, "Hunger and famine will do more to de-stabilize this world; (they are) more explosive than all the atomic weaponry possessed by the big powers. Desperate people do desperate things."

The writer, Isaac Bashevis Singer stated, "If a man has the heart to cut the throat of a chicken or a calf, there's no reason he should not be willing to cut the throat of a man." The Prime Minister of India, Shri Morarji Desai, has stated, "If I am entitled to kill a living creature either by my pleasure or for my nourishment, equally would somebody else be justified in killing me for his enjoyment or sustenance."

Can vegetarianism truly have an impact on the global problems we face today? I believe that it can. The primary thing to keep in mind is the ecological principle that everything is connected to everything else — it is impossible to do just one thing, without affecting many other things.

Our wasteful flesh-centered diets have ramifications related to most, if not all, problems facing the world today.

When we use much land for the production and feeding of beef, it reduces land that could be used to grow nutritious crops for people. When we import cash crops, beef, and sea products from other countries, we utilize some of their badly needed food-producing resources for our own wasteful needs. When we require much pesticides, fertilizer, and energy to meet our food needs, we make these resources scarce and costly for others.

When we require an assured, inexpensive source of supplies from other countries, we have a vested interest in keeping repressive regimes in power, regimes which meet the needs of local elites at the expense of the majority of people. When we have such energy-wasteful agricultural and industrial production methods, we endanger ourselves and the rest of the globe with threats related to nuclear radiation, weapons proliferation, pollution, oil spills, and possible climate modifications.

Our wasteful use of resources leads to scarcity and competition which causes inflation and other economic difficulties, and increases the chances of war.

Vegetarianism cannot completely solve all of the world's problems. But it can greatly help. If the freed resources are used properly, it can be the beginning of the solution.

It should also be noted that people who shift to nutritious vegetarian diets will benefit from reduced food costs and better personal health and vigor.

* * *

(Richard H. Schwartz is a member of the faculty of the College of Staten Island.)

1. What percent of the world's people are either malnourished or undernourished?
2. About 47.5 million people die annually in the world due to all causes. What percents of these deaths are due to hunger and its effects? Give a range for your answer.
3. The average person in the undeveloped countries eats about 400 pounds of grain annually. About how much grain (mostly in the form of animal products) does the average American eat?
4. What percent of U. S. agricultural land is used to grow crops for feeding animals?
5. What is the ratio of land needed on an average non-vegetarian diet compared to land needed on a total vegetarian diet?
6. What percent of beef in world trade is not imported by the United States?
7. What are the ratios of (a) pounds of wheat, (b) pounds of rice, and (c) pounds of potatoes that could be grown on one acre of land compared to the number of pounds of beef that could be raised using one acre of land? Draw a bar chart showing the number of pounds of each type of food that could be grown on one acre of land.
8. How many tons of grease, carcass dressing, intestinal waste, and other animal residues are discharged by Omaha, Nebraska's 18 meat packing companies?
9. How many pounds of fertilizer are necessary to grow food for an average person in Asia or Africa?
10. What is the ratio of water required to provide the standard diet of a person in the U. S. compared to the water needed to provide a pure vegetarian diet?
11. How many gallons of water are necessary to fulfill all the daily needs of an average American?

12. What is the ratio of energy in the U. S. used to grow, process, transport, prepare, and store food to the energy used in the U. S. for all other purposes?
13. Find the ratio of energy used per food calorie in the U. S. to that of some of the poorer countries.
14. About how many people's energy needs could be met by the energy used by the 7,000 slaughterhouses in the U. S. and Canada?

Solutions to Selected Problems

Article 2.1

H.1 Acres lost = 420,000 + 160,000 + 420,000 + 1,000,000 = 2,000,000 acres

H.2 100,000/365 = 274

H.3 Using 10,000,000 people

a) 10,000,000/52 = 192,000/week

b) 10,000,000/365 = 27,400/day

c) 27,400/24 = 1,140/hour

d) 1,140/60 = 19 people/min.

e) 19/60 = 0.32 people/sec.

Double these figures for 20 million people

H4. $100 \frac{\text{farmers}}{\text{week}} \cdot 52 \frac{\text{weeks}}{\text{year}}$

$= 52,000 \frac{\text{farmers}}{\text{year}}$

In 10 years, 10 · 52,000 = 520,000 farmers

D1. (\$900 + \$380 + \$170) million = \$1,450 million

D2, \$25.5 billion / 6 years = \$4.25 billion/year

D3. $24 \frac{\text{hrs.}}{\text{day}} \cdot 365 \frac{\text{days}}{\text{year}} \div 16 \text{ hrs.}$

= 548 times/year

P1. Add →

53,613,000 children under 15

P2. 74.2 − 67.0 = 7.2 yrs.

W1. 220 billion Asians

W2. $\frac{36 \text{ billion containers}}{178 \text{ Containers/American}}$

= 202 million Americans

W3. $250 \frac{\text{cans}}{\text{American}} \cdot 220 \text{ million Americans}$

= 55 billion cans

W4. $11.5 \text{ million } \cancel{\text{tons}} \cdot \frac{17 \text{ trees}}{\cancel{\text{ton}}}$

= 195.5 million trees

W5. $\frac{575 \text{ lb}}{\text{American}} \cdot 220 \text{ million Americans} \cdot \frac{1 \text{ ton}}{2000 \text{ lb}}$

= 63.3 million tons

W6.

(a) $\frac{4,340,000 (2000) \text{ lb}}{365 \text{ days}}$

= 23,800,000,000 lb/day

W7. $14,000,000 \cancel{\text{lb}} \cdot \frac{1 \cancel{\text{ton}}}{2000 \cancel{\text{lb}}} \cdot \frac{17 \text{ trees}}{1 \cancel{\text{ton}}}$

= 119,000 trees

T1. $\frac{220,000,000}{2} = 110,000,000$

T2. (a) $9,000 \text{ miles} \cdot \frac{1 \text{ gallon}}{15 \text{ miles}}$

= 600 gallons

b) 300 gallons

T3. 50,000/365 = 137

137/24 = 5.71

WR1.

a) 5280 ft · 1000 ft · 900 ft

= 4,752 million ft^3

b) $\frac{100 \text{ million tons } (2000 \text{ lb/ton})}{4,752 \text{ million ft}^3}$

= 42.1 lb/ft^3

(seems low for ice)

WR2.

15,000 yrs / 50 yrs

= 300 yrs/year

E1. (a) $700,000 \text{ kw} \cdot 1000 \frac{\text{watts}}{\text{kw}}$

= 700 million watts

(b) 700 megawatts

E2. 700 ÷ 3/4 = 933 football fields

E3. $\frac{(3.14)(400)^2/4}{100 \cdot 300} = 4.2$ fields

E4.

$1200 \text{ watt} \cdot \frac{1 \text{ kw}}{1000 \text{ watt}} \cdot 4 \frac{\text{minutes}}{\text{day}} \cdot 365 \frac{\text{days}}{\text{year}}$

$\cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{\$0.10}{\text{kw-hr.}} = \$2.92$

E5.

$$\frac{30,000 \frac{\text{gallons}}{\text{sec}} \cdot 3600 \frac{\text{secs}}{\text{hour}} \cdot 24 \frac{\text{hrs}}{\text{day}}}{150 \text{ gallons/day} \cdot \text{person}}$$

= 17.3 million people

E6. Let x = no. of inches covered

$30 \frac{\text{ft}^3}{\text{sec}} \cdot (60 \cdot 60 \cdot 24) \frac{\text{secs}}{\text{day}}$

$= (5280 \cdot 5280) \frac{\text{ft}^2}{\text{day}} \left(\frac{x}{12}\right) \text{ft}$

$x = 1.1$ inches

Article 2.2

H1. 1900/400 = 4.75

H2. 15 billion/4 billion = 3.75

H3. $3.5 \div \frac{1}{5} = (3.5)(5) = 17.5$

H4. 1,600,000,000 ÷ 220,000,000 = 7.3

H5. $\frac{7 \text{ lb grain}}{1 \text{ lb beef}} = \frac{x \text{ lb grain}}{500 \text{ lb beef}}$

→ 3,500 lb grain

for $\frac{8 \text{ lb grain}}{\text{lb beef}}$ → 4,000 lb grain

H6. 7.5/2.5 = 3

H7. 3500/475 = 7.37

H8. 3500/350 = 10

H9. $\frac{10}{1} = \frac{2\text{ billion}}{x}$

$x = 200{,}000{,}000$ tons

H10. a) 3,100/2,200 = 1.41

b) 96/58 = 1.66

D1. \$640/\$40 = \$16

D2. a) 14.3, b) 2.4

c) 0.45 d) 1.37 e) 0.20

f) 22.3

D3. a) 17.6 b) 2.3

c) 2.0 d) 36 e) 6.3

f) 1.3

P01. 110/20 = 5.5

Po2. 97/16 = 6.1

Po3.

$\frac{1\text{ abortion}}{2\text{ births}} = \frac{x\text{ abortions}}{120\text{ million births}}$

→ 60 million abortions.

There would have been 180 million births

Article 2.3

H1. $\frac{10}{50}\cdot 100\% = 20\%$

$\frac{20}{50}\cdot 100\% = 40\%$

20 to 40% of all deaths are due to hunger & its effects

H2. $\frac{40}{100}\cdot 220 = 88$ million people

H3. $\frac{4}{5}\cdot 100 = 80\%$

H4. $\frac{500\text{ million}}{4{,}200\text{ million}}\cdot 100 = 11.9\%$

H5. $\frac{10}{4{,}300}\cdot 100 = 0.23\%$

$\frac{20}{4{,}300}\cdot 100 = 0.47\%$

H6. $\frac{1}{3}\cdot 100 = 33.3\%$

$\frac{1}{2}\cdot 100 = 50.0\%$

H7. $\frac{40}{220}\cdot 100 = 18.2\%$

H8. $\frac{650\text{ million}}{1{,}000\text{ million}}\cdot 100 = 65\%$

D1. $\frac{98.7}{149.1}\cdot 100 = 66.2\%$

D2. (a) $\frac{239}{345}\cdot 100 = 69.3\%$

b) $\frac{134}{345}\cdot 100 = 38.8\%$

D3. Asia

a) $\frac{853}{1{,}196}\cdot 100 = 71.3\%$

b) $\frac{499}{1{,}196}\cdot 100 = 41.7\%$

D4. a) $\frac{2.5}{3.8}\cdot 100 = 65.8\%$

b) $\frac{2.7}{3.8}\cdot 100 = 71.1\%$

D5. $\frac{(756-701)}{701}\cdot 100 = 7.8\%$

D6. a) $\frac{60}{1{,}000}\cdot 100 = 6\%$

b) 1/1 (Americans have 1/2 of income)

c) $\frac{(350+370)\cdot 100}{100} = 72\%$

d) $(200/940)\cdot 100 = 21.3\%$

e) 697/303 = 2.30

D7. 30% + 23% = 53%

D8. 95 − 5 = 90%

P1. $\frac{4-2}{2}\cdot 100 = 100\%$

P2. $\frac{(3{,}000{,}000 - 175{,}000)}{175{,}000}\cdot 100$

$= 1610\%$

R1. 11.24

R2. a) 18.92 b) 6.34 c) 5.95

d) 4.17 e) 4.0 f) 3.76

WR1. $\frac{95}{202}\cdot 100 = 47\%$

W1. $\frac{5.0-2.75}{2.75}\cdot 100 = 82\%$

P01. $\frac{4}{100}\cdot 2000 = 80$ ll

Po2. $\frac{43}{57}\cdot 100 = 75\%$

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chapter 3

1. (a) $\frac{220.3}{4,321} \cdot 100 = 5.1\%$ in U.S.

b) $\frac{950}{4,321} \cdot 100 = 22\%$ in China

2. a) $\frac{950}{220.3} = 4.3$

b) $\frac{950}{118.7} = 8.0$

3. (a) U.S. $\frac{24+11}{100-35} = 0.538$

c) Mexico $\frac{46+3}{100-49} = 0.961$

f) India $\frac{41+3}{100-44} = 0.786$

4. (a) U.S. $\frac{70}{0.6} = 116.7$ yrs

d) India $\frac{70}{1.9} = 36.8$ yrs

5. (a) U.S.

$\frac{260.4-220.3}{220.3} \cdot 100 = 18.2\%$

(e) the world

$\frac{6,162-4,321}{4,321} \cdot 100 = 42.7\%$

6. (a) 9.5% (b) 1.4% (c) 14.3%

d) 7% e) 0.8% f) 12.2%

7. highest life expectency = 76 (Iceland)

lowest life expectency = 39 (Angola & Guinea-Bissau)

$76 \div 39 = 1.95$

8. highest per capita GNP = $14,420 (United Arab emirate)

lowest per capita GNP = $50 (Maldives & Bhutan)

$14,420/$50 = 180

9. birth rate (use 35 as cut off point)
high: Asia, Africa, Latin America

Low: North America, Europe, USSR, Oceania

10. a) birth rate

range = $\overset{(Gaza)}{53} - \overset{(Germany)}{10} = 43$

i) PQLI

range = $97 - 14 = 83$

11.

Mid 1979 population

$3,148/1,173 = 2.68$

Birth Rate : $33/16 = 2.06$

Death Rate : $12/9 = 1.33$

years to double: $33/99 = 0.33$

Infant mortality rate: $107/28 = 3.82$

Life expectency: $56/71 = 0.79$

PQLI : $55/92 = 0.60$

per capita GNP: $490/5,210 = 0.094$

12. a) $\frac{36-30}{1,000} = \frac{x}{100}$, $x = 0.6\%$

b) $\frac{34-14}{1,000} = \frac{x}{100}$, $x = 2.0\%$

13. a) $\frac{1.7}{100} \cdot 4,321,000,000$
$= 73,500,000$

b) $73,500,000/365 = 201,000/\text{day}$
$201,000/24 = 8,380/\text{hr}$
$8,380/60 = 140/\text{minute}$
$140/60 = 2.3/\text{second}$

c) 220,300,000 Americans = 73,500,000 $\frac{\text{people}}{\text{yr}}$ · (no. of yrs)

$\frac{220,300,000}{73,500,000} = \underline{3 \text{ years}}$

d) (1) $\frac{609,000}{201,000} \approx 3$ days

(3) just over 1 year

14. $\frac{1.9}{100} \cdot 660.9$ million
= 12.6 million / year

12.6 million / 365 = 34,500/day

15. 38% are under 15 years of age

$\frac{38}{100} = \frac{x}{5000}$

$x = \frac{5000 \cdot 38}{100} = \underline{1900}$

16. $73/39 = 1.87$

17. a) 3,144,000 births — 1,893,000 deaths = 1,251,000

b) $\frac{3,144,000}{213,540,000} \cdot 1000 = 14.7$

c) $\frac{1,893,000}{213,540,000} \cdot 1000 = 8.9$

d) natural inc. + 450,000 immigrants
$= 1,251,000 + 450,000 = 1,701,000$

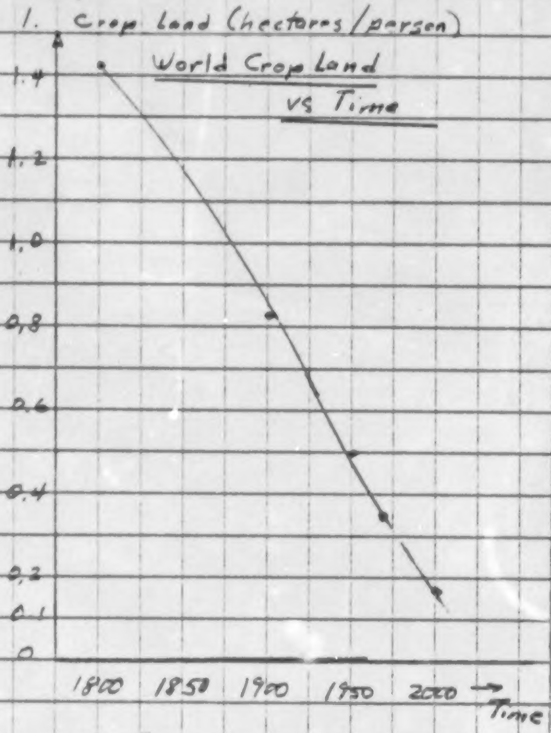
e) $1,701,000 + 800,000$
$= 2,501,000$

f) $\frac{1,701,000}{213,540,000} \cdot 100 = 0.8\%$

g) $\frac{2,501,000}{213,540,000} \cdot 100 = 1.2\%$

h) $\frac{800,000}{450,000} = 1.78$

Chapter 4 Article 4.1



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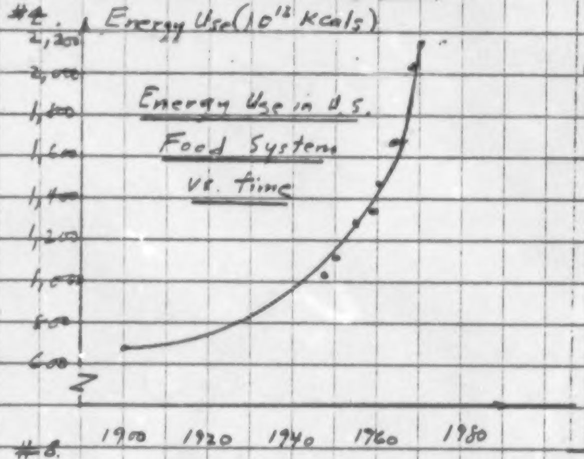

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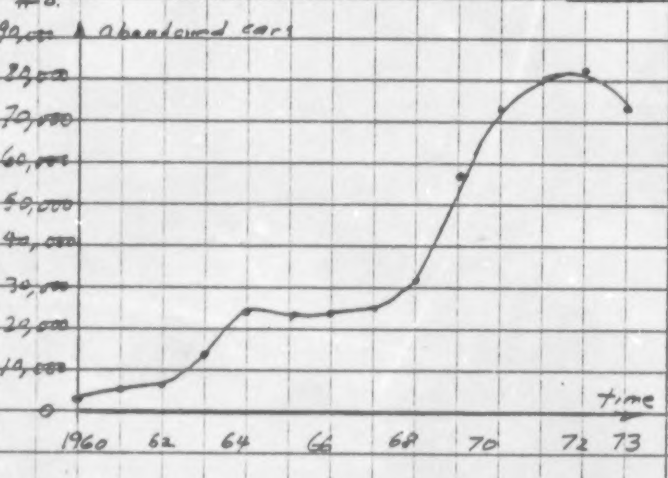


Article 4.1 (cont.)

12. 1960: \$1360/\$120 = 11.3
1970 \$2680/\$190 = 14.1
1974 \$4410/\$360 = 12.3

13. Underdeveloped Countries
1850: (BR−DR)/10 ≐ (41−36)/10 = 0.5%
1970: (BR−DR)/10 ≐ (37−14)/10 = 2.3%

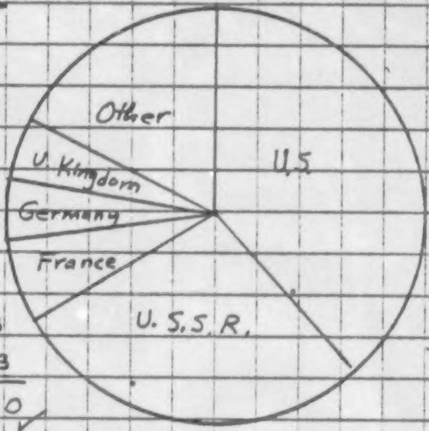
14. oil: 1974 price/1973 price ≐ \$8/\$2 = 4:1

15. 1940: 700/130 ≐ 5.4
1970 2200/190 ≐ 11.6

Article 4.2

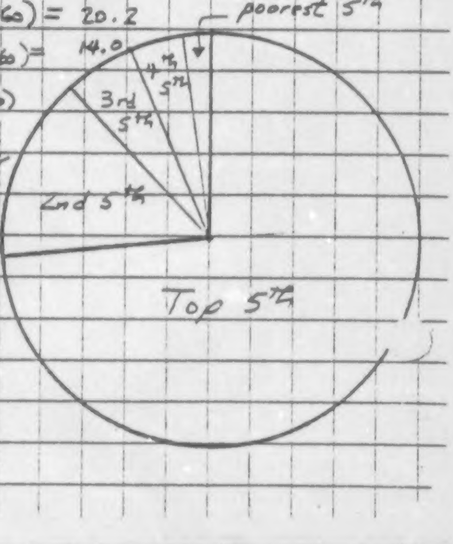
#1 Arms Sales

U.S.
$\frac{38.8}{100}\cdot 360 = 139.7$
USSR
(27.9/100)360 = 100.4
France
(6.3/100)·360 = 22.7
Germany
(4.9/100)·360 = 17.6
U. Kingdom
(4.8/100)·360 = 17.3
Other
(17.3/100)·360 = 62.3
360.0 ✓



#6 Income Gap in Ecuador

%
1st .735·(360) = 264.6
2nd .145(360) = 52.2
3rd .056(360) = 20.2
4th .039(360) = 14.0
5th .025(360)
= 9.0
sum = 360.0 ✓

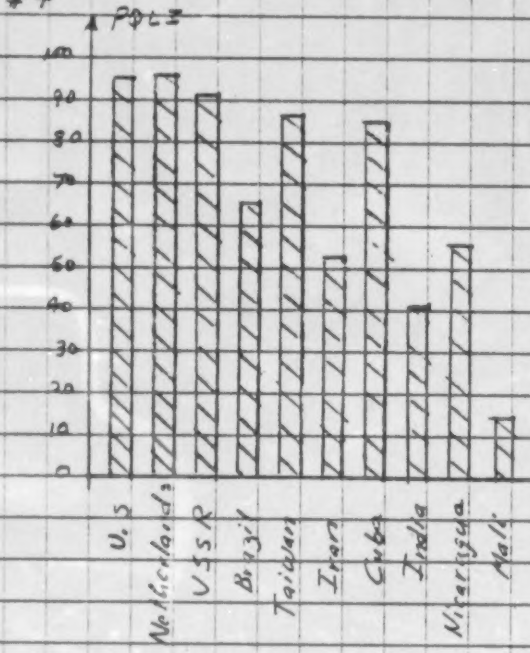


Article 4.2 (cont.)

see next page

Article 4.3

#4



Article 4.4

#2

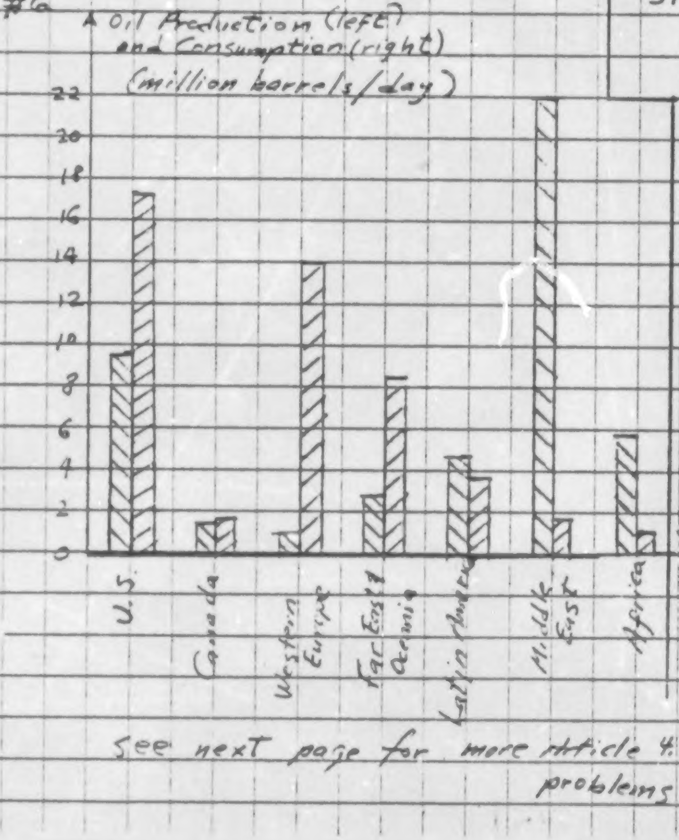
Range = 31 − 17 = 14

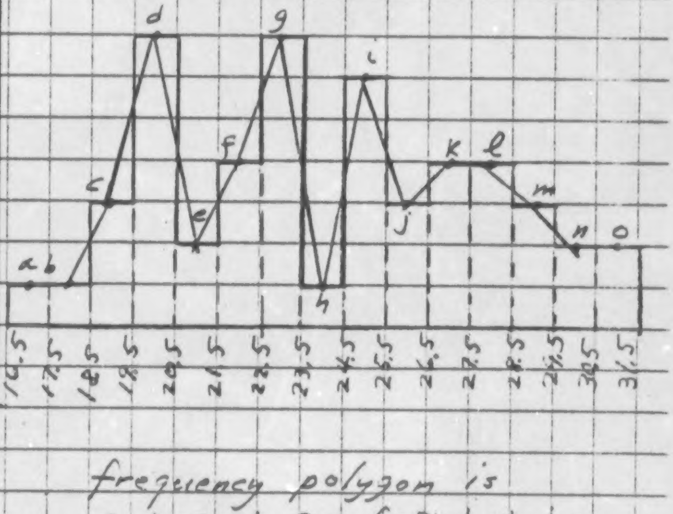
→ Real Boundaries from 16.5 to 31.5

→ 15 intervals, 1 unit each

Value	Real Boundaries	Tally	(frequency) Total
17	16.5−17.5	𝍷	1
18	17.5−18.5	𝍷	1
19	18.5−19.5	𝍷𝍷𝍷	3
20	19.5−20.5	𝍸 𝍷𝍷	7
21	20.5−21.5	𝍷𝍷	2
22	21.5−22.5	𝍷𝍷𝍷𝍷	4
23	22.5−23.5	𝍸 𝍷𝍷	7
24	23.5−24.5	𝍷	1
25	24.5−25.5	𝍸 𝍷	6
26	25.5−26.5	𝍷𝍷𝍷	3
27	26.5−27.5	𝍷𝍷𝍷𝍷	4
28	27.5−28.5	𝍷𝍷𝍷𝍷	4
29	28.5−29.5	𝍷𝍷𝍷	3
30	29.5−30.5	𝍷𝍷	2
31	30.5−31.5	𝍷𝍷	2
			50 ✓

#6





frequency polygon is
a−b−c−d−e−f−g−h−i−j−
k−l−m−n−o

See next page for more Article 4.3
problems

Chapter 4

Article 4.2 (cont.)

#10. a) Forest

b) $20.5\% \approx 1/5$

c) $(31.7/100) \cdot 360 = 114.1°$

11. a) $941 + 152 + 102 + 210 + 126 + 104 + 9 = 1644$

b) $(941/1644) \cdot 100 = 57.2\%$

c) $(102/1644) \cdot 360 = 22.3°$

13. a) $(17/100) \cdot 360 = 61.2\%$

b) $(16/100) \cdot 331{,}700 = 53{,}100$

Article 4.3 (cont.)

7. Brazil (1960 to 1990)

$\% \text{ inc} \doteq \frac{65-42}{42} \cdot 100 = 55\%$

São Paulo (1960 to 1985)

$\% \text{ inc.} \doteq \frac{16-4}{4} \cdot 100 = 300\%$

8. Asia (1965 to 1985)

Increase $\doteq 2800 - 1800 =$ 1,000 million people

$\% \text{ inc.} = \frac{1{,}000}{1{,}800} \cdot 100 = 56\%$

9. a) $(52{,}194/864{,}215) \cdot 100 = 3.4\%$

b) $(853{,}605/864{,}215) \cdot 100 = 98.5\%$

11. (a) $95\% - 5\% = 90\%$ decrease

(b) $38/98 = 0.39$

12. a) Egypt
30%
23%
53%

b) Mexico
30%
19%
49%

Article 4.4 (cont.)

#4

Mexico

a) $\% \text{ under } 15 = \overbrace{8.4 + 8.1 + 6.7}^{\text{males}} + \underbrace{8.2 + 7.9 + 6.4}_{\text{females}} = 45.7\%$

b) $\% \text{ over } 64 \doteq 2.5\%$ (estimate)

c) $DR = \frac{45.7 + 2.5}{100 - (45.7 + 2.5)} = 0.93$

Chapter 5

1. a) geometric sequence ($\cdot \frac{1}{4}$)

$1/256, \ 1/1{,}024, \ldots$

b) not a sequence

c) arithmetic sequence (add 5)

23, 28, ...

d) twice Fibonacci sequence

26, 42, ...

e) arithmetic (add (−5))

5, 0, ...

f) geometric (mult by 3)

81, 243, ...

2.

day	fraction covered
30	1
29	1/2
28	1/4
27	1/8
26	1/16
25	1/32
24	1/64
23	1/128

4.

time	Sq. miles
today	12
in 10 yrs	24
20	48
30	96
40	192
50	384
60	768
70	1,536
80	3072
90	6,144
100	12,288
110	24,576
120	49,152
130	98,304
140	196,608
150	393,216
160	786,432
170	1,572,864
180	3,145,728

Ans. a little less than 180 yrs.

This can be solved more easily with logs (chapter 8)

5. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

6. $220.3/67.7 = 3.25$

In 80 yrs.

U.S. population $= 2(220.3) = 440.6$ million

Mexico's population $= (67.7) \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 1{,}083.2$ million

$440.6/1{,}083.2 = 0.41$

7. $2^{20} = 1{,}048{,}576$

(see chapter 8 for solution using logarithms)

8. a) about 37 folds

b) about 43 folds.

c) about 53 folds

9. \$ 10,740,000

10. see #7

In 20 doublings, there is an increase of a little over 1 million

hence, $20 \cdot 10 = 200$ yrs.

11.		
	now	1/8 polluted
	10 yrs.	1/4 polluted
	20 yrs.	1/2 polluted
	30 yrs	fully polluted

12.

	yrs. from now							
	now	20	40	60	80	100	120	140
A	200			400			800	
B	10	20	40	80	160	320	640	1280

The populations will be equal in less than 140 yrs.

A graph of both countries population growth would give a more accurate answer

Chapter 5 (cont.)

13. simple interest

the increase is \$80/yr.

after 1yr, \$1,080
2yrs., \$1,160
3yrs., \$1,240
4yrs, \$1,320
5yrs., \$1,400

doubling time = $100/8 = 12.5$ yrs.

Compound interest

increase each year is by factor of 1.08

after 1 yr, \$1,080
2yrs, \$1,080 (1.08) = \$1166.40
3yrs, \$1,259.71
4 yrs, \$1360.49
5yrs, \$1,469.33

Doubling time $= \frac{70}{8} \doteq 9$ yrs

14.

today	1
5,700yrs	1/2
11,400 yrs	1/4
17,100 yrs	1/8
22,800yrs.	1/16

alternately $\frac{1}{16} = \left(\frac{1}{2}\right)^4$

→ 4 half lives

$4 \cdot (5{,}700) = 22{,}800$ yrs

15. $(1/8) = (1/2)^3$

→ 3 half lives

3 (5.4 billion) = 16.2 billion yrs

16.

now	1
10 days	1/2
20 days	1/4
30 days	1/8

or, 30 days = $\underline{3} \cdot 10$ →
3 half lifes
$(1/2)^3 = 1/8$

17. $\frac{10}{640} = \frac{1}{64} = \frac{1}{2^6}$

→ 6 half lives of 10 yrs. each

$6 \cdot 10 = 60$ years

18. (a) $(0.95)^n = 0.5$

$n \approx 13.5$ yrs (using a calculator)

(b) $(250)(0.95)^{28} = 59.5$ (million tons)

These are best solved using logarithms (see chapter 8)

Chapter 6

1.

BC	C	R	PR₁	PR₂	Total
0	64	0	0	0	64
1	96	64 (32)	0	0	160
2	144	96 (48)	64	0	304
3	216	144 (72)	96	64	520
4	324	216 (108)	144	96	780
5	486	324 (162)	216	144	1170

In 30 years (2 birth cycles, population more than doubles: $\frac{1170}{520}$

doubling time ≈ 26 yrs.
(Using logarithms (chapt 3) → 25.6 yrs

2.

BC	C	R	PR₁	PR₂	Total
0	80	0	0	0	80
1	40	80 (40)	0	0	120
2	20	40 (20)	80	0	140
3	10	20 (10)	40	80	150
4	5	10 (5)	20	40	75
5					

from BC 3 to BC 4 →
population change is 75 or $\underline{1/2}(150)$
thus, time for population to reach half its value = 15yrs.

3.

BC	C	R	PR₁	PR₂	Total
6	1280	640 (320)	320	160	2400
7a	160	1280 (640)	640	320	2400
8a	320	160 (80)	1280	640	2400
9a	640	320 (160)	160	1280	2400

BC 8a $\frac{320}{80}$ = 4 children per couple

BC 9a $640/160$ = 4 $\frac{\text{children}}{\text{couple}}$

Chapter 6 (continued)

5.

BC	C	R	PR₁	PR₂	total
0	512	0	0	0	512
1	640	512 (256)	0	0	1152
2	800	640 (320)	512	0	1952
3	1,000	800 (400)	640	512	2952
4	1,250	1000 (500)	800	640	3690
5	1562	1,250 (625)	1000	800	4612
6	1952	1562 (781)	1250	1000	5765

From birth cycle 3 to 6, the population has nearly doubled → 3(15+) yrs

D.T. $\simeq$ 46 yrs

(Using logs → D.T. $\doteq$ 46.6 yrs).

6.

BC	C	R	PR₁	PR₂	Total
0	512	0	0	0	512
1	384	512 (256)	0	0	896
2	288	384 (192)	512	0	1184
3	216	288 (144)	384	512	1400
4	162	216 (108)	288	384	1050
5	122	162 (81)	216	288	788
6	92	122 (61)	162	216	592

7. Do some thinking.
Look for patterns.

8. a) 20,000,000 + 600,000 − 350,000 + 100,000 − 50,000 = 20,300,000

b) $\frac{300,000}{20,000,000} \cdot 100 = 1.5\%$

c) $\frac{50,000}{300,000} \cdot 100 = 16.7\%$

chapter 8

P. 8−11

1. a) 3.64×10^6
b) 3.2×10^{-4}
c) 3.25×10^6
d) 2.14×10^{-3}

2. a) 3,000,000
b) 0.00032
c) 119,000

3. a) 10^8 b) 10^6 c) 10.6×10^7
d) $10^{15-7} = 10^8$ e) $12 \times 10^{13} / 2 \times 10^6$
$= 6 \times 10^7$

4.(a) $x = 326 \cdot 24$

$\log x = \log 326 + \log 24$ (Eq. 8.8)

$= 2.5132 + 1.3802$

$\log x = 3.8934$

$x = 7823.5$

true answer = 7824
(discrepancy due to round off)

Note: Using a calculator, don't round off until the end.

(b) $x = 473/36$

$\log x = \log 473 - \log 36$ (Eq. 8.9)

$\log x = 2.67486 - 1.55636$

$= 1.11856$

$x = 13,14$

(c) $x = (3.12)^{4.71}$

$\log x = 4.71 \log(3.12)$ (Eq. 8.10)

$= 4.71\,(0.49415)$

$\log x = 2.3274$

$x = 212,55$

(d) $x = \frac{28(35)}{12(7)}$

$\log x = \log 28 + \log 35 - \log 12 - \log 7$

$x = 11,67$

4.(e) $x = \sqrt{7} = 7^{1/2}$

$\log x = \frac{1}{2} \log 7$

$= \frac{1}{2}(0.8451)$

$= 0.42255$

$x = 2.6458$

P. 8−18

1. (a)

$t = \frac{\log 2}{\log(1+r)} = \frac{0.30103}{\log(1.015)}$ (E

$t = \frac{0.30103}{0.006466}$

$t = 46.6$ yrs

check $70/r = \frac{70}{1.5} = 46.7$ yrs

(b) $P = P_0(1+r)^t$

$500 = 200\,(1.015)^t$

$2.5 = (1.015)^t$

$\log 2.5 = t \log(1.015)$

$t = \frac{\log 2.5}{\log 1.015}$

$t = \frac{0.39794}{0.006466}$

$t = 61.5$ yrs

2.

$P = P_0(1+r)^t$

$380 = 240\,(1+r)^{20}$

$\log 380 = \log 240 + 20 \log(1+r)$

$\frac{\log 380 - \log 240}{20} = \log(1+r)$

$\frac{2.5798 - 2.3802}{20} = \log(1+r)$

$\log(1+r) = 0.009979$

$1+r = 1.0232$

$r = 0.0232$
$= 2.32\%$

b)
$D.T = 70/2.32 = 30.2$ yrs

Article 8.4 (cont.)

3. $R = R_0 (1+r)^t$

$\frac{1}{2} R_0 = R_0 (1+(-0.04))^t$

$1/2 = 0.96^t$

$\log 0.5 = t \log (0.96)$ Eq. 8.10

$t = \frac{\log 0.5}{\log 0.96} = \frac{-0.30103}{-0.017729}$

$t = 17.0$ yrs

b) $t = 2000 - 1975 = 25$ yrs

$R = 300 (1+(-0.04))^{25}$

$\log R = \log 300 + 25 \log (0.16)$

$= 2.47712 + 25 (-0.017729)$

$= 2.47712 - 0.44323$

$\log R = 2.0339$

$R = 108$ tons

4. $R = R_0 (1+r)^t$

$\frac{1}{2} R_0 = R_0 (1+r)^{50}$

$0.50 = (1+r)^{50}$

$\log 0.50 = 50 \log (1+r)$

$-\frac{0.30103}{50} = \log (1+r)$

$-0.0060206 = \log (1+r)$

$(1+r) = 0.9862$

$r = -0.0138$

$= -1.38\%$

5. For Brazil, in 1979

population = 118.7 million people

doubling time = 25 yrs.

a) $P = P_0 \cdot 2^{t/k}$

for $t = 50$ yrs,

$P = 118.7 \cdot 2^{50/25}$

$= 118.7 \cdot 2^2 = 118.7(4)$

$= 474.8$ million people

5(b)

$P = P_0 \cdot 2^{t/k}$, $t = 70$ yrs

$P = 118.7 \cdot (2^{70/25})$

$= 118.7 \cdot (2^{2.8})$

$\log P = \log 118.7 + 2.8 \log 2$

$= 2.07445 + 2.8 (.30103)$

$= 2.91733$

$P = 826.7$ million people

6. $M = \frac{M_0}{2^{t/h}}$ $h = 20$ yrs

$M_0 = 500$ curies

a) $t = 20$ yrs

$M = \frac{500}{2^{20/20}} = \frac{500}{2} = 250$ curies

b) $t = 40$ yrs

$M = \frac{500}{2^{40/20}} = \frac{500}{2^2} = \frac{500}{4}$

$M = 125$ curies

c) $t = 50$ yrs

$M = \frac{500}{2^{50/20}} = 500 \cdot 2^{-2.5}$

$\log M = \log 500 - 2.5 \log 2$

$= 2.69897 - 2.5(.30103)$

$= 1.9464$

$M = 88.4$ curies

7. use $P = P_0 \cdot 2^{t/k}$

where $K = 10$ yrs doubling time

$3,000,000 = 12 \cdot 2^{t/10}$

$\log 3,000,000 - \log 12 = t/10 \log 2$

$6.47712 - 1.07918 = t/10 (0.30103)$

$t/10 = 17.93$

$t = 179.3$ yrs

Ans. in chapter 5: a little less than 150 yrs

9. $P = P_0 \cdot 2^{t/k}$

here $P_0 = 1¢$ $t = 30$ days, & $K = 1$ day

$P = 1¢ \cdot 2^{30/1} = 2^{30}$ ¢

$\log P = 30 \log 2$

$= 30 (0.30103)$

$= 9.0309$

$P = 1,073,742,100$ ¢

$= \$ 10,740,000$

10. $1,000,000 = 1 \cdot 2^{t/10}$ ← $K = 10$ yrs

$\log 1,000,000 = t/10 \log 2$

$6.0 = t/10 (0.30103)$

$t/10 = 19.93$

$t = 199$ yrs

Article 8.5

1. Every increase of 10 dB means that a sound is 10 times louder

$100,000 = 10^5$

→ 50 dB

2, 100 million $= 10^8$

80 dB

3.

From Eq. 8.15

$80 \text{ dB} = 10 \log \frac{I}{10^{-12}}$

$8 = \log \frac{I}{10^{-12}}$ $I = 10^{-4} \frac{w}{m^2}$

4. For each jet

$100 = 10 \log (I/10^{-12})$

$I = 10^{-2}$

for 8 jets

$\text{dB} = 10 \log \frac{(8 \times 10^{-2})}{10^{-12}}$

$= 10 (10.90) = 109.9$ dB

Chapter 9

Article 9.3

5. (a) students in an environmental class probably wouldn't be representative of the general population

(b) National Rifle Association members are not representative of the general public on the gun control issue.

c) sample is too small

d) sample too small

e) Only successful graduates might answer. People may not tell the truth about their incomes

f) Not everyone has a telephone & not all phone numbers are listed

g) people may be intimidated with the police officer there

h) People who write letters to public officials may not be representative of the general public

Article 9.4

1. use Eq. 9.1

$$\frac{300 \text{ tagged}}{\text{total \# of bass}} = \frac{30}{200}$$

$$\text{total \# of bass} = \frac{300 \cdot 200}{30}$$

$$= 2{,}000 \text{ bass}$$

2. $$\frac{300 \text{ tagged}}{\text{total}} = \frac{10}{100}$$

$$\text{total} = \frac{300 \cdot 100}{10}$$

$$= 3{,}000$$

3. a)

$$\frac{400}{\text{total}} = \frac{25}{200}$$

$$\text{total} = 400 \cdot \frac{200}{25} = 3200 \text{ bass}$$

b) $$\frac{200}{\text{total}} = \frac{15}{50}$$

$$\text{total} = \frac{200\,(50)}{15} \doteq 660 \text{ bass}$$

Chapter 10

4.

children per family	number of families	total children
0	8	0
1	20	20
2	34	68
3	20	60
4	11	44
5	4	20
6	3	18
sums	100	230

$$\mu = \Sigma x/n = 230/100$$

$$= 2.30$$

mode = 2 children

median = 2 children

5. $\mu = \Sigma x/n$

$\mu = 234/10 = 23.4$

6.

no. of children	no. of couples	total children
0	4	0
1	24	24
2	20	40
3	12	36
4	9	36
5	6	30
6	5	30
sum	80	196

a) mode = 1 child

b) mean = $\Sigma x/n = \frac{196}{80} = 2.45$

c) μ > replacement level

d) for 100 couples →

$100\,(2.11) = 211$ children

$211 - 196 = \underline{15}$ children for the additional 20 couples

9. Tropical South America

Country	(1) population (millions)	(2) life expectancy	(1)·(2)
Bolivia	5.2	48	249
Brazil	118.7	61	7,240.7
Columbia	26.1	59	1,539.9
Ecuador	8.0	60	480.0
Guyana	0.8	68	54.4
Paraguay	3.0	62	186.0
Peru	17.3	55	951.5
Suriname	0.4	66	26.4
Venezuela	13.5	65	877.5
sums	193.0	544	11,606.0

for people

$$\mu = \frac{11{,}606.0}{193.0} = 60.1 \text{ yrs}$$

(checks 60 yrs. on data sheet)

for countries $\mu = \frac{544}{9} = 60.4$ yrs.

Chapter 11

1. Ranges

a) mid-1979 population estimate

China, 950 million

Maldives, 0.1 million

Range = 950 − 0.1 = 949.9 million people

e) Infant mortality rate

Afghanistan, 226

Sweden, 8

Range = 226 − 8 = 218

f) % under 15

Gaza, 51

Luxembourg, 20%

Range = 51% − 20% = 31%

2. For each case, $\mu = 70$, and $\sigma = 5$; except for set 5, where $\sigma = 10$.

3.

x	$x-\mu$	$(x-\mu)^2$
4	−6	36
5	−5	25
6	−4	16
7	−3	9
8	−2	4
9	−1	1
10	0	0
12	2	4
12	2	4
14	4	16
16	6	36
17	7	49
sums 120	0 ✓	200

range = 17 − 4 = 13

$\mu = \Sigma x / n = 120/12 = 10$

$\sigma = \sqrt{200/12} = 4.08$

6.

(1) f families	(2) x children per family	(3) $f \cdot x$	(4) $(x-\mu)$	(5) $(x-\mu)^2$	(6) $f(x-\mu)^2$
16	0	0	−2	4	64
18	1	18	−1	1	18
26	2	52	0	0	0
18	3	54	1	1	18
6	4	24	2	4	24
4	5	20	3	9	36
2	6	12	4	16	32
sums 90		180			192

$\mu = \dfrac{\text{children}}{\text{families}} = \dfrac{180}{90} = 2$ median = 2

mode = 2

$\text{Variance} = \dfrac{192}{90} = 2.133$

$\sigma = \sqrt{\text{variance}} = 1.46$

Chapter 12

1. a) $p = F/T = 5/10 = 1/2$

b) $p = F/T = 3/10$

c) $p = 2/10 = 1/5$

d) $p = p_1 \cdot p_2 = 5/10 \cdot 5/10 = 1/4$

e) $P = 8/10 \cdot 8/10 = 64/100 = 16/25$

2. See Pascal's Triangle P.12-12

Row 8 $T = 256$

a) P (all boys) = 1/256

b) P (5 girls) = 56/256 = 7/32

c) P (6 or more boys) = $\dfrac{(28+8+1)}{256}$ = 37/256

3. $P = P_1 \cdot P_2 \cdot P_3 \cdot P_4 \cdot P_5$

$= .01(.02)(.004)(.012)(.005)$

$= 4.8 \times 10^{-11}$

Evidentally, based on the Three Mile Accident & other incidents, the probabilities are higher

4. See Pascal's Triangle row 6, $T = 64$

a) $P = 15/64$

b) $P = (15+6+1)/64 = 22/64 = 11/32$

c) $P = \dfrac{(15+20+15+6+1)}{64} = 57/64$

5. (see next page)

6. $P = 35p^4q^3 + 21p^5q^2 + 7p^6q + 1 \cdot p^7$

$= 35(1/3)^4(2/3)^3 + 21(1/3)^5(2/3)^2 + 7(1/3)^6(2/3)^1 + 1(1/3)^7$

$= \dfrac{280+84+14+1}{2187}$

$= 379/2187$

Chapter 12 (cont.)

#5

p = prob. air is polluted = 1/3

$q = 2/3$

Let x = no. of polluted days

$P(x=0) = 1\cdot q^7 = 1\left(\frac{2}{3}\right)^7 = \frac{128}{2187}$

$P(x=1) = 7pq^6 = 7\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^6 = \frac{448}{2187}$

$P(x=2) = 21p^2q^5 = 21\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^5 = \frac{672}{2187}$

$P(x=3) = 35p^3q^4 = 35\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^4 = \frac{560}{2187}$

$P(x=4) = 35p^4q^3 = 35\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^3 = \frac{280}{2187}$

$P(x=5) = 21p^5q^2 = 21\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^2 = \frac{84}{2187}$

$P(x=6) = 7p^6q = 7\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^1 = \frac{14}{2187}$

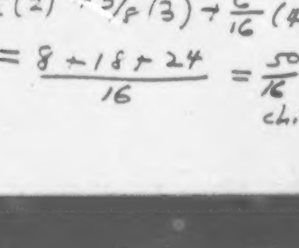
$P(x=7) = 1p^7 = 1\left(\frac{1}{3}\right)^7 = \frac{1}{2187}$

sum = $\frac{2187}{2187}$ ✓

8. a) $1/3 \cdot 1/3 = 1/9$
   b) $1/3 \cdot 2/3 + 2/3 \cdot 1/3 = 4/9$
   c) sum of (a) & (b) = 5/9
   d) $(1/3)^3 = 1/27$
   f) $(2/3)^3 = 8/27$

9. a) $1/5 \cdot 1/5 = 1/25$
   b) $4/5 \cdot 1/5 = 4/25$
   c) $1/5 \cdot 4/5 + 4/5 \cdot 1/5 = 8/25$
   d) (a) + (c) = 9/25

10.



$\frac{1}{4}(2) + \frac{3}{8}(3) + \frac{6}{16}(4)$

$= \frac{8 + 18 + 24}{16} = \frac{50}{16} = 3\frac{1}{8}$ children/couple

Chapter 13

1. a) 0.1915
   b) 0.1151
   c) .3849 − .0793 = .3056
   d) .2734 − .2422 = .0312
   f) 100 + 10 (0.52) = 105.2

3. a) 43.3%
   b) 15.9%
   c) 69.2%
   d) 34.13 + 43.32 → 77.5%
   e) 20 + (−0.25) (2) = 19.5

Chapter 14

1.

cell	O	E	O − E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1	25	22.5	2.5	6.25	0.28
2	25	27.5	−2.5	6.25	0.23
3	20	22.5	2.5	6.25	0.28
4	30	27.5	−2.5	6.25	0.23
			0 ✓		1.02

$\chi^2 = \Sigma\frac{(O-E)^2}{E} = 1.02$

d.o.f. = (2 − 1)(2 − 1) = 1

α = 5% = 0.05

$\chi_{cr}^2 = 3.84$

$\chi^2 < \chi_{cr}^2$

We cannot reject H_0, that children wanted is independent of children in the family

5.

cell	O	E	O − E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1	28	25	3	9	0.36
2	36	40	−4	16	0.40
3	11	10	1	1	0.10
4	22	25	−3	9	0.36
5	44	40	4	16	0.40
6	9	10	−1	1	0.10
			0 ✓		

$\chi^2 = 1.72$

$\alpha = 0.05$ d.o.f. = (3 − 1)(2 − 1) = 2

→ $\chi_{cr}^2 = 5.99$

$\chi^2 < \chi_{cr}^2$ → we can't reject H_0

6.

cell	O	E	O − E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1	40	18.6	21.4	458.0	24.6
2	4	9.9	−5.9	34.8	3.5
3	4	19.5	−15.5	240.3	12.3
4	17	17.8	.8	0.6	0.0
5	20	9.5	10.5	110.3	11.6
6	9	18.7	−9.7	94.1	5.0
7	3	23.6	−20.6	424.4	18.0
8	8	12.6	−4.6	21.2	1.7
9	50	24.8	25.2	635.0	25.6
			0 ✓		102.3

d.o.f. = (3 − 1)(3 − 1) = 4

$\alpha = 0.01$

$\chi_{cr}^2 = 13.28$

$\chi^2 > \chi_{cr}^2$

→ PQLI is dependent on per capita GNP

Chapter 15

1.

x	y	xy	x^2	y^2
0	9	0	0	81
1	8	8	1	64
2	6	12	4	36
3	4	12	9	16
4	3	12	16	9
5	2	10	25	4
6	2	12	36	4
7	0	0	49	0
28	34	66	140	214

$r = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{n(\Sigma x^2)-(\Sigma x)^2}\sqrt{n(\Sigma y)^2-(\Sigma y)^2}}$

$= \frac{8(66) - 28(34)}{\sqrt{8(140)-(28)^2}\sqrt{8(214)-(34)^2}}$

$= \frac{528 - 952}{\sqrt{336}\sqrt{556}}$

$= \frac{-424}{432.2} = -0.981$

→ negative linear corr.

Chapter 16

Basic Calculations

1 2($0.70)(365) = $511

2. Add → 34,043,000

Percent

1. $\frac{3985-1285}{1,285} \cdot 100 = 210\%$

2. $\frac{34 \text{ million}}{220 \text{ million}} \cdot 100\% = 15.5\%$

3. 13.6% 4. 50% 5. 17%

6. 24%

Ratio

1. 1.6 2. 16

General Problems

1. a) 20% b) 3 c) omit

2. a) 380 b) 2.1%

3. a) 43.9 − 38.4 = 5.5 yrs lost

b) 48.6 − 42.4 = 6.2 yrs. lost

c) 5.5 yrs lost

$\rightarrow \frac{5.5(365)(24)(60)}{43.1(15)(365)} = 12 \frac{\text{min}}{\text{cig.}}$

Graphs

3. (b) 7.5 yr. avg. doubling time.

4. (b) D.T.$_{avg} \approx$ 13 yrs

averages & variability

$\mu = 1470/70 = 21$ Range = 40

Normal Curve

a) 16% b) 82% c) 14%

Logarithms

a) 70/5 = 14 yrs

b) price = 70¢ $(1.05)^{21}$

= 195¢, or $1.95

Chapter 17

Basic Computations

1. $126 per person

2. $200 per person

3. 365 · 24/18 = 548

4. 1268 yrs

631,000 miles

Ratios

2. 2960 3. 19

4. US:

$\frac{\text{military}}{\text{education}} = \frac{453}{181} = 2.50$

military/health = 6.47

V. S. a) 1.02 b) 1.75

5. 2.14

6. a) 2,500,000 b) 1920

Percents

1. 66.6% 2. 700%

3. 100% 4. 133% 5. 50%

Sequences

1. Avg. D.T. ≈ 9 yrs

Logarithms

$\$400 \text{ billion} = \$107 \text{ billion} (1+r)^{17}$

r = 0.08 → about 8%

by 1990 — $\$400 \text{ billion} (1.08)^{13}$

→ $1,089 billion

Chapter 18

Basic Computations

1. 15.7% 2. 1.42 miles

percents & ratios

1. (a) compared to B'klyn

$\frac{38.7-30.0}{30.0} \cdot 100 = 29\%$

b) 38.7/30.0 = 1.29

2. a) 37.5% b) 1.30

3. 52.3% 4) a) 18,379 b) 15.1

Averages

1. 0.062/4 = 0.0155

2. .091/4 = 0.0228

3. mode = 1

μ = 57,610/48,718 = 1.18

4. 59.00/12 = 4.92

5. 3.45

Chapter 19

Basic Calculations

1. Add → 53,613,000 children

2. 122 million

percents

1. 48% 2. b) 16% c) 41%

3. 1,157 million 4. 9.9%

Ratios

1. 158/14 = 11.3 2. 0.55

3. 0.89 3. 15:1

Review of Selected Mathematical Topics

The following review covers only certain selected mathematical topics, which you need for solving problems in this book. For more extended treatments, consult your teacher or a mathematics refresher book.

R.1 Fractions

A fraction involves one number (a numerator) divided by a 2nd number (denominator). In the fraction 3/4, 3 is the numerator, and 4 is the denominator.

In the fraction -5/7, -5 is the numerator, and 7 is the denominator.

To convert a fraction to a decimal, we divide the denominator into the numerator.

Example R.1

Convert 5/8 into a decimal.

Solution

We divide 8 into 5

```
   .625
 8/5.000
   4 8
     20
     16
      40
      40
       0
```

5/8 = 0.625. A hand calculator could have been used to speed up calculations.

R.2 Ratios

A ratio compares two quantities. It is always originally in the form of a fraction.

Examples of ratios are:

$$\frac{\text{16 miles}}{\text{1 gallon of gas}}$$ (or 16 miles per gallon of gas)

$$\frac{\text{20,000 people}}{\text{1 square mile}}$$ (or 20,000 people per square mile)

$$\frac{\$5{,}000}{\text{1 person}}$$ (or $5,000 per person)

Example R.2

Simplify the ratio 20:4.

Solution

20:4 means 20/4. We divide numerator and denominator by 4 and get 5/1 or 5:1.

R.3 Proportions

A proportion is a statement that two ratios are equal, when the statement is true. Examples are:

$$\frac{3}{4} = \frac{6}{8}$$

$$\frac{1}{5} = \frac{2}{10}$$

A general form for a proportion is

$$\frac{a}{b} = \frac{c}{d}$$

where a and d are called the extremes and b and c are called the means. For a proportion (two ratios being equal), the product of the means must equal the product of the extremes, or

$$a \cdot d = b \cdot c$$

That is, the products obtained from cross-multiplication are equal.

Example R.3

Compute x (the unknown) in the following proportion.

$$\frac{3}{5} = \frac{x}{15}$$

Solution

We cross-multiply

$$5 \cdot x = 3 \cdot 15$$

Dividing both sides by 5,

$$x = \frac{3 \cdot 15}{5} = 3 \cdot 3 = 9$$

We can check our work by substituting 9 for x.

$$\frac{3}{5} = \frac{9}{15}$$

$9 \cdot 5 = 45$ and $3 \cdot 15 = 45$. It checks.

R.4 Percents

Percent is simply a ratio of some number to 100. Another way of looking at it is percent is a fraction whose denominator (lower part) is 100. The percent sign is %; hence 5% means 5 percent.

Examples of percent are:

$$50\% = \frac{50}{100} = 0.50$$

$$3\% = \frac{3}{100} = 0.03$$

$$125\% = \frac{125}{100} = 1.25$$

To convert from a decimal to a percent, we multiply by 100.

Hence 0.24 = 24%

0.129 = 12.9%

Another way to put this is, to change a decimal to a percent, move the decimal point two places to the right and write the % sign.

Example R.4

What is 20% of 64?

Solution

Let x = the unknown.

We can rewrite the given problem as

$$x = \frac{20}{100} \cdot 64$$

$$x = 0.2 \cdot 64 = \underline{12.8}$$

Example R.5

What percent of 60 is 45?

Solution

We rewrite the question in mathematical terms.

$$\frac{x}{100} \cdot 60 = 45$$

Note the replacement of "what percent" by x/100, since x represents an unknown and percent means per 100.

Multiplying both sides by 100 and dividing by 60, we solve for x.

$$x = \frac{100 \cdot 45}{60} = 75\%$$

Alternate solution

We wish to convert the fraction 45/60 to a percent. We use the following proportion: $\frac{45}{60} = \frac{x}{100}$

Cross multiplying, $45 \cdot 100 = 60x$

$$x = \frac{45}{60} \cdot 100 = 75\%, \text{ as before.}$$

Note that, to convert a fraction to a percent, we need merely multiply by 100, and simplify.

Example R.6

What percent of 80 is 16?

Solution

$$x = \frac{16}{80} \cdot 100\% = \frac{1}{5} \cdot 100\% = 20\%$$

R.5 Exponents

An exponent is a number that tells how many times a base is to be multiplied by itself.

For example,

5^3 means 5 (the base) should be multiplied by itself 3 times, since the exponent is 3. Thus

$$5^3 = 5 \cdot 5 \cdot 5 = 125$$

In a similar way,

$$7^2 = 7 \cdot 7 = 49$$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

An exponent is often called a power. Hence 5^3 can be read as 5 cubed or 5 to the 3rd power.

Any number with a zero exponent is equal to 1. Thus,

$$5^0 = 1$$

$$(-3)^0 = 1$$

$$(127)^0 = 1$$

R.6 The Decimal System (Base Ten); Reading Numbers

The decimal system is a place value system that uses 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) and the value of each place is a power of ten. Here are the values of some powers of ten:

$10^{-3} = 0.001$ = one-thousandth
$10^{-2} = 0.01$ = one-hundredth
$10^{-1} = 0.1$ = one-tenth
$10^{0} = 1$ = one
$10^{1} = 10$ = ten
$10^{2} = 100$ = 1 hundred
$10^{3} = 1{,}000$ = 1 thousand
$10^{4} = 10{,}000$ = 10 thousand
$10^{5} = 100{,}000$ = 100 thousand
$10^{6} = 1{,}000{,}000$ = 1 million
$10^{7} = 10{,}000{,}000$ = 10 million
$10^{8} = 100{,}000{,}000$ = 100 million
$10^{9} = 1{,}000{,}000{,}000$ = 1 billion

Let us review how to read some large numbers.

Example R.6

Write 53,254 in English word equivalents.

Solution

$$53{,}254 = 50{,}000 + 3{,}000 + 200 + 50 + 4$$

$$= 53{,}000 + 200 + 54$$

In words, our number is fifty thousand + three thousand + two hundred + fifty + four or fifty three thousand, two hundred fifty-four.

Example R.7

Write 12,634,271 in words.

Solution

This can be considered as 12,000,000 + 634,000 + 271. In words, this is twelve million, six hundred and thirty-four thousand, two hundred seventy-one.

Example R.8

Write 23,249,371,000 in words.

Solution

This number can be first expressed as 23,000,000,000 + 249,000,000 + 371,000. In words, this is twenty three billion, two hundred forth-nine million, and three hundred seventy-one thousand.

Example R.9

Write 21.245 in words.

Solution

We have a whole number equal to 21 plus two-tenths, four-hundredths, and five-thousandths. Note that

0.04 = 4 hundredths = 40 thousandths

0.02 = 2 tenths = 20 hundredths = 200 thousandths

Hence, 0.245 = two hundred and forty-five thousandths.

The complete number, in words, is twenty-one and two hundred and forty-five thousandths.

R.7 Rounding Off Numbers

When we round off a number, we replace the given number with one that is nearby. This is best shown by examples.

Example R.10

Round off 213,748 to the nearest thousand.

Solution

The given number is between 213,000 and 214,000. We focus on the 748. Since this is more than 500, our number is closer to 214,000 than it is to 213,000. Hence, our rounded off number is 214,000.

The key digit in this round off is the one to the right of the thousands place, that is the one in the hundreds place. When this digit is 5 or more, we round to the next higher number. Since we have a 7 in the hundreds place, we round off to the higher possibility, or 214,000.

Example R.11

Round off 217,354,961 to the nearest million.

Solution

The given number is between 217 million and 218 million. Since it is nearer to 217 million, we round it off to 217,000,000.

Example R.12

Round off 217,354,961 to the nearest (a) ten, (b) hundred, (c) thousand, (d) ten thousand

Solutions

a) The given number is between 217,354,960 and 217,354,970. 61 is nearer to 60 than to 70, hence we round it to 217,354,960.

b) The given number is between 217,354,900 and 217,355,000. It is nearer to 217,355,000.

c) The answer is 217,355,000 (check this). Note that 217,354,961 rounded off to the nearest hundred is the same as the number rounded off to the nearest thousand.

d) The answer is 217,350,000 (check this).

The place to which a number should be rounded off depends on the accuracy with which the number is known. For example, since there is much uncertainty with regard to the population of China, it should be given only to the nearest million people. If 150 people were questioned to get some idea of the opinions of a population of 10,000 people, it would not be correct to report that 42.38% of the population had a certain opinion, because of the uncertainties inherent in sampling (see chapter 9); the result should be rounded off to 42%.

Generally, results should not have more significant figures than that of any number used in the calculation. Significant figures for several numbers are indicated below:

234.7	4 significant figures
219.460	6 significant figures; a final zero after a decimal point is significant.
0.00352	3 significant figures (3, 5, and 2), the two zeros before the 3 are not significant - they are place markers which locate the decimal point.
950,000,000	It is unclear how many of the zeros are significant. If this number represents the population of China, probably only the first three numbers are significant, since the population is not known closer than to the nearest million people.

Example R.13

Compute $\frac{218.37}{9.42}$, and round off your answer.

Use of a calculator (see the next article) gives an answer of 23.181529. Since the denominator has only 3 significant digits, our answer should be rounded of to 23.2

More examples of round off of results are given in the next article.

Generally, answers in this book have been rounded off to be consistent with the data used in the calculations. More digits are generally used in intermediate calculations, to insure accuracy in final results. In some cases, an additional significant figure has been used in a solution, when it was felt to be meaningful. For example, a mean value (chapter 10) generally has one significant figure more than the data used to calculate it. Often, judgment has to be used in deciding how many significant figures should be used in an answer. The scope of this book does not permit a full treatment of significant digits. What we do want to convey is that you should not use 7 or 8 digits in your answer, because you can easily read them off your hand calculator, when only three digits, for example, are significant.

R.8 Use of a hand calculator

The solution of problems in this book will be made much easier if you use a hand calculator. One that can compute square roots and logarithms will be especially valuable. Luckily, a good calculator can be obtained relatively inexpensively, and it is a good investment for the future.

Since there are so many different calculators on the market, with many variations in their modes of operation, we won't give a detailed discussion on hand calculator use here, but will just present a few examples. For more information, consult the manual that comes with your calculator, and/or your teacher.

Multiplication

Example R.14

Multiply 328.42 times 24.715.

Solution

This could take some time to do by hand, and there would be a greater chance of error.

Using a hand calculator, it is very simple.

Press 328.42

Press x

Press 24.715

The answer is 8116.9003

Since the numbers multiplied have 5 significant digits, we use 5 digits in the answer, and make it 8116.9.

Examples for practice: Calculate

a) 28.43(227.53) ans. 6469

b) 512.49(193.46) ans. 99146

c) 218.31·(0.00273) ans. 0.596

Division

Example R.15

Compute $\frac{329.71}{42.86}$

Solution

Again, this would take much time and have a greater possibility of error when done by hand.

With a calculator,

Press 329.71

Press ÷

Press 42.86

Press =

The result is 7.6927205. We round it off to 7.693 (since the denominator has only 4 significant digits, we use 4 digits in the answer).

Examples for practice: Calculate

a) $\frac{294.73}{28.914}$ ans. 10.193

b) $\frac{191.7}{2483.2}$ ans. 0.07720

c) $\frac{212.8}{.00341}$ ans. 62,400 (3 significant figures)

Finding square roots

It is advisable to have a hand calculator which computes square roots directly. If you have one which doesn't, here is a trial-and-error method for computing square roots, using a calculator.

Example R.16

Compute $\sqrt{93.62}$, to the nearest hundredth.

Solution

Since $9^2 = 81$, and $10^2 = 100$, the answer is between 9 and 10. Our first guess could be 9.6 (since 93.62 is closer to 100, than it is to 81).

Divide 9.6 into 93.62.

$$93.62/9.6 = 9.752$$

Average 9.6 and 9.752 (add them and divide by 2).

$$\frac{9.6 + 9.752}{2} = 9.676$$

Divide 9.676 into 93.62

$$\frac{93.62}{9.676} = 9.675$$

The answer is between 9.675 and 9.676, thus 9.68 to the nearest hundredth.

It is hoped that these calculations have shown you

1) the advantage of having a hand calculator that can give you square roots

directly (in one step), and

2) the fact that the application of creativity along with a hand calculator can solve many problems.

Even when a hand calculator is used, mistakes can be made; it is important to check results.

Let us square 9.68:

$$(9.68)\cdot(9.68) = \underline{93.70}$$

This is close to 93.62, but we might want to check 9.67, to be sure that 9.68 is the correct answer, to the nearest hundredth.

$$(9.67)\cdot(9.67) = 93.51$$

Since 93.62 is closer to 93.70 than it is to 93.51, we have checked that 9.68 is the required answer.

Appendix

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Jackson, Herbert L., *Mathematics of Radiology and Nuclear Medicine*, St. Louis, Missouri, W. H. Green, Inc., 1971.

Good coverage of exponents, logarithms, exponential laws of growth and decay, and word problems.

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Applications of probability to problems in biology. This book would be valuable as background for students who wish to apply probability to problems beyond the scope of this text.

Pollution: Problems, Projects and Mathematical Exercises, Bulletin 1082, Wisconsin Department of Public Instruction, Madison, Wisconsin.

A wide variety of mathematical problems related to population, air, water, and noise pollution, solid waste, hunger, etc. Aimed at students in 6th to 9th grades, but can be adapted for other grades. Primarily word problems.

Runyan, Richard, Winning With Statistics, A Painless First Look at Numbers, Ratios, Percentages, Means, and Inference, Reading, Massachusetts, Addison-Wesley Publishing Co., 1977.

Basic principles of statistics presented in a humorous, "relaxing" way.

Tanur, Judith M., et al (editors), Statistics: A Guide To The Unknown, San Francisco, Holden-Day, Inc., 1972.

Applications of statistics and probability to many issues such as polling, smoking, Salk Vaccine testing, census taking, and vanishing wildlife.

Global Issues Books

Boughey, Arthur S. Strategy for Survival, An Exploration of the Limits to Further Population and Industrial Growth, Menlo Park, California, W. A. Benjamin, Inc., 1976.

A detailed analysis of The Limits To Growth and other social systems simulation models and their implications as to possible limits to growth. Excellent for supplementing The Limits To Growth.

Brown, Lester, The 29th Day, New York, W. W. Norton & Company, 1978.

Title based on riddle of water lilies doubling daily in the pond. Demonstrates that we may be in "the 29th day" with regard to threats to our ecosystems - our forests, croplands, grasslands, and fishing areas.

Brown, Lester, World Without Borders, New York, Vintage Books, Random House, Inc., 1972.

Comprehensive analysis of global issues including pollution, population growth, the rich-poor gap, increasing unemployment, hunger, and the arms race.

Callenbach, Ernest, Ecotopia, Berkely, CA 94709, Banyan Tree, 1975.

Very good science fiction version of an ecologically sane future. Fine for interdisciplinary classes.

Chart Book on Smoking, Tobacco, and Health, U. S. Dept. of Health, Education, and Welfare, Center for Disease Control, Atlanta, Ga.

Many graphs, tables, and data related to cigarette smoking. Excellent source for mathematical problems related to smoking.

Commoner, Barry, The Closing Circle; Nature, Man, and Technology, New York, Knopf, 1971.

An excellent presentation of the effects of technology on the environmental crisis.

Ehrlich, Paul R. and Anne H., Population, Resources, and Environment, Issues in Human Ecology, San Francisco, W. H. Freeman & Co., 1970.

An extremely comprehensive review of global issues. Very well documented.

Falk, Richard A., A Study of Future Worlds, New York, Free Press, 1975.

George, Susan, How the Other Half Dies, The Real Reasons for World Hunger, Montclair, New Jersey, Allanheld, Osmun & Co., 1977.

Shows that famine and hunger result primarily from inequitable distribution of power and resources and unjust economic and social systems. Stresses need for food self-reliance rather than dependency.

Lappe, Frances Moore, Diet For a Small Planet, New York, Ballantine Books/ Random House, 1971.

Shows the wastefulness and ecological consequences due to our heavy consumption of meat. Indicates how we can have sufficient proteins from non-meat sources.

Lappe, Frances Moore and Joseph Collins, Food First - Beyond the Myth of Scarcity, Boston, Houghton Mifflin Company, 1977.

Explodes the many myths behind global hunger. States that hunger is not due primarily to population, scarcity of land, weather, lack of technology, but rather primarily due to inequalities in wealth and power, and unjust social and economic systems.

Laszlo, Ervin, et al, Goals for Mankind, New York, Signet, 1977.

A report to the Club of Rome on the new horizons of the global community. Successor to The Limits To Growth, Mankind at the Turning Point, and RIO.

Lerza, Catherine and Michael Jacobson, editors, Foor for People, Not for Profit, New York, Ballantine Books, 1975.

Official food day handbook. A sourcebook on the food crisis. An excellent collection of essays on many aspects of food and hunger. Preface by Ralph Nader.

Lyons, Stephen (editor), Sun! A Handbook for the Solar Decade, San Francisco, Friends of the Earth, 1978.

A call for an International Solar Decade. Essays on a wide variety of issues related to solar energy.

McHale, Magda C., John McHale with Guy F. Streatfeild, Children In The World, Population Reference Bureau, Inc., Washington, D. C. 20036, 1979.

Over 80 tables and charts, with brief accompanying text, covering children's health, welfare, and education, numbers and proportions of children in the world, and more. Published as a background source of information for the International Year of the Child.

McLaughlin, Martin M., and the Staff of the Overseas Development Council, The United States and World Development, Agenda 1979, New York, Praeger, 1979.

Up-to-date analysis of current international problems and what needs to be done to reduce poverty and hunger, to cut inflation and unemployment, and to improve conditions in all countries. Appendices include many graphs, charts, and data that are useful for setting up mathematical problems.

Meadows, Donella et al., The Limits to Growth: A Report For the Club of Rome's Project on the Predicament of Mankind, Universe Books, 1972.

Shows world headed for disaster within 100 years unless exponential growth of population and industrialization is stopped. Study Guide in chapter 7 of this book.

Medlovitz, Saul H., ed., On the Creation of a Just World Order, New York, Free Press, 1975

Mesarovic, Mihajilo and Eduard Pestel, Mankind at the Turning Point, New York, E. P. Dutton & Co., 1974.

The second report to the Club of Rome. Successor to The Limits To Growth. Uses computer model to simulate growth. Predicts disaster unless there are changes in values and relations between countries.

Miles, Rufus, Man's Population Predicament, Bulletin of the Population Reference Bureau, Inc., Vol. 27, No. 2, 1775 Massachusetts Avenue, N. W., Washington, D. C., 1971.

Good summary of population growth issues; many graphs and charts which are available as slides.

Miles, Rufus E., Jr., Awakening From the American Dream, The Social and Political Limits To Growth, New York, Universe Books, 1976.

States that the world is in danger, but not primarily due to physical limits as argued in The Limits To Growth, but mainly due to social and political limits.

Miller, G. Tyler, Jr. Replenish The Earth - A Primer in Human Ecology, Belmont, Cal., Wadsworth, 1972.

An excellent background book on population problems, the world food crisis, and resource depletion. Very easy to follow.

Mische, Gerald and Patricia, Toward A Human World Order, New York, N. Y./Ramsey, N. J., Paulist Press, 1977.

Shows the powerlessness of nations caught in a national security straightjacket due to military, economic, and resource competition. Points to need for a new international order.

Nobile, P. and J. Deedy, The Complete Ecology Fact Book, Garden City, N. Y., Doubleday and Co., Inc., 1972.

Collection of statistics, charts, graphs on many environmental issues; very good for supplementing other texts.

Population and the American Future, Commission on Population and the American Future, 1972, U. S. Government Printing Office, Washington, D. C. 20402 or Signet (New York).

A report of the U. S. Population Commission findings which considers U. S. population growth and its effects. Recommendations on how the nation can best cope with the impact of population growth. Based on a two-year study of research and public hearings.

Sauer, Robert, Voyages: Scenario for a Ship Called Earth, New York, ZPG/Ballantine, 1971.

An anthology of science fiction short stories with population and environmental themes.

Schaefer, Lawrence M., Population, Environment, and Society: A Teacher's Manual, E-P Education Services, 1976, c/o ACES, 800 Dixwell Ave., New Haven CT 06511.

Designed to help teachers present material related to critical issues. Has many mathematics applications. Reprints of articles are valuable for student reports and debates. Includes discussion questions.

Schumacher, E. F., Small Is Beautiful, Economics as if People Mattered, New York, Harper and Row, 1973.

The need for intermediate technology and production by the masses rather than mass production. Shows potential disasterous effects from rapid resource depletion, nuclear power, and excessive use of large-scale technology.

Sewell, John W. and the Staff of the Overseas Development Council, The United States and World Development, Agenda 1977, New York, Praeger Publishers, 1977.

Analysis of the current rapidly changing world situation and recommendations for changes in U. S. policy to improve conditions in developing countries. Appendices include many graphs, charts, and data that can be used for creating mathematical problems.

Simon, Arthur, Bread For The World, New York, Paulist Press, 1975.

The need for justice and a change in U. S. public policy to reduce global hunger. Stresses the need for individual and group citizen action.

Sivard, Ruth Leger, World Military and Social Expenditures 1978, WMSE Publications, Box 1003, Leesburg, Virginia 22075.

Very complete analysis of the arms race and its effects on social expenditures. Many graphs and charts. Excellent background resource for the chapter on Mathematics and the Arms Race.

Stadler, John (editor), Eco-Fiction, Washington Square Press, 1971.

Excellent for presenting ecological issues through fiction. Collection of short stories on the environmental crisis.

Terry, Mark, Teaching for Survival: A Handbook for Environmental Education, New York, Friends of the Earth/Ballantine, 1971.

Presents skillful case for environmental education in all our schools. Many valuable classroom ideas.

Tinbergen, Jan (coordinator), RIO, Reshaping the International Order, New York, E. P. Dutton, 1976.

An all-inclusive working plan that is designed to help the world win its global race for survival. The 3rd report to the Club of Rome.

AUDIOVISUAL MATERIALS

Films

Alone In The Midst of The Land. 1970, 27 minutes, color, NBC Educational Enterprizes, 30 Rockefeller Plaza, N. Y. 10020.

A warning of disasters to come if population and pollution are not controlled.

Beyond the Next Harvest. 1975, 28 minutes, color, Northeast CROP office, 87 Shaker Road, East Longmeadow, Massachusetts 01028.

The nature of global hunger, its causes, and some possible solutions. Interviews with delegates to the U. N. World Food Conference in 1974.

Bottle Babies. 1975, 28 minutes, color, Tricontinental Film Center, 333 Sixth Ave., New York, N. Y. 10014.

Shows the very negative impact on infants in the underdeveloped countries of the milk formulas which are marketed by multinational corporations.

Diet For A Small Planet. 1974, 28 minutes, color, Bullfrog Films, Inc., Box 114, Milford Square, PA, 18935.

Discusses the need to reduce our consumption of meat in order to free protein for the world's hungry people. Shows the ecological costs of our heavy meat consumption and how to get sufficient protein from non-meat sources. Similar to book of the same title.

Limits To Growth. 1973, 30 minutes, color, Lane Council of Governments, 135 Sixth Avenue East, Eugene, Oregon 97401.

A visual version of the book, Limits To Growth, by D. Meadows. Clearly shows the effects of exponential growth with regard to population, industrialization, resource consumption, pollution, and use of arable land. Stresses need for changes if we are to avoid disaster within 100 years.

Sorry No Vacancy. 1973, 27 minutes, color, Malibu Films, Inc., P. O. Box 428, Malibu, CA, 90265.

Dramatically shows the effects of rapid population growth, industrialization, and wasteful lifestyles on hunger, energy shortages, pollution, and social and economic problems. Interviews with several experts on population and environmental issues.

Tilt. 1972, 23 minutes, color, Audio Visual Department, World Bank, 1818 H Street, N. W., Washington, D. C. 20433 (free).

Shows the uneven distribution of global wealth and results of this. Cartoon form used.

Filmstrips and Slides

Bread For The World, Bread For the World, 4600 North Kilpatrick Avenue, Chicago, IL, 60630.

Filmstrip with cassette tape on the present hunger crisis. It stresses the need for citizen groups to lobby to affect U. S. public policy in order to reduce world hunger.

Ecology, Understanding the Crisis. 1972, Encyclopedia Britannica Educational Corporation, Chicago, IL, 60611.

Series of six filmstrips with accompanying casettes dealing with current ecological issues.

1) Environments and Ecosystems. Basic ecological concepts.
2) Man in Ecosystems. How man's extravagant consumption affects environmental systems.
3) Human Communities, Simple and Complex. Contrasts primitive cultures with our modern industrial world with regard to impact on ecosystems.
4) Creating Imbalances. Imbalances created by rapid industrialization.
5) Destroying the Future. Harm done to the environment and the reduction of open space.

6) Creating The Future. Steps people are taking to restore a balance with nature.

The filmstrips may be rented or purchased separately.

Guess Who's Coming to Breakfast. 40 minutes, slideshow/flimstrip, with casette tape, Clergy and Laity Concerned (CALC).

The negative effects of multinational corporations on our eating habits, our economy, and on Third World countries.

Hunger on Spaceship Earth. American Friends Service Committee, 160 North 15th St., Philadelphia, PA, 19102.

Very comprehensive slideshow with accompanying casette tape on global hunger. Discusses causes, effects, and what individuals and groups can do.

Population Reference Bureau Slide Series. 1970-78, Population Reference Bureau, 1755 Massachusetts Ave., Washington, D. C. 20036.

A series of slides keyed to detailed discussions in Population Bulletins which accompany the slides. Each series focuses on a particular aspect of global population growth and its effects. Particularly recommended is Series No. 5, Man's Population Predicament (25 slides) which provides an overview of many aspects of the population problem.

Toward a New Economic Order, American Friends Service Committee, 160 North 15th Street, Philadelphia, PA, 19102.

The negative effects of our present global conditions and the need for a new international economic order. Slideshow with casette tape.

General Sources of Information and Problems

1. Directory of the Population - Related Community of the Washington, D. C. Area, 1978, World Population Society, D. C. Chapter, 1650 North 21st Road, No. 1, Arlington, Virginia 22209.
2. The Environmental Film Review, published annually by Environment Information Center, Inc., Film Reference Dept., 292 Madison Ave., New York, N. Y.
   A critical guide to environmental films under 21 categories including energy, land use and misuse, solid waste, and air, water, and noise pollution.
3. New Internationalist, monthly magazine, 112 Atlantic Avenue, Brooklyn, N. Y. 11201 or 62A High Street, Wellington, Oxford, England.
   Excellent coverage of international issues. Back issues on disarmament, hunger, population, poverty, and energy are well worth reading. Explodes many myths.
4. Peace and World Order Studies - A Curriculum Guide, 1978, Transnational Academic Program, Institute For World Order, 1140 Avenue of the Americas, New York, N. Y. 10036.
   Excellent collection of course outlines in peace and world order studies. Contains introductory essays and a very comprehensive listing of learning packages, books and periodicals, multimedia materials, and programs in peace and world order studies.
5. People Packet and Hunger Packet, National Association for Foreign Student Affairs, 1860 19th Street, N. W., Washington, D. C. 20009.
   Much valuable information on population and hunger issues.
6. Population Education Activities For The Classroom. 1977, selected and developed by Judith M. Schultz and Herbert L. Coon, ERIC/SMEAC Center for Science, Mathematics, and Environmental Education, The Ohio State University, 1200 Chambers Road, 3rd floor, Columbus, Ohio 43212.
   A wide variety of exercizes for elementary through senior high school classes related to population issues. Many involve mathematical concepts.

7. Population Education Packet, prepared by Judith Kunofsky, 1975, Sierra Club, 1050 Mills Tower, San Francisco CA 94104.

   Various population education materials including a 48-page teacher's manual with lesson plans and classroom projects.

8. Population Education Resources, 1973, Zero Population Growth, Inc., 1346 Connecticut Avenue, N. W., Washington, D. C. 20036

   A folder containing many valuable resources for population education.

9. Population education: sources and resources, 1975, Population Reference Bureau, 1775 Massachusetts Ave., N. W., Washington, D. C. 20036 (single copy free)

   Excellent summary of groups, books, teacher resources and audio-visual materials related to population issues.

10. Worldwatch Papers (occasional), Worldwatch Institute, 1776 Massachusetts Ave., N. W., Washington, D. C. 20036.

    Short paperbacks on crucial global issues such as hunger, population growth and its effects, energy, and waste.

Global Issues Groups

There are many groups working on such global issues as pollution, population growth, hunger, resource depletion and peace. Here is a list of several representative groups.

1. Center For War/Peace Studies, 218 East 18th Street, New York, N. Y. 10003.
   Attempts to stimulate interest in global problems of conflict and social change.
2. ERIC (Educational Resources Information Center) Information Analysis Center for Science, Mathematics and Environmental Education, The Ohio State University, 1200 Chambers Road, 3rd Floor, Columbus, Ohio 43212.
   Provides bibliographic and reference service on all aspects of environmental education.
3. Institute for World Order, 777 United Nations Plaza, New York, N. Y. 10017.
   Considers ways in which global institutions can be structured to achieve social justice and peace.
4. Overseas Development Council, 1717 Massachusetts Avenue, N. W., Washington, D. C. 20036.
   Provides information on U.S. relations with regard to the developing countries.
5. Population Reference Bureau, 1755 Massachusetts Ave., N. W., Washington, D. C. 20036.
   Many publications on world population growth and its impact. Publishes the annual World Population Data Sheet.
6. Sierra Club, 1050 Mills Tower, San Francisco, CA 94104.
   Devoted to protection of our ecosystems, natural areas and resources.
7. World Watch Institute, 1776 Massachusetts Avenue, N. W., Washington, D. C. 20036.
   Publishes a series of papers on global issues such as hunger, population, energy, waste, and resources.

8. Zero Population Growth, Inc. (ZPG), 1346 Connecticut Ave., N. W., Washington, D. C. 20036.

Aim is to educate the public on the negative effects of population growth and the need for a stabilization of population.

The following questionnaire may be used for a pre-test, a post-test, the basis of a class discussion, or the basis of a lecture to a school or community group.

Global Survival Questionnaire

Please take a few minutes to answer these questions.

1. About _______ percent of the world's people are malnourished or undernourished.

   a. 5 b. 10 c. 50 d. 80

2. About _______ million people die annually due to starvation or its effects.

   a. 1 b. 5 c. 10 d. 30

3. In 1976, the population of the world reached _____ billion people. This number will double in about _______ years.

   a. 4, 100 b. 4, 40 c. 2, 40 d. 3, 80

4. About _______ percent of people in undeveloped areas such as India, Latin America, etc. are under 15 years old.

   a. 20 b. 30 c. 40 d. 50

5. The United States population has __________ (increased, decreased, or remained about the same) in the last few years.

6. The average person in the U. S. has an effect on life support systems in terms of pollution and resource depletion equal to about ______ people in an undeveloped country.

   a. 5 b. 10 c. 50 d. 100 e. 500

7. The U.S. has about ______ percent of the world's population. We use about _____ percent of the world's resources and cause about ______ percent of the world's pollution.

   a. 10, 33, 50 b. 5, 10, 20 c. 5, 33, 50 d. 5, 25, 35

8. The U.S. diet requires about ______ times as much arable lands as that of a person in an undeveloped country.

   a. 6 b. 13 c. 17 d. 25

9. The amount of grain needed to produce one pound of beef in a feedlot is about ______ pounds.

   a. 2 b. 4 c. 6 d. 8

10. Historically, which has been more effective in reducing the rate of population growth?

    a. Agricultural self-sufficiency with old age security

    b. Family planning and birth control.

11. Mexico City had about 3 million people in 1950. By the year 2000, its projected population is about ______ million people.

    a. 5 b. 10 c. 20 d. 30

12. In 1978, the amount of money spent by the world's countries on weapons and other military related expenditures was about ______ billion dollars.

    a. 10 b. 50 c. 150 d. 250 e. 400

13. Use of electrical energy has been doubling every _____ years, until recently.

    a. 5 b. 10 c. 20 d. 30

14. Imagine you own a pond with a water lily growing in it. The lily plant doubles in size every day and if left unchecked would cover the pond in 30 days killing all other forms of life in the pond. For a long time, the water lily is small and you decide not to take action until the pond is half covered. How much time will you then have (after the pond is half covered) to save the pond? ____________

Answers 1) c, 2) c, 3) b, 4) c, 5) increasing, 6) c, 7) c, 8) c,
9) d, 10) a, 11) d, 12) e, 13) b, 14) 1 day

The following questions can be used as the basis of class discussions or debates. Class results can be used to compute averages and to draw bar or circle charts.

Survey of Attitudes Related to Global Issues

Place the number which best expresses your feeling next to each statement below.

1 = strongly agree
2 = agree
3 = neutral
4 = disagree
5 = strongly disagree

_____ 1. We should simplify our lifestyles to reduce pollution, resource depletion, and hunger.

_____ 2. There should be a mandatory deposit on all beverage containers in the United States.

_____ 3. Since we can always migrate to other planets, there is no need to worry about population growth.

_____ 4. There is no need to worry about shortages of food, energy, and resources because science and technology will provide solutions to these problems.

_____ 5. The United States can afford to have many more people because we have so much open space in our country.

_____ 6. There is very little that I can do with regard to the problem of world hunger since it is such an enormous and complex problem.

_____ 7. The prime cause for the widespread hunger in the world today is due to people in the poorer countries having too many children.

_____ 8. Since we face so many external threats we must continue to spend very large amounts for weapons to help us defend ourselves.

_____ 9. We should provide food and other aid to poor countries only if they take major steps to reduce population and they back our foreign policy goals.

_____ 10. The United States is the most wasteful country in the world and this contributes to poverty and hunger in the world.

Teachers and students can add similar types of questions to stimulate thoughts, discussion, and debate.

POSSIBLE TERM THEME TOPICS

Here are some topics that can be used for student term themes. Teachers can make additional suggestions, or a student can select another topic that he or she is interested in, with the teacher's approval.

Students should try to utilize mathematics in their report - for example, applications of various types of graphs, surveys, sequences, calculations, computation of averages, variations, etc.

1. Are we running out of water?
2. Is the U. S. overpopulated?
3. Noise pollution analysis in terms of decibels.
4. Benefits of requiring deposit bottles in terms of resources, energy, litter, jobs, etc.
5. Mathematics and smoking.
6. Triage: the case for and against.
7. Zero population growth: prospects and possible effects.
8. Effects of recent technology on pollution.
9. Energy usage on U. S. Farms.
10. Sample people's opinions on some environmental issue.
11. A new international economic order.
12. Fuel efficiency in cars.
13. Solar energy.
14. Recycling.
15. Vanishing wildlife.
16. Review of a book related to a global issue.
17. Food from the sea.
18. The Green Revolution.
19. Scrapbook of pertinent articles, with commentary
20. Population growth in cities.

21. Benefit and dangers of nuclear energy.

22. Population changes and/or demography of a local area.

23. Population growth and its effects in

    a) China b) India c) Japan d) Mexico

    or some other country of your choice.

24. Mathematics applications related to

    a) Staten Island
    b) vegetarianism
    c) energy
    d) waste
    e) hunger
    f) another issue of your choice

25. Finding more examples related to global issues and involving one or more of the following:

    a) sampling b) normal curves c) Chi-square analysis d) correlation